This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.

Potential Games for Energy-Efficient Power Control and Subcarrier Allocation in Uplink Multicell OFDMA Systems

Stefano Buzzi, *Senior Member, IEEE*, Giulio Colavolpe, *Senior Member, IEEE*, Daniela Saturnino, and Alessio Zappone, *Student Member, IEEE*

***Abstract*—The problem of non-cooperative resource allocation in multicell uplink OFDMA systems is considered in this paper. Non-cooperative games for subcarrier allocation and transmit power control are considered, aiming at maximizing the users' SINRs and, most notably, the users' energy-efficiency, measured in bit/Joule and representing the number of error-free delivered bits for each Joule of energy used for transmission. The theory of potential games is used to come up with several non-cooperative games admitting Nash equilibrium points. Since the proposed resource allocation games exhibit a computational complexity that may be in some cases prohibitive, approximate, reduced-complexity, implementations are also considered. For comparison purposes, some considerations on social-optimum solutions are also discussed. Numerical results confirm that the proposed resource allocation schemes are effective in increasing the network energy-efficiency (as compared to rate-maximizing schemes), thus permitting to optimize the use of the energy stored in the battery. Moreover, the proposed approximate implementations exhibit a performance very close to that of the exact procedures.**

***Index Terms*—OFDMA, Potential games, power control, subcarrier allocation, Nash equilibrium, social optimum, energy efficiency.**

I. Introduction and Work Motivation

Resource allocation is one of the most critical building blocks of a wireless network. Indeed, through a wise design of resource allocation schemes the performance of a wireless network may be optimized according to a number of relevant parameters such as the data-rate, the radiated power, the number of supported users, and so on. Among these parameters, one that has been gaining momentum in the last decade is the energy-efficiency, which is measured in bit/Joule, and is defined here as the number of successfully (i.e., error-free) delivered bits for each energy-unit used for transmission. Indeed, especially when considering the uplink of a wireless network, wherein the transmitting devices are mobile terminals powered by a battery, maximizing energy-efficiency results in a prolonged battery life and in a greater amount of data that can be transmitted for a given amount of energy to be used for transmission. Nonetheless, maximizing energy-efficiency may be of interest also in the downlink of a wireless network, given the worldwide growing concern about the energy-consumption of data networks.

Regarding energy-efficiency maximization, the paper [1], adopting a game-theoretic approach to implement a non-cooperative power control scheme, is a pioneering work that has paved the way to many interesting subsequent works both aiming at energy efficiency maximization and using the tools of game theory. Indeed, game theory, a branch of mathematics studying the interactions among several autonomous subjects with contrasting interests, can be well used in multiuser wireless networks to model the interactions between selfish active users, who are indeed in mutual competition for the available bandwidth and, in general, for the shared available network resources [2]. As examples, the reader is referred to [1], [3], [4]. Here, for a multiple access wireless data network using code division multiple access (CDMA), non-cooperative and cooperative games are introduced, wherein users choose their transmit powers in order to maximize their energy efficiency. While the above studies consider the issue of power control assuming that a conventional matched filter is available at the receiver, the paper [5] considers for the first time the problem of joint linear receiver design and power control so as to maximize the utility of each user. In particular, it is shown here that the inclusion of receiver design in the considered game brings remarkable advantages. The approach of [5] is then extended in [6], [7], wherein transmitter optimization, i.e., spreading code allocation, is also considered in addition to power allocation and linear receiver choice, and in [8], wherein joint spreading code adaptation and power control for a multicell CDMA system is considered.

While all of the above cited works refer to the CDMA multiplexing strategy, non-cooperative resource allocation algorithms for energy efficiency maximization in orthogonal frequency division multiple access (OFDMA), the leading multiple access strategy for the forthcoming fourth generation of wireless networks, is a much less investigated subject. Regarding resource allocation for OFDMA systems, in [9], [10] the downlink of multiuser OFDM systems is considered; in particular, in [9] the problem of joint subcarrier, bit, and power allocation is tackled. Instantaneous CSI is assumed and subcarrier reuse is not allowed. The resources are allocated

The research leading to these results has received funding from the European Union Seventh Framework Programme (FP7/2007-2013) under grant agreement n. 257740 (Network of Excellence "TREND"). This paper has been partly presented at the 2nd ICST Conference on Game Theory for Networks (GAMENETS 2011), Shanghai, China, April 2011. S. Buzzi and A. Zappone are with Consorzio Nazionale Interuniversitario per le Telecomunicazioni (CNIT) and University of Cassino, Via G. Di Biasio, 43, I-03043 Cassino (FR), Italy (e-mail: buzzi@unicas.it, alessio.zappone@unicas.it); G. Colavolpe is with CNIT and Università di Parma, Dip. di Ingegneria dell'Informazione, Viale G. P. Usberti 181A, I-43100 Parma, Italy (e-mail: giulio@unipr.it); D. Saturnino was with Scuola Superiore Sant'Anna di Pisa, I-56124 Pisa, Italy (e-mail: daniela.saturnino@sssup.it)

in order to minimize the overall transmit power from all users, subject to individual rate constraints, and an adaptive algorithm is proposed that first assigns the subcarriers based on instantaneous CSI, and then performs bit and power allocation. In [10], the constraint that each subcarrier must be assigned to no more than one user is relaxed, and subcarrier reuse is allowed. In this context, a two-step resource allocation algorithm is proposed: in the first step, subcarrier assignment is performed for individual rate maximization, whereas in the second step, transmit power allocation for each subcarrier is carried out for system's sum-rate maximization subject to total transmit power and bit-error rate (BER) constraints. Both studies indicate that significant performance improvements are obtained with respect to static resource allocation schemes, due to the multiuser diversity achieved when instantaneous CSI information is exploited. In [11] the downlink of a multicell OFDMA system is analyzed. The CSI assumption is relaxed, and subcarrier and transmit power allocation algorithms are proposed for individual outage probability minimization with rate constraints. Resource allocation in the uplink of a single-cell OFDMA systems is analyzed in [12]. There, a low-complexity greedy subcarrier allocation algorithm and an iterative waterfilling algorithm are proposed for system's sum-rate maximization. The results indicate that the optimal solution for the downlink case is not optimal for the uplink, thus motivating a number of subsequent studies on the uplink of single-cell and multicell OFDMA networks. In [13], the uplink of multicell OFDMA network is considered, and a resource allocator aiming at minimizing the total transmitted power subject to individual rate constraints is studied. The results indicate that the proposed algorithm converges to a stable resource allocation policy only when the interference load is below a certain threshold. In [14], a game-theoretic approach to resource allocation in the uplink of multicell OFDMA wireless networks is taken. First, a non-cooperative game in which each user selfishly tries to minimize his own transmitted power subject to a transmission rate constraint is proposed. The resources that each user can allocate are the subcarriers to use, the modulation format, and the transmit power on each subcarrier. However, the proposed game is not guaranteed to converge to a Nash Equilibrium, and for this reason a virtual referee is introduced to dictate the resource allocation and force it to a stable and efficient equilibrium point. In [15], an auction approach to subcarrier, modulation, and coding scheme allocation in single-cell and multicell OFDMA networks is proposed. As for multicell networks, the users in each cell are divided in interior and edge users, and the simplification is made to assume that the inner users in each cell do not interfere with adjacent cells. Several low-complexity algorithms for sum-rate maximization are proposed that exhibit near-optimal performance. Other relevant papers dealing with resource allocation (mainly aimed at rate maximization) and the game-theoretic framework are [16], [17]. Finally, in the recent paper [18], non-cooperative transmit power control and cooperative subcarrier allocation are jointly performed for energy-efficiency maximization in a multicell OFDMA system.

This paper considers the problem of resource allocation for the uplink of a multicell wireless network. In particular, we consider the problem of subcarrier allocation and transmit power control on the chosen subcarriers. In order to obtain resource allocation games converging to a Nash equilibrium (NE), we resort here to the theory of potential games [20]. Roughly speaking, in a potential game each change in the utility enjoyed by a given player due to an *unilateral* change of strategy by that player is paired by a similar change in a global function called the potential function. In a potential game, the best response strategy always leads to a Nash equilibrium (NE), and users, by acting selfishly, serve the greater good without knowing it. Potential games are a quite recent discovery for the communications and signal processing scientific community, and very few papers have considered their application to resource allocation problems in this area [22]–[26].

This paper is organized as follows. Next section contains some background material on potential games and a description of the considered multicell OFDMA wireless network. Section III focuses on a simple 2-user system and provides some useful theoretical insights for asymptotic regimes. Section IV discusses subcarrier selection games for maximizing the received Signal-to-Interference plus Noise-ratio (SINR) for fixed transmit power, whereas Section V describes a non-cooperative transmit power control game for energy efficiency maximization, assuming that subcarrier allocation has already taken place. In Section VI we consider instead the more challenging and interesting case of joint power control and subcarrier choice for the maximization of the users' energy efficiency. Numerical results are shown and commented in Section VII, while, finally, concluding remarks are given in Section VIII.

II. Preliminaries and system model

In this section we give brief details on potential games, introduce the general form of the system model for a multicell OFDMA wireless network and formulate the problem statement.

A. Strategic games and potential games

In its strategic form, a game $\mathcal{G}$ can be described as a triplet $\mathcal{G} = [\mathcal{K}, \{\mathcal{S}_k\}, \{u_k\}]$, wherein $\mathcal{K}$ is the set of players (e.g., the communicating devices in a multiple access network), $\mathcal{S}_k$ is the set of all possible strategies (i.e. the parameters' choices that may be taken in response to other players' choices) for the k-th player, and u_k represents the utility function or payoff of the k-th player; u_k is a scalar function, to be maximized, which depends on the strategies taken by all players of the game.

Thus, a change in strategy from one player affects all other players as well, and triggers a dynamic process, in which players iteratively update their own strategies as a reaction to changes in the strategies of the other players. This process is usually referred to as best-response dynamics (BRD), since in each iteration, given the strategies of the other players, each player responds by choosing the strategy that maximizes his own utility function. In this context, a crucial question is whether the BRD converges to an equilibrium point, or if the players indefinitely go on changing their strategies in a restless fashion. Here a key concept is the notion of *Nash equilibrium* (NE). Let

$$(s_1, s_2, \dots, s_K) \in \mathcal{S}_1 \times \mathcal{S}_2 \times \dots \mathcal{S}_K$$

denote a certain strategy K-tuple for the active users. The point $(s_1^*, s_2^*, \dots, s_K^*)$ is an NE if for every user k, we have

$$u_k(s_k^*, \boldsymbol{s}_{-k}^*) \geq u_k(s_k, \boldsymbol{s}_{-k}^*) \ ,$$

$\forall s_k \neq s_k^*$, wherein the vector $\boldsymbol{s}_{-k}$, as customary in the game-theoretic literature, denotes the vector of the strategies of all users but the k-th one. Otherwise stated, at the NE, no user can *unilaterally* improve its own utility by taking a different strategy. Thus, at the NE, each user, provided that the other users' strategies do not change, is not interested in changing its own strategy, which implies that any NE is also a fixed point of the BRD. Thus, in a strategic-form game an NE may be reached by running the BRD until convergence. However, this is not always the case since the BRD is not always guaranteed to be convergent.

We give now the formal definition of a potential game [20]. A strategic game $\mathcal{G} = [\mathcal{K}, \{\mathcal{S}_k\}, \{u_k\}]$ is called an *exact potential game* if there exists a function $V : \mathcal{S}_1 \times \mathcal{S}_2 \times \dots \mathcal{S}_K \rightarrow \mathcal{R}$ such that for any $k \in \mathcal{K}$ and for any $(s_k, \boldsymbol{s}_{-k}), (s_k^*, \boldsymbol{s}_{-k}) \in \mathcal{S}_1 \times \mathcal{S}_2 \times \dots \mathcal{S}_K$, we have

$$u_k(s_k, \boldsymbol{s}_{-k}) - u_k(s_k^*, \boldsymbol{s}_{-k}) = V(s_k, \boldsymbol{s}_{-k}) - V(s_k^*, \boldsymbol{s}_{-k}) \ . \quad (1)$$

Likewise, the game $\mathcal{G}$ is an *ordinal potential game* if the aforementioned function V is such that

$$u_k(s_k, \boldsymbol{s}_{-k}) > u_k(s_k^*, \boldsymbol{s}_{-k}) \Rightarrow V(s_k, \boldsymbol{s}_{-k}) > V(s_k^*, \boldsymbol{s}_{-k}) \ . \quad (2)$$

The function V is called the exact (respectively, ordinal) potential of the game.

In an exact potential game, Nash equilibria include maximizers of the potential function (note that generally, the reverse is not true), and, if the utility functions are continuous and the strategy spaces are compact, the BRD will converge to an NE of the game [20]. Roughly said, in a potential game wherein the potential function is bounded from above, any BRD will always converge to an NE: this is a very attractive property that can be used, as we will be showing in the sequel of the paper, to obtain convergent noncooperative games.

B. System model

Consider the uplink of a multicell OFDMA network with B base stations (BSs); each cell consists of mobile users and their assigned BS. Different links among cells are assumed to be synchronized. Let N be the number of subcarriers associated to the whole system and K_j the number of active users in the j-th cell, such that $\sum_{j=1}^{B} K_j = K$, where K is the total number of active users in the network.

Since we are considering a multicell environment, each user is affected by interference from users' terminals outside its own cell in addition to the interference of the ones within the same cell. When a BS is designated to detect the received signal from a given user's transmitter, we say that the user has been assigned to that BS. The BS assignment is denoted by the K-dimensional vector $\boldsymbol{a} = (a_1, \dots, a_K)$, whose entry $a_i \in \{1, \dots, B\}$. We can note that there are B^K different possible assignments. We assume here that the BS assignment vector $\boldsymbol{a}$ has been determined in a previous phase and we focus on the resource allocation problem only. As an example, each user can be assigned to his nearest BS.

Let us denote by $h_{k,j}(n)$ the channel gain between the k-th user and the j-th BS on the n-th subcarrier and by $p_k(n)$ the transmit power of the k-th user on the n-th subcarrier. For simplicity, we assume a real channel model; in particular, in the numerical simulations the channel coefficients will be modeled as Rayleigh-distributed random variables with a mean square value tied to the distance between transmitter and receiver. Note also that all of the subsequent developments may be trivially extended to the case of complex-valued channel gains. Full channel information is assumed here and in the sequel of the paper. Let us assume that each user can transmit on L subcarriers.[1] Let $\mathcal{F}_k$ be the set of L subcarriers allocated to the k-th user, i.e., $\mathcal{F}_k = \{F_k(1), \dots, F_k(L)\}\ , k = 1, \dots, K$, where $F_k(l) \in \{1, \dots, N\}$ is the index of the ℓ-th subcarrier allocated to user k. Otherwise stated, the set $\mathcal{F}_k$ contains L different indexes in $\{1, \dots, N\}$. Denote by $r_{n,i}$ the observable received at the i-th BS on the n-th subcarrier frequency; we have that

$$r_{n,i} = \sum_{k=1}^{K} \sum_{\ell \in \mathcal{F}_k \cap \{n\}} \sqrt{p_k(\ell)} h_{k,i}(\ell) b_k(\ell) + w_{n,i} \ , \quad (3)$$

wherein $b_k(\ell)$ is the information symbol transmitted by the k-th user on the ℓ-th subcarrier frequency, and $w_{n,i}$ is the additive noise term on the n-th subcarrier and at the i-th BS, modeled as a zero-mean Gaussian random variate with variance σ^2.

Given the fact that each user transmits on L subcarriers, a set of L distinct SINRs (one for each used subcarrier) can be defined for each user as follows, for any $\ell \in \mathcal{F}_k$:

$$\gamma_{k,a_k}(\ell) = \frac{p_k(\ell) h_{k,a_k}^2(\ell)}{\sigma^2 + \sum_{j=1, j \neq k}^{K} \sum_{n \in \mathcal{F}_j \bigcap \{\ell\}} p_j(n) h_{j,a_k}^2(n)} \ . \quad (4)$$

III. A closer look at a simple 2-user system

Before considering the general case, we examine a simple system wherein the number of users is $K = 2$, the number of available subcarriers is $N = 2$, and each user may transmit on $L = 1$ subcarrier[2]. Each user selfishly chooses to transmit on the subcarrier that grants the largest SINR. We consider the case that the two users are served by two different BSs (otherwise this would be a single-cell system), and, with no loss of generality, we assume that $a_1 = 1$ and $a_2 = 2$, i.e.,

[1] In particular, of primary interest is the case in which $KL > N$. It might be also questioned why each user should exactly transmit on L subcarriers; we note, however, that this is a very practical situation encountered in many of the current wireless standards. The case in which each user also optimizes the number of active subcarriers is an interesting topic that is however out of the scope of this paper.

[2] For additional details on this scenario, see also the recent paper [27].

the first and second user are assigned to the first and second BS, respectively. Although this is clearly a toy example, it permits us to extrapolate some key features of a multicell OFDMA system with universal frequency reuse, and, also, to give immediate evidence of the fact that subcarrier allocation based on greedy SINR-maximization games may not converge to a stable equilibrium in a multicell system.

Denote by p_1 and p_2 the transmit power of the 1st and 2nd user, respectively. Regarding the problem of subcarrier choice for SINR maximization, there may be four possible outcomes: letting (s_i, s_j), with $i, j = 1, 2$ denote the case in which the first user transmits on subcarrier i and the second user transmits on subcarrier j, we may have the following configurations: (s_1, s_1), (s_1, s_2), (s_2, s_1), and (s_2, s_2). For the case in which each user chooses to transmit on the subcarrier ensuring the largest SINR, it is readily seen that:

- (s_1, s_1) is an NE of the game if the equations

$$\begin{cases} \dfrac{h_{1,1}^2(1)}{\sigma^2 + h_{2,1}^2(1)p_2} \geq \dfrac{h_{1,1}^2(2)}{\sigma^2}, \\ \dfrac{h_{2,2}^2(1)}{\sigma^2 + h_{1,2}^2(1)p_1} \geq \dfrac{h_{2,2}^2(2)}{\sigma^2}, \end{cases} \quad (5)$$

  are both satisfied;
- (s_1, s_2) is an NE of the game if the equations

$$\begin{cases} \dfrac{h_{1,1}^2(1)}{\sigma^2} \geq \dfrac{h_{1,1}^2(2)}{\sigma^2 + h_{2,1}^2(2)p_2}, \\ \dfrac{h_{2,2}^2(2)}{\sigma^2} \geq \dfrac{h_{2,2}^2(1)}{\sigma^2 + h_{1,2}^2(1)p_1}, \end{cases} \quad (6)$$

  are both satisfied;
- (s_2, s_1) is an NE of the game if the equations

$$\begin{cases} \dfrac{h_{1,1}^2(2)}{\sigma^2} \geq \dfrac{h_{1,1}^2(1)}{\sigma^2 + h_{2,1}^2(1)p_2}, \\ \dfrac{h_{2,2}^2(1)}{\sigma^2} \geq \dfrac{h_{2,2}^2(2)}{\sigma^2 + h_{1,2}^2(2)p_1}, \end{cases} \quad (7)$$

  are both satisfied; and, finally,
- (s_2, s_2) is an NE of the game if the equations

$$\begin{cases} \dfrac{h_{1,1}^2(2)}{\sigma^2 + h_{2,1}^2(2)p_2} \geq \dfrac{h_{1,1}^2(1)}{\sigma^2}, \\ \dfrac{h_{2,2}^2(2)}{\sigma^2 + h_{1,2}^2(2)p_1} \geq \dfrac{h_{2,2}^2(1)}{\sigma^2}, \end{cases} \quad (8)$$

  are both satisfied.

Now, a careful inspection of Eqs. (5)-(8) reveals the following.

- For vanishingly small σ^2 (i.e., *low thermal-noise regime* or *interference-dominated regime*), depending on the channel coefficient realizations, either (s_1, s_2) or (s_2, s_1) is the NE point, while, at the same time, (s_1, s_1) and (s_2, s_2) can never be Nash equilibria points. Otherwise stated, in the case in which the thermal noise power vanishes, the two users transmit on orthogonal channels.
- For increasingly large σ^2 (i.e., *large thermal-noise regime*), each user, regardless of the behavior of the other one, chooses the subcarrier with the largest channel coefficient. In this case the four possible outcomes (s_i, s_j), $i, j = 1, 2$ may all be an NE point, however, for a given realization of the channel coefficients, a unique NE exists.
- In the case in which the channel coefficients $h_{i,j}^2(\ell) \to 0$, $\forall i \neq j$ and for $\ell = 1, 2$, each user, regardless of the behavior of the other one, again chooses the subcarrier with the largest channel coefficient. In the considered scenario, indeed, the system decouples in two isolated cells, and each user has to pick one of two interference-free subcarriers.
- In the general case in which no one of the above regimes holds, no general conclusions can be given. It is however evident that there may be certain realizations of the channel coefficients which lead to a system with no NE point. As an example, for the values reported in Table I, one can easily verify that, if each user greedily maximizes its SINR with respect to the subcarrier choice, no NE point exists.

Table I
AN EXAMPLE OF PARAMETERS, FOR THE 2-USER CHANNEL, LEADING TO A SYSTEM WHEREIN GREEDY SINR MAXIMIZATION WITH RESPECT TO THE CHOICE OF THE SUBCARRIERS DOES NOT LEAD TO AN EQUILIBRIUM.

σ^2	10^{-3}
$h_{1,1}^2(1)$	0.2881
$h_{1,1}^2(2)$	1.1413
$h_{1,2}^2(1)$	0.1958
$h_{1,2}^2(2)$	0.0017
$h_{2,1}^2(1)$	0.0638
$h_{2,1}^2(2)$	0.0023
$h_{2,2}^2(1)$	0.9975
$h_{2,2}^2(2)$	1.7823

The above analysis reveals that, even in a simple 2-user scenario, there may be conditions wherein non-cooperative SINR maximization has no equilibrium. Numerical simulations reveal that the occurrence of situations with no equilibrium increases as the number of users increases, especially in the case, of primary practical interest, in which $KL > N$. Moreover, even if an equilibrium exists, convergence of the BRD is not guaranteed.

In the following we will show how, using the potential games framework, non-cooperative games *always* converging to an equilibrium can be conceived.

IV. SUBCARRIER ALLOCATION FOR SINR-MAXIMIZATION GAMES

We are now interested in non-cooperative maximization of each user's SINR with respect to the choice of the allocated subcarriers, assuming, for the moment, that the transmit powers are arbitrary and fixed. In particular, each user may transmit on the N available subcarriers with a random, but fixed, power; of course, at each iteration of the games that we are going to discuss, each user only transmits on its chosen subcarriers, and the transmit power on the $N-L$ subcarriers is set to zero. The considered non-cooperative game can be formally expressed as the triplet $\mathcal{G} = [\mathcal{K}, \{\mathcal{S}_k\}, \{u_k\}]$, where

- $\mathcal{K} = \{1, 2, \ldots, K\}$ is the set of player, i.e., the active users in the network;
- u_k is the utility function the k-th player seeks to maximize; it must be a scalar increasing function of the k-th user SINRs $\{\gamma_{k,a_k}(\ell)\}_{\ell \in \mathcal{F}_k}$. As an example, we might consider the quantity $\sum_{\ell \in \mathcal{F}_k} \gamma_{k,a_k}(\ell)$.
- $\mathcal{S}_k$ is the set of strategies for the k-th user, i.e., the $\binom{N}{L}$ possible configurations of the set $\mathcal{F}_k$.

Otherwise stated, at each step the k-th user chooses the L subcarriers (i.e., the set $\mathcal{F}_k$) such that his utility u_k is maximized, given that the set of subcarriers for all the other users are fixed. Unfortunately, numerical evidence, as corroborated also by the discussion of the previous section, shows that if $u_k(\mathcal{F}_k) = \sum_{\ell \in \mathcal{F}_k} \gamma_{k,a_k}(\ell)$, the considered game does not always admit an NE.

In order to obtain a non-cooperative game always admitting an equilibrium, we can resort to the theory of potential games. Based on the concepts of Section II, we have to design a potential function V such that:

(a) $\forall k = 1, \ldots, K$, V can be expressed as $V = g_k(\zeta_k, \omega_k)$, wherein the scalar-valued argument ζ_k depends on the k-th player strategy (and possibly on the other players' strategies as well), the scalar-valued argument ω_k is independent of the k-th player strategy, and $g_k(\cdot, \cdot)$ is a monotonically increasing function of its former argument;

(b) V is an increasing function of the SINRs $\gamma_{k,a_k}(\ell)$, for all $\ell \in \mathcal{F}_k$ and $k = 1, \ldots, K$: note that this property is not strictly related to the theory of potential games, but we are considering it here in order to ensure that maximizing the potential V indirectly results in a maximization of the users' SINRs, and, thus, in improved performance.

If we are able to find a function V fulfilling properties (a) and (b), then choosing as utility function for the k-th user the quantity ζ_k leads to an ordinal potential game. Moreover, if, for any k, the decomposition $V = \zeta_k + \omega_k$ holds, then letting $u_k = \zeta_k$ leads to an exact potential game.

In this paper we propose to use as potential function the negative of the inverse of the SINRs, summed over all subcarriers and all the users[3], i.e.:

$$V = -\sum_{k=1}^{K} \sum_{\ell \in \mathcal{F}_k} \frac{1}{\gamma_{k,a_k}(\ell)} = -\sum_{k=1}^{K} \sum_{\ell \in \mathcal{F}_k} \left(\frac{\sigma^2 + \sum_{j \neq k} \sum_{n \in \mathcal{F}_j \bigcap \{\ell\}} p_j(n) h^2_{j,a_k}(n)}{p_k(\ell) h^2_{k,a_k}(\ell)} \right). \qquad (9)$$

It is easily shown that, for any k, the potential V can be expressed as shown in Eq. (10) at the top of the next page, wherein ζ_k contains all the quantities depending on $\mathcal{F}_k$ (the strategy set for the k-th user), and ω_k is an additive term (whose expression is not reported for the sake of brevity) independent of the strategy of the k-th user. Given decomposition (10), the following result follows.

Proposition 1: *Consider a non-cooperative game $\mathcal{G}$ wherein the k-th user aims at maximizing, with respect to the choice of the set of subcarriers $\mathcal{F}_k$, the utility function reported in Eq. (11) at the top of the next page. The game $\mathcal{G}$ is an exact potential game with potential function V in (9). Hence, the BRD associated to $\mathcal{G}$ always converges to an NE.*

Proof: The proof is simply given by decomposition (10) for the potential function V, which fulfils both properties (a) and (b), and by the properties of potential games. ■

A couple of remarks are now in order. First of all, the potential V may have (actually has) local maxima, all of these resulting in Nash equilibria, and there is no guarantee that at a certain NE the global optimum has been achieved. Otherwise stated, several Nash equilibria may exist, and we may have suboptimal performance; numerical results, however, will show that at equilibrium points performance is generally better than that achieved before the game was played, so this game results in a genuine performance improvement. Secondly, despite the fact that the game considered in Proposition 1 is obviously convergent, there is no guarantee that, upon maximization of the utility function in (11), this actually results in an increase for the SINRs achieved by the k-th user. As a consequence, in playing the game, we assume that each user chooses the strategy that maximizes the utility $u_k(\mathcal{F}_k)$ in (11), but it really adopts such a strategy only if this brings an increase to the sum of the SINRs on its used subcarriers, i.e., to the performance metric $\sum_{\ell \in \mathcal{F}_k} \gamma_{k,a_k}(\ell)$; otherwise, the user keeps on using its previous strategy. Note that this behavior still results in a game that converges to an equilibrium (at each stage of the game the potential function is either constant or increasing), although the improved performance is at the price of slower convergence speed, wherein by "improved performance" we mean that adopting such a strategy (i.e. changing strategy only when a user gains for himself) gives better performance with respect to the case in which each user always adopts its utility maximizing strategy.

A. Approximate implementations and a simple upper bound

The main drawback of the subcarrier assignment scheme of the previous subsection is its computational complexity; indeed, at each maximization step, each user is to perform an exhaustive search over all the possible $\binom{N}{L}$ choices of L out of N subcarriers. This may be a prohibitive task and approximate implementations of the game, with lower computational complexity are to be sought. We suggest here two possible strategies for complexity reduction.

At each maximization step, each user can make, in place of an exhaustive search over the $\binom{N}{L}$ possible configurations, a search on a reduced set of configurations. Among the others, we comment here on two possible alternatives:

a) the search can be made over a certain number, say Q, of randomly selected configurations; and

[3]Indeed it is easy to check that with such a choice both properties (a) and (b) are fulfilled. Interestingly, since V is an increasing function of the SINRs $\gamma_{k,a_k}(\ell)$, its maximization results on the average in a maximization of the SINRs of the users. See [8], [22] to have other scenarios wherein such a function has been used.

$$V = \underbrace{-\sum_{\ell \in \mathcal{F}_k} \frac{\sigma^2 + \sum_{j=1, j\neq k}^{K} \sum_{n \in \mathcal{F}_j \bigcap \{\ell\}} p_j(n) h_{j,a_k}^2(n)}{p_k(\ell) h_{k,a_k}^2(\ell)} - \sum_{i=1, i\neq k}^{K} \sum_{\ell \in \mathcal{F}_i} \left(\frac{\sum_{n \in \mathcal{F}_k \bigcap \{\ell\}} p_k(n) h_{k,a_i}^2(n)}{p_i(\ell) h_{i,a_i}^2(\ell)} \right)}_{\zeta_k} + \omega_k \ , \tag{10}$$

$$u_k(\mathcal{F}_k) = -\sum_{\ell \in \mathcal{F}_k} \frac{\sigma^2 + \sum_{j=1, j\neq k}^{K} \sum_{n \in \mathcal{F}_j \bigcap \{\ell\}} p_j(n) h_{j,a_k}^2(n)}{p_k(\ell) h_{k,a_k}^2(\ell)} - \sum_{i=1, i\neq k}^{K} \sum_{\ell \in \mathcal{F}_i} \left(\frac{\sum_{n \in \mathcal{F}_k \bigcap \{\ell\}} p_k(n) h_{k,a_i}^2(n)}{p_i(\ell) h_{i,a_i}^2(\ell)} \right) . \tag{11}$$

b) given the fact that there is a high probability that the subcarriers with the lowest channel coefficients will be discarded, the user can make a search over the $\widetilde{N}$ subcarriers (with $L \leq \widetilde{N} < N$), with the largest channel coefficients. Otherwise stated, the k-th user looks at the $\widetilde{N}$ carriers with the largest values of the coefficients $h_{k,a_k}^2(\cdot)$, and maximizes its utility over all the $\binom{\widetilde{N}}{L}$ possible configurations of L out of $\widetilde{N}$ subcarriers.

Note that both the above approximate strategies lead, at each maximization step, to an increase of the potential function, and are thus convergent to an equilibrium.

Finally, a moment's thought gives a simple upper bound to the performance, in terms of achieved SINR, of any resource allocation procedure that can be conceived for the considered scenario. Indeed, for the case in which each user transmits with the same power p_k on all its L assigned subcarriers, any resource allocation procedure cannot beat the case in which there is no interference and each user transmits on its L best channels. Otherwise stated, letting $\mathcal{M}_k \subset \{1, 2, , \dots, N\}$ contain the indices of the subcarriers with the L best channel coefficients for the k-th user, i.e., $|\mathcal{M}_k| = L$ and, $\forall \ell \in \mathcal{M}_k, q \notin \mathcal{M}_k$, $h_{k,a_k}^2(\ell) \geq h_{k,a_k}^2(q)$, then any subcarrier assignment procedure achieves a set of SINRs $\{\gamma_{k,a_k}(\ell)\}_{\ell \in \mathcal{F}_k}$ such that

$$\sum_{\ell \in \mathcal{F}_k} \gamma_{k,a_k}(\ell) \leq \sum_{\ell \in \mathcal{M}_k} \frac{p_k h_{k,a_k}^2(\ell)}{\sigma^2} \ . \tag{12}$$

Although the above bound reveals to be not so tight, especially for increasingly large number of users, it is very simple to evaluate and may be helpful to obtain an approximate and rough indication of the performance frontier that a subcarrier assignment scheme cannot beat.

V. Power allocation game for energy-efficiency maximization

Let us now assume that subcarrier allocation has already taken place, and let us focus on the problem of transmit power control. The utility function that we consider here is the so-called energy efficiency, which is usually defined as (see [1], [3]- [6])

$$u(p) = R \frac{D}{M} \frac{f(\gamma)}{p} \ , \tag{13}$$

wherein R is the transmit data-rate, M is the packet length, $D \leq M$ is the number of information symbols contained in each packet, p is the transmit power, while, finally, $f(\gamma)$ approximates the probability of correct reception for a packet of length M, and is usually chosen as [1], [3]- [6]

$$f(\gamma) = \left(1 - e^{-\gamma}\right)^M \ , \tag{14}$$

with γ the received SINR. It is easy to realize that the utility in (13) is measured in bit/Joule. In our context, first of all note that if user k is using subcarrier ℓ, the corresponding SINR can be written as

$$\gamma_{k,a_k}(\ell) = \frac{p_k(\ell) h_{k,a_k}^2(\ell)}{\sigma^2 + \sum_{j=1, j\neq k}^{K} \sum_{n \in \mathcal{F}_j \cap \{\ell\}} p_j(n) h_{j,a_k}^2(n)} \ . \tag{15}$$

As a consequence, the energy efficiency function for the k-th user on the ℓ-th subcarrier is

$$u_{k,\ell}\left(p_k(\ell)\right) = R \frac{D}{M} \frac{f(\gamma_{k,a_k}(\ell))}{p_k(\ell)} \tag{16}$$

where $f(\cdot)$ is reported in (14). Since each user transmits on L subcarriers, it is reasonable to take as utility function for the k-th user the function

$$u_k\left(\{p_k(\ell)\}_{\ell \in \mathcal{F}_k}\right) = \sum_{\ell \in \mathcal{F}_k} u_{k,\ell}\left(p_k(\ell)\right) \ . \tag{17}$$

We thus consider a non-cooperative game wherein each user aims at maximizing his energy efficiency (17) by tuning the transmit power on his allocated subcarriers. To complete the game formulation, a constraint on the maximum transmit power is needed. To this end, two customary approaches are usually followed:

(a) a constraint may be posed on the maximum transmit power on a subcarrier basis, i.e., $p_k(\ell) \in [0, P_{\max}/L]$, $\forall \ell \in \mathcal{F}_k$.

(b) a constraint may be posed on the total transmitted power by each user, i.e., $\sum_{\ell \in \mathcal{F}_k} p_k(\ell) \in [0, P_{\max}]$;

Approach (b) is the most commonly encountered, but (a) also appears in the literature (see, for instance, [28]), and it turns out to be useful when a power mask constraint is to be fulfilled due to standard compliance; it should be also noted that the set of transmit powers fulfilling constraint (a) is strictly included in the set of powers fulfilling constraint (b), which thus defines a larger set of transmit powers. In the remainder of this Section, we choose to adopt constraint (a), since the utility (17) is not quasi-concave in the domain defined by[4] (a), which implies that a NE is not guaranteed to exist, and hence computing the best response of each player would require the solution of a non-convex problem. The more customary constraint (b) will be instead used in the subsequent section on joint subcarrier allocation and power control for energy efficiency maximization. We have now the following result.

Proposition 2: *Consider a non-cooperative game wherein the k-th user tunes its transmit power (constrained to be not larger than $P_{\max}/L$) on his subcarriers in order to maximize the utility function reported in (17). This game admits a unique NE point; at the NE, the transmit power for the k-th user on the ℓ-th subcarrier, $\widetilde{p}_k(\ell)$ say, is expressed as $\widetilde{p}_k(\ell) = \min\{p_k(\ell)^*, P_{\max}/L\}$, with $p_k(\ell)^*$ the transmit power such that $\gamma_{k,a_k}(\ell) = \gamma^*$, with γ^* the unique solution of the equation*

$$\gamma f'(\gamma) = f(\gamma) \ . \tag{18}$$

The best response dynamics converges to the unique NE point, whatever the initial transmit power is.

Proof: Since the power transmitted by the k-th user on the ℓ-th subcarrier does not affect the SINR on the remaining subcarriers of the same user, and given the additive nature of the utility (17), it follows that each summand in (17) can be optimized separately. The considered game for the k-th user is thus decomposed in L parallel subgames, one for each of the used subcarriers, having the utility function reported in (16). The proof that each of these subgames admits a unique NE point, as well as that the best response dynamic converges to such NE point, as detailed in the text of this proposition, can be given by replicating the same steps reported in [29]. We omit the details for the sake of brevity. ■

VI. Joint Power and Subcarrier Allocation for Energy-Efficiency Maximization

We consider now the more relevant and more challenging scenario wherein each user can jointly allocate its transmit powers and subcarrier frequencies so as to maximize his energy-efficiency.

First of all, we should note that this scenario is much more involved than the power allocation problem for fixed subcarriers that has been discussed in Section V, and direct extension of the approach employed in Section V to the case at hand, is not possible. To see this, assume we employ the approach of Section V, and consider for simplicity the case in which each user transmits on only $L = 1$ subcarrier. Then, for all $k = 1, \ldots, K$ the k-th player's utility function is given by

$$u_k\left(\mathcal{F}_k, \{p_k(\ell)\}_{\ell\in\mathcal{F}_k}\right) = \frac{f(\gamma_{k,a_k}(\ell))}{p_k(\ell)} \ , \tag{19}$$

i.e., (17) with only $L = 1$ term in the sum. In the considered scenario, for all $k = 1, \ldots, K$, player k computes his best response by maximizing (19) with respect not only to $p_k(\ell)$, but also with respect to the choice of the transmit subcarrier. Now, since the choice of the subcarrier only affects the numerator of (19) through the efficiency function $f(\gamma_{k,a_k}(\ell))$, and since $f(\cdot)$ is an increasing function of $\gamma_{k,a_k}(\ell)$, it follows that, for all $k = 1, \ldots, K$, in order to maximize (19), player k will choose the subcarrier that maximizes his individual SINR $\gamma_{k,a_k}(\ell)$. Therefore, following the approach of Section V leads to a BRD in which greedy SINR maximization is to be carried out. But it has been shown in Section III that even in the very simple scenario in which $L = 1$, greedy SINR maximization is not guaranteed to converge, or even admit NE points. Consequently, even in the simple scenario in which each player transmits on only $L = 1$ subcarrier, the conventional approach that has been employed in Section V fails. Therefore, a different approach than that of Section V is needed.

In order to devise a non-cooperative energy-efficient game whose BRD is guaranteed to converge to an NE, again we will make use of the framework of potential games. To this end, a natural choice for the potential function would be

$$V = \sum_{k=1}^{K} \sum_{\ell\in\mathcal{F}_k} u_{k,\ell}(p_k(\ell)) \ , \tag{20}$$

with $u_k(\ell)$ the energy efficiency function defined in (16). However, also this approach fails, since it is readily seen that using the classical efficiency function $f(\gamma) = (1 - e^{-\gamma})^M$ it is not possible to write the potential function as the sum of terms depending on the strategy of a certain user plus other terms independent of the said strategy. In order to circumvent this difficulty, we propose the following approach.

First of all, we consider a different efficiency function, namely:

$$\widetilde{f}(\gamma) = \left(e^{-\beta/\gamma}\right)^M \ , \tag{21}$$

with β a suitable constant to be specified in the sequel. Note that, although $\widetilde{f}(\gamma)$ in (21) does no longer approximate the probability of correct reception of a data-packet of M symbols, just as the classical efficiency function it is still an S-shaped increasing function of γ, approaching zero for $\gamma \to 0$ and approaching unity for $\gamma \to +\infty$. Plugging (21) into the energy efficiency definition (16), we thus obtain the following energy efficiency function on the generic subcarrier ℓ

$$\widetilde{u}_{k,\ell}(p_k(\ell)) = \frac{\left(e^{-\beta/\gamma_{k,a(k)}(\ell)}\right)^M}{p_k(\ell)} \ . \tag{22}$$

Note that $\widetilde{u}_{k,\ell}(p_k(\ell))$ is still a quasi-concave function, and letting $\beta = \gamma^*/M$, with γ^* the unique solution of (18) we are guaranteed that the maximizer of $\widetilde{u}_{k,\ell}(p_k(\ell))$ coincides with that of the classical energy-efficiency $u_{k,\ell}(p_k(\ell))$ in (16). From now on we thus embrace such a choice for the constant β. A further justification of the validity of the efficiency function $\widetilde{f}(\gamma)$ as a substitute for $f(\gamma)$ is obtained by considering the ratio

$$\left[\frac{f(\gamma)}{\widetilde{f}(\gamma)}\right]^{1/M} = \frac{1 - e^{-\gamma}}{e^{-\beta/\gamma}} \ . \tag{23}$$

[4] Note that even if each $u_{k,\ell}(p_k(\ell))$ is quasi-concave with respect to $p_k(\ell)$, $\sum_{\ell\in\mathcal{F}_k} u_{k,\ell}(p_k(\ell))$ is in general not jointly quasi-concave with respect to $\{p_k(\ell)\}_{\ell\in\mathcal{F}_k}$, since the sum of quasi-concave functions is not guaranteed to preserve quasi-concavity.

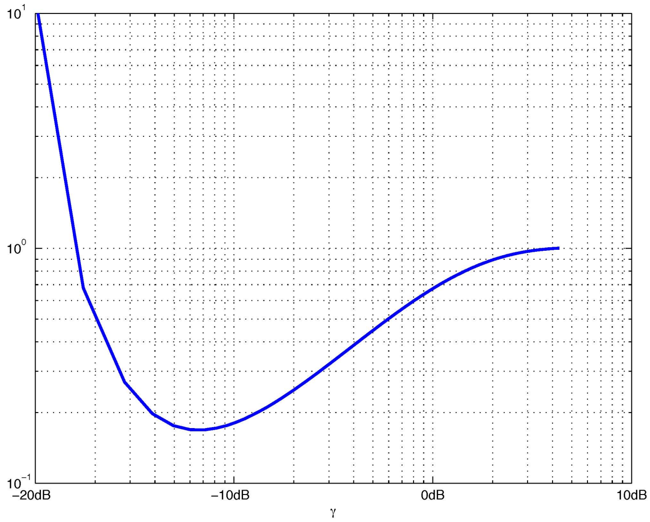


Figure 1. The quantity $(1-e^{-\gamma})/e^{-\beta/\gamma}$ is plotted here versus γ for the case that $M=100$.

It is seen that for large γ, (23) converges to one. Moreover, in order to address how large γ is required to be for (23) to approach unity, in Fig. 1 (23) has been plotted for the case in which $M=100$ (which leads to $\gamma^*=6.4$dB) versus γ ranging from -20dB to 10dB. It is seen that for $\gamma > 0$dB (that is the region of interest) (23) is very close to 1.

Next, instead of the sum in (20), we consider the product of energy efficiencies (22)

$$\prod_{k=1}^{K}\prod_{\ell\in\mathcal{F}_k}\widetilde{u}_{k,\ell}(p_k(\ell))\ . \tag{24}$$

As will be shown in the sequel, unlike (20), (24) lends itself to the derivation of a non-cooperative potential game. Moreover, just as (20), (24) is linked to the global performance of the system, since it is the product of the utilities of all players. Therefore, it is of interest to reach NE where (24) is maximized. Finally, a potential function can be defined as the natural logarithm of (24). Otherwise stated, we consider the following potential function

$$\begin{aligned} V = \ & \ln\left(\prod_{k=1}^{K}\prod_{\ell\in\mathcal{F}_k}\widetilde{u}_{k,\ell}(p_k(\ell))\right) = \\ & -\sum_{k=1}^{K}\sum_{\ell\in\mathcal{F}_k}\left(\frac{\beta M}{\gamma_{k,a_k}(\ell)}+\ln p_k(\ell)\right)\ . \end{aligned} \tag{25}$$

Note that since the natural logarithm is a monotonically increasing function, maximizing (25) is equivalent to maximizing (24). It is also interesting to remark that we have now the term $-\ln p_k(\ell)$, which is a pricing factor discouraging users from transmitting at a too large power. Upon straightforward manipulation, the potential in (25) can be written as shown in Eq. (26) at the top of the next page, wherein ω_k is an additive term independent of the strategies of the k-th user.

As for the constraint on the maximum transmit power, as already discussed in Section V, two approaches are usually considered:

(a) a constraint on the maximum transmit power on each subcarrier, i.e., $p_k(\ell)\in[0,P_{\max}/L]$, $\forall\ell\in\mathcal{F}_k$.
(b) a constraint on the total transmitted power by each user, i.e., $\sum_{\ell\in\mathcal{F}_k} p_k(\ell)\in[0,P_{\max}]$;

Here, both kinds of power constraints will be addressed. To begin with, we give the following proposition.

Proposition 3: *Consider a non-cooperative game $\mathcal{G}$ wherein the k-th user aims at maximizing, with respect to the choice of the set of subcarriers $\mathcal{F}_k$, and of the transmit powers $\{p_k(\ell)\}_{\ell\in\mathcal{F}_k}$, the utility function in Eq. (27) at the top of the next page. When either power constraint (a), or power constraint (b) is employed, $\mathcal{G}$ is an exact potential game with potential function V in (25). Hence the BRD associated to $\mathcal{G}$ is guaranteed to converge to an NE.*

Proof: Independently of the particular power constraint that is employed, decomposition (26) shows that V is an exact potential function for $\mathcal{G}$. Hence, with both power constraint (a) and (b), $\mathcal{G}$ is an exact potential game, and the thesis follows from the properties of potential games. ■

Having established that the BRD associated to $\mathcal{G}$ is guaranteed to converge to an NE, we turn our attention to deriving the best response of player k to the strategies of other players, for all $k=1,\ldots,K$. To elaborate, for any $m\in\mathcal{F}_k$, let us define the (subcarrier-dependent) coefficients

$$a_k(m)=\sum_{i=1,i\neq k}^{K}\ \sum_{\ell\in\mathcal{F}_i\cap\{m\}}\frac{h^2_{k,a_i}(m)}{p_i(\ell)h^2_{i,a_i}(\ell)}\ , \tag{28}$$

and[5]

$$c_k(m)=\frac{\sigma^2+\displaystyle\sum_{j=1,j\neq k}^{K}\ \sum_{n\in\mathcal{F}_j\cap\{m\}}p_j(n)h^2_{j,a_k}(n)}{h^2_{k,a_k}(m)}\ , \tag{29}$$

which allows us to rewrite (27) in the more compact form

$$\begin{aligned} & u_k\left(\mathcal{F}_k,\{p_k(\ell)\}_{\ell\in\mathcal{F}_k}\right) = \\ & -\sum_{m\in\mathcal{F}_k}\left(\beta M\frac{c_k(m)}{p_k(m)}+\beta M a_k(m)p_k(m)+\ln(p_k(m))\right) = \\ & \sum_{m\in\mathcal{F}_k} g_{m,k}(p_k(m))\ , \end{aligned} \tag{30}$$

wherein for all $m\in\mathcal{F}_k$ we have defined the function

$$g_{m,k}(p_k(m))=-\beta M\frac{c_k(m)}{p_k(m)}-\beta M a_k(m)p_k(m)-\ln(p_k(m))\ . \tag{31}$$

Determining the best response of player k involves maximizing (30) with respect to the L transmit powers and subcarriers. In the rest of this section, the two considered power constraints will be treated separately.

A. Subcarrier-based power constraint

Assume the power constraint $p_k(m)\in[0,P_{\max}/L]$, $\forall m\in\mathcal{F}_k$ is adopted. Then, for each of the $\binom{N}{L}$ possible subcarrier allocations, the L transmit powers $\{p_k(m)\}_{m\in\mathcal{F}_k}$ that

[5]Note that $a_k(m)=0$ if no user other than user k is using subcarrier m.

$$V=\omega_k+\underbrace{\beta M\left(\sum_{\ell\in\mathcal{F}_k}\frac{\sigma^2+\sum_{j=1,j\neq k}^{K}\sum_{n\in\mathcal{F}_j\cap\{\ell\}}p_j(n)h^2_{j,a_k}(n)}{p_k(\ell)h^2_{k,a_k}(\ell)}-\sum_{i=1,i\neq k}^{K}\sum_{\ell\in\mathcal{F}_i}\frac{\sum_{n\in\mathcal{F}_k\cap\{\ell\}}p_k(n)h^2_{k,a_i}(n)}{p_i(\ell)h^2_{i,a_i}(\ell)}\right)-\sum_{\ell\in\mathcal{F}_k}\ln(p_k(\ell))}_{\zeta_k} \tag{26}$$

$$\begin{aligned}u_k\left(\mathcal{F}_k,\{p_k(\ell)\}_{\ell\in\mathcal{F}_k}\right)= & -\beta M\sum_{\ell\in\mathcal{F}_k}\frac{\sigma^2+\sum_{j=1,j\neq k}^{K}\sum_{n\in\mathcal{F}_j\cap\{\ell\}}p_j(n)h^2_{j,a_k}(n)}{p_k(\ell)h^2_{k,a_k}(\ell)}\\ & -\beta M\sum_{i=1,i\neq k}^{K}\sum_{\ell\in\mathcal{F}_i}\frac{\sum_{n\in\mathcal{F}_k\cap\{\ell\}}p_k(n)h^2_{k,a_i}(n)}{p_i(\ell)h^2_{i,a_i}(\ell)}-\sum_{\ell\in\mathcal{F}_k}\ln(p_k(\ell))\ ,\end{aligned} \tag{27}$$

maximize (30) have to be determined. In this case, note that, for any fixed $\mathcal{F}_k$, the vector problem of maximizing (30) with respect to $\{p_k(m)\}_{m\in\mathcal{F}_k}$, decouples into L scalar maximization problems wherein for all $m\in\mathcal{F}_k$, $p_k(m)$ is determined as the maximizer of (31), subject to the constraint $p_k(m)\in[0,P_{\max}/L]$. Computing the first derivative of (31), it readily follows that for positive $p_k(m)$, (31) has a unique stationary point, which is also its unconstrained maximizer[6]. Then, setting to zero the first derivative of (31), and taking into account the subcarrier-based power constraint $p_k(m)\in[0,P_{\max}/L]$, for all $m\in\mathcal{F}_k$, the constrained maximizer of (31) is obtained as

$$p_k(m)=\min\left\{P_{\max}/L,\frac{-1+\sqrt{1+4\beta^2M^2a_k(m)c_k(m)}}{2a_k(m)\beta M}\right\}\ , \tag{32}$$

if $a_k(m)\neq 0$, and

$$p_k(m)=\min\left\{P_{\max}/L,\beta Mc_k(m)\right\}\ , \tag{33}$$

if $a_k(m)=0$, respectively. Summing up, for all $k=1,\ldots,K$, player k determines his best response by performing the following steps:

1) Choose one of the possible sets $\mathcal{F}_k$, and compute the associated coefficients $a_k(m)$ and $c_k(m)$, $m\in\mathcal{F}_k$.
2) Maximize the resulting utility (30) with respect to the transmit powers, subject to the power constraint $p_k(m)\in[0,P_{\max}/L]$, $\forall m\in\mathcal{F}_k$, by setting $\{p_k(m)\}_{m\in\mathcal{F}_k}$ according to (32) or (33).
3) Repeat steps 1) and 2) for each of the possible $\binom{N}{L}$ sets $\mathcal{F}_k$, and choose the set and the associated transmit powers that yield the largest utility.

B. Total transmitted power constraint

Similarly as in Section VI-A, for each of the $\binom{N}{L}$ possible subcarrier allocations, the L transmit powers that maximize (30) have to be determined. However, if the power constraint $\sum_{m\in\mathcal{F}_k}p_k(m)\in[0,P_{\max}]$ is adopted, maximization of (30) with respect to the transmitted powers is more involved because the vector maximization problem can not be decoupled into scalar problems. Instead, for any fixed $\mathcal{F}_k$, the following optimization problem needs to be solved.

$$\begin{cases}\max\limits_{\{p_k(m)\}_{m\in\mathcal{F}_k}}\ \sum_{m\in\mathcal{F}_k}g_{m,k}(p_k(m))\\ \text{s.t.}\ \sum_{m\in\mathcal{F}_k}p_k(m)\leq P_{max},\ \ p_k(m)\geq 0,\ \forall m\in\mathcal{F}_k\end{cases} \tag{34}$$

Unfortunately, problem (34) is not convex, because $\sum_{m\in\mathcal{F}_k}g_{m,k}(p_k(m))$ is not a concave function, as can be easily verified by direct computation of its Hessian. However, in the following we will show that by adding an additional constraint on $\{p_k(m)\}_{m\in\mathcal{F}_k}$, problem (34) can be reformulated as a convex problem, and that no loss of optimality is incurred by adding such a constraint. This is accomplished in the following proposition

Proposition 4: *The objective function of problem (34) is concave when restricted to the set*

$$\mathcal{C}=\{\{p_k(m)\}_{m\in\mathcal{F}_k}\ :\ p_k(m)\in[0;2\beta Mc_k(m)]\}\ . \tag{35}$$

Moreover, any solution of (34), belongs to $\mathcal{C}$.

Proof: Consider the Hessian of $\sum_{m\in\mathcal{F}_k}g_{m,k}(p_k(m))$. It is easy to realize that all the off-diagonal components equal zero, whereas for all $m\in\mathcal{F}_k$ we have

$$\frac{\partial^2\left(\sum_{m\in\mathcal{F}_k}g_{m,k}(p_k(m))\right)}{\partial p_k^2(m)}=\frac{p_k(m)-2\beta Mc_k(m)}{p_k^3(m)}\ . \tag{36}$$

From (36) it readily follows that the Hessian of the objective function of (34) is a non-positive matrix if $\{p_k(m)\}_{m\in\mathcal{F}_k}\in\mathcal{C}$. Hence, $\sum_{m\in\mathcal{F}_k}g_{m,k}(p_k(m))$ is concave when restricted to $\mathcal{C}$. In order to complete the proof, we have to show that any solution of (34) lies within $\mathcal{C}$. To this end, consider the generic summand of $\sum_{m\in\mathcal{F}_k}g_{m,k}(p_k(m))$, as given by (31), for all $m\in\mathcal{F}_k$. As noted in Section VI-A, (31) has a unique stationary point $p_k^*(m)$, which is also its global, unconstrained

[6]The term unconstrained is used to stress that the said maximizer might be unfeasible, i.e. might be larger than $P_{\max}/L$.

maximizer. Thus, such an unconstrained maximizer can be found by setting to zero the first derivative of (31) and solving for $p_k(m)$, which yields, for all $m \in \mathcal{F}_k$

$$p_k^*(m) = \frac{-1 + \sqrt{1 + 4\beta^2 M^2 a_k(m) c_k(m)}}{2a_k(m)\beta M} \ , \qquad (37)$$

if $a_k(m) \neq 0$, and

$$p_k^*(m) = \beta M c_k(m) \ , \qquad (38)$$

if $a_k(m) = 0$, respectively. Now, note that it holds

$$\begin{aligned} \sum_{m \in \mathcal{F}_k} g_{m,k}(p_k(m)) \quad &\leq \quad \sum_{m \in \mathcal{F}_k} \max_{p_k(m)} \left(g_{m,k}(p_k(m))\right) \\ &= \quad \sum_{m \in \mathcal{F}_k} g_{m,k}(p_k^*(m)) \ . \end{aligned} \qquad (39)$$

Thus, $\boldsymbol{p}_k^* = \{p_k^*(m)\}_{m \in \mathcal{F}_k}$ clearly is the unique, global, unconstrained maximizer of $\sum_{m \in \mathcal{F}_k} g_{m,k}(p_k(m))$. Of course, there is no guarantee that $\boldsymbol{p}_k^*$ is a feasible point for problem (34). Otherwise stated, it can happen that $\sum_{m \in \mathcal{F}_k} p_k^*(m) > P_{\max}$. In the following, we will first prove the result in case in which $\boldsymbol{p}_k^*$ satisfies the power constraint. Next, we will address the case in which $\boldsymbol{p}_k^*$ is not a feasible power vector.

When $\sum_{m \in \mathcal{F}_k} p_k^*(m) \leq P_{\max}$, then clearly $\boldsymbol{p}_k^*$ is also the unique solution of (34). In this case the proof is completed if we show that $\boldsymbol{p}_k^* \in \mathcal{C}$, i.e. that for all $m \in \mathcal{F}_k$, $p_k^*(m) \in [0; 2\beta M c_k(m)]$. Now, for any $m \in \mathcal{F}_k$, if $a_k(m) = 0$, $p_k^*(m)$ is given by (38), thus trivially implying that $p_k^*(m) \in [0; 2\beta M c_k(m)]$, whereas if $a_k(m) \neq 0$, $p_k^*(m)$ is given by (37) and we need to show that

$$\frac{-1 + \sqrt{1 + 4\beta^2 M^2 a_k(m) c_k(m)}}{2a_k(m)\beta M} \leq 2\beta M c_k(m) \ . \qquad (40)$$

Multiplying both sides of (40) by $2a_k(m)\beta M$, and since $\sqrt{x} \leq x \ , \forall x \geq 1$, the thesis follows.

Instead, when $\boldsymbol{p}_k^*$ does not satisfy the power constraint, it clearly can not be the solution to (34). Therefore, even if it is still true that $\boldsymbol{p}_k^* \in \mathcal{C}$, this does not directly imply the result, but a further step is needed. Denote by $\widetilde{\boldsymbol{p}}_k = \{\widetilde{p}_k(m)\}_{m \in \mathcal{F}_k}$ a solution of (34). We have to show that $\widetilde{\boldsymbol{p}}_k \in \mathcal{C}$. To this end, note that $\widetilde{p}_k(m) \leq p_k^*(m)$, for all $m \in \mathcal{F}_k$. To see this, note that if it existed an $\bar{m} \ : \widetilde{p}_k(\bar{m}) > p_k^*(\bar{m})$, then we could define a new power vector $\widehat{\boldsymbol{p}}_k$, which has the same components as $\widetilde{\boldsymbol{p}}_k$, except the $\bar{m}$-th component, which is set to $\widehat{p}_k(\bar{m}) = p_k^*(m)$. Now, clearly $\sum_{m \in \mathcal{F}_k} \widehat{p}_k(m) \leq \sum_{m \in \mathcal{F}_k} \widetilde{p}_k(m)$, thus implying that $\widehat{\boldsymbol{p}}_k$ is feasible. Moreover, note that we have

$$\sum_{m \in \mathcal{F}_k} g_{m,k}(\widehat{p}_k(m)) > \sum_{m \in \mathcal{F}_k} g_{m,k}(\widetilde{p}_k(m)) \ . \qquad (41)$$

Equation (41) holds because $g_{\bar{m},k}(\widehat{p}_k(\bar{m})) = g_{\bar{m},k}(p_k^*(\bar{m})) > g_{\bar{m},k}(\widetilde{p}_k(\bar{m}))$, (recall that $p_k^*(\bar{m})$ is the unique global maximizer of $g_{\bar{m},k}(\cdot)$), while any other summand in the left-hand-side of (41) is equal to the corresponding summand in the right-hand-side, because $\widehat{p}_k(m) = \widetilde{p}_k(m)$ for all $m \neq \bar{m}$. Thus, if it exists an $\bar{m} \ : \widetilde{p}_k(\bar{m}) > p_k^*(\bar{m})$, it would be possible to increase the value of the objective of (34) by replacing $\widetilde{p}_k(\bar{m})$ with $p_k^*(\bar{m})$, without violating the power constraint. Clearly, this contradicts the fact that $\widetilde{\boldsymbol{p}}_k$ is a solution of (34). Then, we have $\widetilde{p}_k(m) \leq p_k^*(m)$ for all $m \in \mathcal{F}_k$, and since we have already proved that $p_k^*(m) \leq 2\beta M c_k(m)$ for all $m \in \mathcal{F}_k$, we finally have $\widetilde{p}_k(m) \leq p_k^*(m) \leq 2\beta M c_k(m)$ for all $m \in \mathcal{F}_k$, which is equivalent to saying that $\widetilde{\boldsymbol{p}}_k \in \mathcal{C}$, as we wanted to prove. ■

Proposition 4 ensures that no loss of optimality is incurred in problem (34) by constraining the transmit powers to lie in the set $\mathcal{C}$, and that within such a set, the objective function is concave. Therefore, the non-convex problem (34) can be recast as the convex problem

$$\begin{cases} \max\limits_{\{p_k(m)\}_{\ell \in \mathcal{F}_k}} \sum\limits_{m \in \mathcal{F}_k} g_{m,k}(p_k(m)) \\ \text{s.t.} \sum\limits_{m \in \mathcal{F}_k} p_k(m) \leq P_{max} \\ \boldsymbol{p}_k \in \mathcal{C} \end{cases} , \qquad (42)$$

which can be solved by means of any standard convex optimization method, such as the interior point algorithm, or numerically solving the associated KKT conditions, [21]. To summarize, for all $k = 1, \ldots, K$, player k determines his best response according to the following steps:

1) Choose one of the possible sets $\mathcal{F}_k$, and compute the associated coefficients $a_k(m)$ and $c_k(m)$, $m \in \mathcal{F}_k$.
2) Maximize the resulting utility (30) with respect to the transmit powers $\{p_k(m)\}_{m \in \mathcal{F}_k}$, subject to the power constraint $\sum\limits_{m \in \mathcal{F}_k} p_k(m) \in [0, P_{\max}]$, by solving the convex problem (42).
3) Repeat steps 1) and 2) for each of the possible $\binom{N}{L}$ sets $\mathcal{F}_k$, and choose the set and the associated transmit powers that yield the largest utility.

C. Suboptimal implementations and a simple upper bound

Similarly to the discussion of Section IV.A, also in this case approximate implementations are needed in order to avoid the $\binom{N}{L}$ computational complexity of subcarrier allocation in the maximization step of the games in the previous section.

The techniques of Section IV.A can be applied to the case at hand too, so a possible strategy is to resort to a search on a reduced set of configurations. Again, the search may be either made over a certain number, say Q, of randomly selected configurations, or assuming that each user selects its subcarriers by choosing L out of its $\widetilde{N}$ best[7] subcarriers.

An approximated game may be then obtained by constraining each user to transmit on its L best channels, so that only transmit power is to be tuned, which can be done according to the game of Section V. Otherwise stated, in situations where computational complexity is a critical issue, one could skip the subcarrier allocation phase, by letting the users transmit on their best channels, and just focus on transmit power control. We will see in the forthcoming section on numerical results that this strategy provides quite satisfying results in lightly to moderately loaded systems.

[7]Here best means with the largest channel coefficients.

Finally, also in this case a simple upper bound to the achieved energy efficiency of any resource allocation procedure may be obtained assuming that there is no interference, each user transmits on its best channels, and on each channel the transmit power control is set so as to achieve the target SINR γ^* (in case that such a SINR cannot be reached the transmit powers are obviously to be determined so as to satisfy the power constraint as explained in Sections VI-A or VI-B, according to which power constraint is enforced).

D. Two social optimum solutions

For the sake of comparison, in the following we derive alternative power control strategies based on cooperative optimization and inspired by social optimum solutions [19], wherein global (and not user-centric) performance measures are optimized subject to some fairness constraints. We assume that each user transmits on its L best channels (i.e., we skip the subcarrier assignment task); although this may seem a limiting assumption, numerical results shown in the following will confirm that, when subcarrier allocation takes place, with high probability each user ends up transmitting on his best channels.

To begin with, we consider here the centralized (i.e., cooperative) social optimum solution, which is the solution to the following optimization problem:

$$\max_{\{p_k(\ell)\}_{k=1,\dots,K,\ \ell\in\mathcal{F}_k}} \sum_{k=1}^{K}\sum_{\ell\in\mathcal{F}_k} u_{k,\ell}(p_k(\ell)) \; . \tag{43}$$

Substituting $u_{k,\ell}(p_k(\ell))$ with its expression in (16), and denoting by $\mathcal{U}_\ell$ the set of users transmitting on the ℓ-th subcarrier, the above optimization problem may be re-stated as

$$\begin{aligned}\max_{\{p_k(\ell)\}_{\ell=1,\dots,N,\ k\in\mathcal{U}_\ell}} \sum_{\ell=1}^{N}\sum_{k\in\mathcal{U}_\ell} R\frac{D}{M}\frac{f(\gamma_{k,a_k}(\ell))}{p_k(\ell)} = \\ \sum_{\ell=1}^{N} \max_{\{p_k(\ell)\}_{k\in\mathcal{U}_\ell}} \sum_{k\in\mathcal{U}_\ell} R\frac{D}{M}\frac{f(\gamma_{k,a_k}(\ell))}{p_k(\ell)} \; .\end{aligned} \tag{44}$$

The equality in (44) reveals that the considered optimization problem decouples in N separate optimization problems, one for each of the available subcarriers: for the ℓ-th subcarrier, the powers $p_k(\ell)$, with $k\in\mathcal{U}_\ell$ are to be found so as to maximize the quantity

$$\sum_{k\in\mathcal{U}_\ell} \frac{f(\gamma_{k,a_k}(\ell))}{p_k(\ell)} \; ; \tag{45}$$

such a maximization can be carried out through a numerical search on the domain $[0, P_{\max}]^{|\mathcal{U}_\ell|}$.

Another possible resource allocation strategy may be obtained by looking for a social optimum solution with a fairness constraint. More precisely, following [5], we assume that all the users transmitting on a given subcarrier, say the ℓ-th one, are received with the same power $P_R(\ell)$, $\forall \ell = 1,\dots,N$ (thus leading to $p_k(\ell) = P_R(\ell)/h^2_{k,a_k(\ell)}$), and that there is no maximum transmit power constraint. Based on the above assumptions, a social optimum solution is obtained through maximization of the following quantity:

$$\sum_{\ell=1}^{N}\sum_{k\in\mathcal{U}_\ell} R\frac{D}{M}\frac{\left(1-e^{-\gamma_{k,a_k}(\ell)}\right)^M}{p_k(\ell)} \; . \tag{46}$$

Given the orthogonal nature of different subcarriers, the problem decouples in N separate maximization problems, one for each subcarrier. On the ℓ-th subcarrier, recalling that $p_k(\ell) = P_R(\ell)/h^2_{k,a_k}(\ell)$, and given the fact that the SINR $\gamma_{k,a_k(\ell)}$ can be easily shown to be expressed as

$$\gamma_{k,a_k}(\ell) = \frac{P_R(\ell)}{\sigma^2 + \displaystyle\sum_{j\in\mathcal{U}_\ell-\{k\}} P_R(\ell)\frac{h^2_{j,a_k}(\ell)}{h^2_{j,a_j}(\ell)}} \tag{47}$$

the social optimum solution is obtained by maximizing the quantity

$$\sum_{k\in\mathcal{U}_\ell} h^2_{k,a_k}(\ell)\frac{\left[1-\exp\left\{-\frac{P_R(\ell)}{\sigma^2+\sum_{j\in\mathcal{U}_\ell-\{k\}} P_R(\ell)\frac{h^2_{j,a_k}(\ell)}{h^2_{j,a_j}(\ell)}}\right\}\right]^M}{P_R(\ell)} \tag{48}$$

with respect to $P_R(\ell)$. Once the maximizer $P_R^*(\ell)$ of (48) has been found, the users in the set $\mathcal{U}_\ell$ tune their transmit power as follows: $p_k(\ell) = P_R^*(\ell)/h^2_{k,a_k(\ell)}$.

VII. Numerical results

We provide here some numerical results showing the performance, at the NE, of the proposed non-cooperative resource allocation games. We consider a square area of 2800×2800 square meters, with four BSs (each equipped with an omnidirectional antenna) regularly placed at points with coordinates $(700, 700)$, $(700, 2100)$, $(2100, 700)$, and $(2100, 2100)$, and users placed randomly inside this area, subject to the constraint that the distance of each user from the BSs is not smaller than 20m. It is assumed that the number of available subcarriers is $N = 10$, and that each user transmits on $L = 3$ subcarriers. A system with a transmit rate $R = 100$ kbit/s, packet length $M = D = 120$, $\frac{P_{\max}}{L} = -10$ dBW, and $\sigma^2 = 10^{-11}$ Watt/Hz, has been considered. The squared channel coefficient $h^2_{i,j}(\ell)$ linking the i-th user with the j-th BS on the ℓ-th carrier frequency is an exponential random variate with mean $d_{i,j}^{-3}$, with $d_{i,j}$ the distance between the i-th user and the j-th BS. With regard to the game of Section IV we report, as performance measure, the average achieved SINR, at the equilibrium, versus the number K of active users. The average achieved SINR is the mean of the SINRs of the users on all their used subcarriers, i.e.

$$\frac{1}{KL}\sum_{k=1}^{K}\sum_{\ell\in\mathcal{F}_k} \gamma_{k,a_k}(\ell) \; ; \tag{49}$$

although we would be interested in the SINR achieved by each user on its used subcarriers, we believe that the average achieved SINR is a compact performance measure able to give a reliable picture of how well the proposed resource allocation algorithms perform. We assume that, at the beginning of each

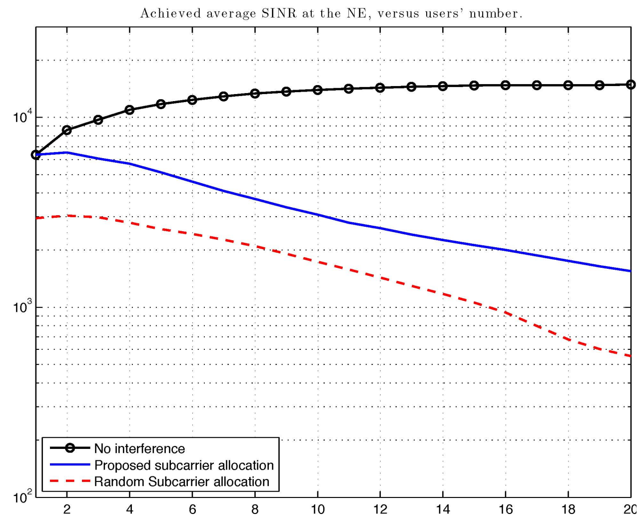


Figure 2. Average achieved SINR at the equilibrium, versus the number of active users, for the game of Section IV.

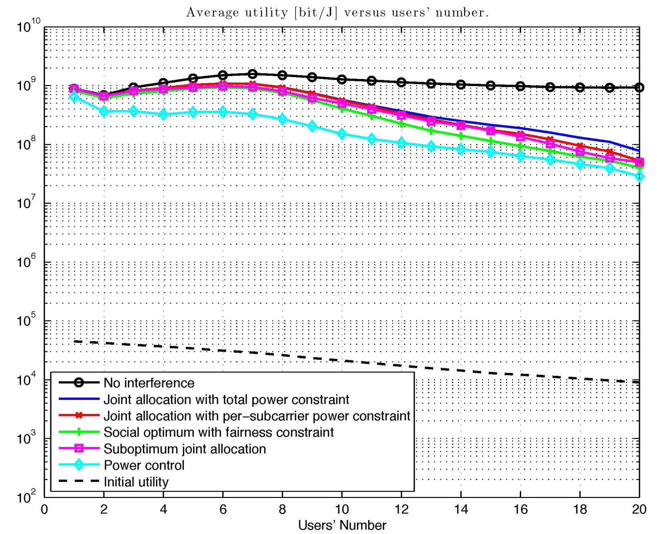


Figure 3. Average achieved energy efficiency at the equilibrium, versus the number of active users, for the games of Sections V and VI.

game, users are assigned and L random subcarriers and random transmit powers (one for each transmit subcarrie satisfying the transmit power constraint. Each user is assign to the BS with the best channel coefficients, and the sho plots are the result of an average over $2 * 10^4$ independe realizations of the channel coefficients and users' locatio Fig. 2 reports the performance of the game considered Section IV; for comparison purposes, we also report t initial average SINR, resulting from a random choice of t subcarriers to be used, and the upper bound correspondi to the case that each user transmits on his L best chann assumed free of interference. Results clearly show that t achieved SINR at the equilibrium, for both the conside games, are largely superior to those achieved with the init random assignment. It is also worth pointing out that t simulation scenario is extremely overloaded, since the pl are shown up to $K = 20$ users, and the product $KL =$ against a total of only $N = 10$ available subcarriers.

Fig. 3 reports the performance for the energy efficiency games of Sections V and VI. When considering the games of Section V (no subcarrier allocation), we assume that users choose, at the beginning of the game, their active subcarriers in a random fashion. For comparison purposes, we also report: (a) the initial average energy efficiency, namely the achieved performance before the resource allocation scheme comes into play, (b) the energy efficiency of an ideal system wherein each user may transmit on his L best channels with no interference, (c) the approximate implementation (with the constraint on the total transmitted power of each user) wherein the maximization step with respect to the subcarrier choice is made considering only the $\widetilde{N} = 5$ best carriers for each user, and (d) the performance of the social optimum solution with the fairness constraint. Results show that, in the considered scenario, the joint power control and subcarrier allocation games achieve an energy efficiency that is roughly three times

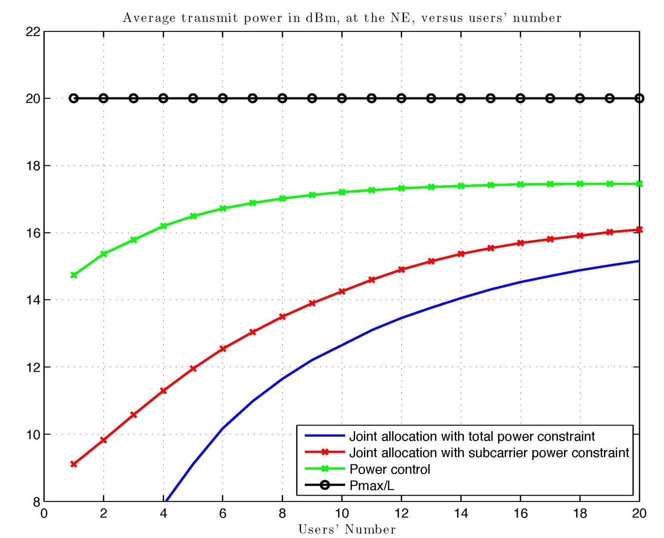


Figure 4. Average transmit power per subcarrier at the equilibrium, versus the number of active users, for the games of Sections V and VI.

larger than the energy efficiency of the power control game (the one of Section V). Also, among the two proposed joint allocation games, the one with a power constraint on the total transmitted power of each user performs better than that with per-subcarrier power constraint, especially for an increasingly number of active users. This is expected, since, as already noted, per-subcarrier power constraint is a particular case of the more general constraint on the total transmitted power. However, especially for low and intermediate network load, it is seen that the gap between the two games is negligible. Thus, the computationally simpler per-subcarrier power constraint proves to be a valid substitute of the more complex game with a constraint on the total power of each user. Interestingly, it is also seen that the approximate implementation achieves a

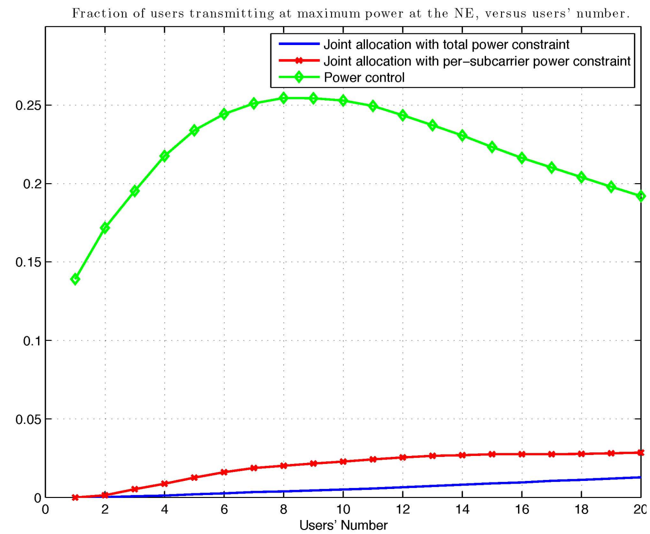


Figure 5. Fraction of users transmitting at maximum power at the equilibrium, versus the number of active users, for the games of Sections V and VI.

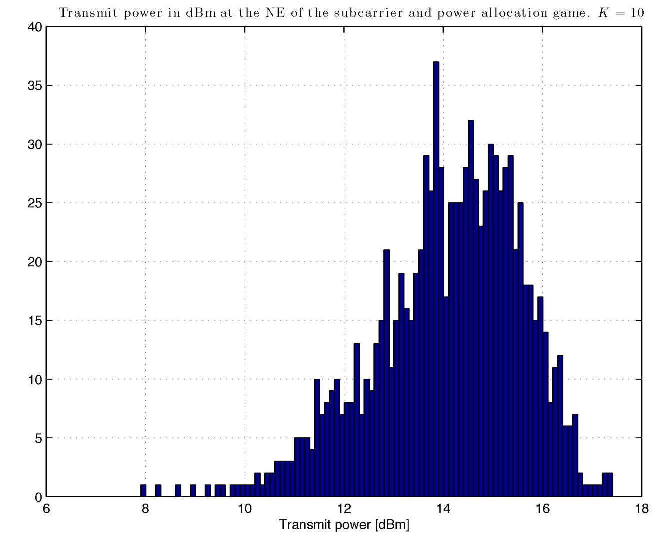


Figure 6. Empirical distribution (at the NE) of the transmit power for each user, for $K = 10$, for the game of Section VI with per-subcarrier power constraint.

performance that is very close to that of the optimum soluti thus showing that close-to-optimality results may be obtain also with a reduced complexity, as well as that the soc optimum solution with the fairness constraint achieves simi results. In Fig. 4 the average transmit power per subcarrier the NE is reported for the games of Sections V and VI. expected, it is seen that the needed transmit power increa as the number of users increases, as well as that the jo power control and subcarrier allocation game permits savi a significant amount of power with respect to the pov control game of Section V, while at the same time granting larger energy efficiency (Fig. 3). It is also seen that wher power constraint on the total transmitted power of each user enforced, better performance is obtained with respect to p subcarrier power constraint. Fig. 5 reports, again for the gar of Sections V and VI, the fraction of users that at the NE a transmitting at the maximum power: these are the users that are not able to reach the optimal target SINR and are thus suffering not-optimal performance. It is seen that this fraction is sufficiently small, especially for the games of Section VI. Moreover, it is seen that joint allocation with per-subcarrier power constraint has only a slight gap with respect to the joint allocation with the total transmitted power constraint.

Now, since the results shown in Figs. 2 - 5 refer to averaged values, in Fig. 6 we look at how the actual values may be spread around the average value. In particular, we report the histogram (i.e., the empirical distribution) of the transmit power realizations, at the NE, for the case in which $K = 10$, for the joint subcarrier and power allocation game of section VI, with subcarrier-based power constraint. Results show that the actual values of the transmit powers are quite concentrated around their average values, and that heavy tails are absent. Similar results, that are omitted for the sake of brevity, show that similar results hold for the other proposed games.

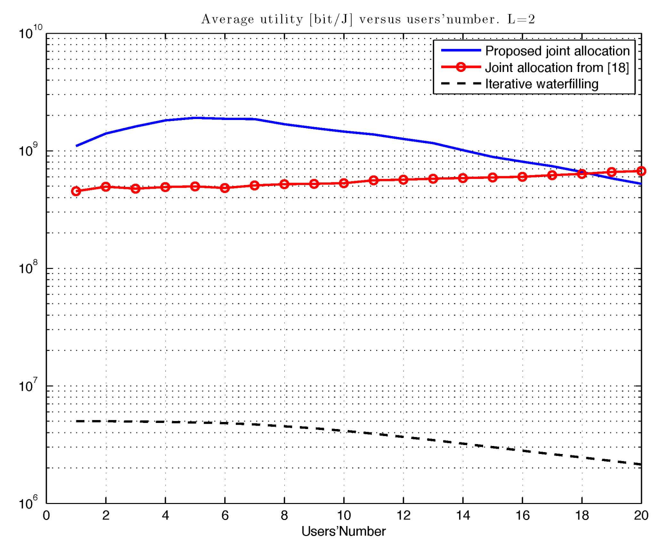


Figure 7. Average achieved energy efficiency at the equilibrium, versus the number of active users, for the proposed subcarrier and power allocation game of Section VI with total power control, for the joint allocation algorithm provided in [18], and for the iterative-waterfilling algorithm.

A. A performance comparison with competing alternatives

Although we have not been able to find in the literature papers dealing with a scenario similar to the one that we are considering, i.e., non-cooperative energy efficiency maximization in the uplink of a multicell OFDMA cellular system, in Fig. 7 we compare the performance of the resource allocation game of Section VI with some similar alternatives. We set $L = 2$, while the other parameters are the same as for previous figures, and we contrasted the average energy-efficiency achieved by the following algorithms:

(a) Our proposed game of section VII, which performs non-cooperative joint subcarrier assignment and power control for energy efficiency maximization, with a power constraint on the total power transmitted by each user on its subcarriers.

(b) The energy-efficiency maximization algorithm proposed in [18]; note that this algorithm considers a scenario very similar to ours, with the only difference that the subcarrier assignment is made in a centralized fashion. Otherwise stated, the paper [18] considers energy efficiency maximization in a multicell uplink OFDMA system wherein power control is performed non-cooperatively, but subcarrier assignment is a centralized task carried out by the network. In particular, the algorithm in [18] assigns the subcarrier with the constraint that the same subcarrier can not be used by more than one user per cell, thus completely suppressing intra-cell interference. Clearly, this is possible only because a centralized subcarrier assignment has been performed. Instead, in our work we do not pose such a constraint, and allow for universal frequency reuse even inside each cell. As the authors of [18] state themselves, the constraint to use each subcarrier only once per cell, may result in unfair situations in which some users are not assigned any transmit subcarrier. To cope with this issue, in [18] a parameter $\lambda \in [0;1]$ is introduced in the subcarrier assignment subroutine, which trades off between network's overall energy-efficiency, and fairness. When $\lambda = 1$, the best performance in terms of average energy-efficiency is obtained, but the resulting subcarrier assignment is most unfair, whereas the opposite situation takes place when $\lambda = 0$. In Fig. 7, the performance of [18] are plotted for $\lambda = 1$, i.e. in the best case as far as average energy-efficiency is concerned.

(c) In order to point out how the classical rate maximizing algorithms totally disregard the energy-efficiency issue, we consider a system wherein each user transmits on the subcarriers selected at the equilibrium by our algorithm (a), but each user sets his L transmit powers in order to maximize the sum of his own achievable rates on the used subcarrier, which has been done by means of the standard iterative water-filling algorithm. Otherwise stated, algorithms (a) and (c) use the same subcarriers (as decided by algorithm (a)), but the transmit power is assigned according to different policies. We point out that, iterative water-filling is known to be not always convergent in multicell systems (see for example [16] and references therein), and for this reason in our simulations we averaged only with respect to those channel realizations that led to a convergent process.

Remarkably, even if the parameter λ of the algorithm in [18] has been set to 1, the results indicate that the proposed algorithm from Section VI performs better than the algorithm from [18], even in moderately overloaded networks, which shows how the proposed method is able to ensure remarkable overall energy efficiency performance, despite being a totally non-cooperative algorithm. Of course, as the number of active users K increases, the gap between the two algorithms tends to disappear, and for heavily overloaded networks (recall that in the presented numerical results, the number of total available subcarriers in the system has been set to $N = 10$, while up to $K = 20$ users are considered, each one transmitting L streams) the algorithm from [18] performs slightly better. This is expected, since for increasing K, our approach has to cope with a stronger and stronger intra-cell interference, whereas in [18] intra-cell interference is suppressed thanks to the centralized subcarrier assignment and at the expense of fairness. Finally, as expected, the iterative waterfilling algorithm performs poorly in terms of energy-efficiency.

VIII. Conclusions

This paper has dealt with the problem of non-cooperative resource allocation in the uplink of OFDMA multicell networks. Using a game-theoretic approach, we have considered both the problems of SINR maximization with respect to the active subcarriers, and of energy efficiency maximization, with respect to the transmit powers and to the choice of the active subcarriers. Differently from the usual approach in the literature, we have posed no constraint on the subcarrier choice algorithm (so that a user is allowed to pick a subcarrier used by another user in the same cell), namely a system with frequency-reuse factor equal to one has been considered. The framework of potential games has been used as a tool to obtain non-cooperative games convergent to a NE, although, on the contrary, it has not been possible to show uniqueness of the NE for the considered games. Numerical results, showing the performance at the NE of the proposed resource allocation schemes, have proven the effectiveness of the proposed solutions.

Acknowledgement

The authors are grateful to the Editor, Prof. Erik Larsson, and to the reviewers; their comments and constructive criticism helped to improve the quality of the paper.

References

[1] D. Goodman and N. Mandayam, "Power control for wireless data," *IEEE Personal Commun.*, Vol. 7, pp. 48-54, April 2000.

[2] Special Issue on "Game theory in communication systems," *IEEE J. Select. Areas Commun.*, Vol. 26, September 2008.

[3] C. U. Saraydar, N. B. Mandayam, and D. J. Goodman, "Efficient power control via pricing in wireless data networks," *IEEE Trans. Commun.*, vol. 50, pp. 291–303, Feb. 2002.

[4] C. U. Saraydar, "Pricing and power control in wireless data networks," Ph.D. dissertation, Dept. Elect. Comput. Eng., Rutgers University, Piscataway, NJ, 2001.

[5] F. Meshkati, H. V. Poor, S. C. Schwartz, and N. B. Mandayam, "An energy-efficient approach to power control and receiver design in wireless data networks," *IEEE Trans. Commun.*, vol. 53, no. 11, pp. 1885 - 1894, Nov. 2005.

[6] S. Buzzi and H.V. Poor, "Joint receiver and transmitter optimization for energy-efficient CDMA communications," *IEEE J. Sel. Areas Commun., Special Issue on Multiuser Detection for Advanced Communications Systems and Networks*, vol. 26, pp. 459 - 472, April 2008.

[7] S. Buzzi, H. V. Poor, and A. Zappone, "Transmitter waveform and widely-linear receiver design: Non-cooperative games for wireless multiple-access networks," *IEEE Trans. Inf. Th.*, Vol. 56, pp. 4874 - 4892, October 2010.

[8] S. Buzzi, "Joint waveform adaptation and power control in multicell wireless data networks: A game-theoretic approach," *Proc. of the 8th Intl. Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOpt '10)*, Avignon (France), May 2010.

[9] C. Wong et al., "Multiuser OFDM with adaptive subcarrier, bit, and power allocation," *IEEE Journal on Selected Areas in Communications*, Vol. 17, pp. 1747 - 1758, Oct. 1999.

[10] J. Jang and K. Lee, "Transmit power adaptation for multiuser OFDM systems," *IEEE Journal on Selected Areas in Communications*, vol. 21, pp. 171 - 178, Feb. 2003.

[11] S. V. Hanly, L. L. H. Andrew, and T. Thanabalasingham, "Dynamic allocation of subcarriers and transmit powers in an OFDMA cellular network," *IEEE Transactions on Information Theory*, vol. 55, pp. 5445 - 5462, Dec. 2009.

[12] K. Kim, Y. Han, and S. Kim, "Joint subcarrier and power allocation in uplink OFDMA systems," *IEEE Communications Letters*, vol. 9, pp. 526 - 528, June 2005.

[13] M. Moretti and A. Todini, "A resource allocator for the uplink of multicell OFDMA systems," *IEEE Trans. Wireless Communications*, vol. 6, pp. 2807 - 2812, Aug. 2007.

[14] Z. Han, Z. Ji, and K. Liu, "Non-cooperative resource competition game by virtual referee in multicell OFDMA networks," *IEEE Journal on Selected Areas in Communications*, vol. 25, pp. 1079 - 1090, Aug. 2007.

[15] K. Yang, N. Prasad, and X. Wang, "An auction approach to resource allocation in uplink OFDMA systems," *IEEE Trans. Sig. Proc.*, vol. 57, pp. 4482 - 4496, Nov. 2009.

[16] G. Scutari, D. P. Palomar and S. Barbarossa, "Asynchronous iterative water-filling for Gaussian frequency-selective interference channels," *IEEE Trans. Inf. Th.*, Vol. 54, pp. 2868 - 2878, Jul. 2008.

[17] P. von Wrycza, M.R.B. Shankar, M. Bengtsson and B. Ottersten, "Game theoretic approach to spectrum allocation for weak interference systems," *Proc. of the 2008 IEEE Global telecomm. Conf. (GLOBECOM 2008)*, New Orleans (LA), Dec. 2008.

[18] G. Bacci, A. Bulzomato and M. Luise, "Uplink power control and subcarrier assignment for an OFDMA multicellular network based on game theory," *Proc. Int. Conf. on Performance Evaluation Methodologies and Tools (ValueTools)*, Paris, France, May 2011.

[19] D. Fudenberg and J. Tirole, *Game Theory*, Cambridge, MA: MIT Press, 1991.

[20] D. Monderer and L. Shapley, "Potential games," *J. Games Economic Behavior*, Vol. 14, no. 0044, pp. 124 - 143, 1996.

[21] S.P. Boyd, and L. Vandenberghe, *Convex optimization*, Cambridge University Press, 2004.

[22] R. Menon, et al., "Interference avoidance in networks with distributed receivers," *IEEE Trans. Commun.*, Vol. 57, pp. 3078 - 3091, October 2009.

[23] R. Menon, et al., "A game-theoretic framework for interference avoidance," *IEEE Trans. Commun.*, Vol. 57, pp. 1087 - 1098, Apr. 2009.

[24] G. Scutari, S. Barbarossa, and D. P. Palomar, "Potential games: a framework for vector power control problems with coupled constraints," *Proc. of the 2006 IEEE Int. Conf. Acoustics, Speech and Sig. Proc.*, Toulouse (France), May 2006.

[25] Q. Zhu, "A Lagrangian approach to constrained potential games: Theory and examples," *Prof. of the 47th IEEE Conf. on Decision and Control*, Cancun (Mexico), Dec. 2008.

[26] E. Altman, Y. Hayel, and H. Kameda, "Evolutionary dynamics and potential games in non-cooperative routing," *Proc. of the 5th Int. Symp. on Modeling and optimization in Mobile, Ad Hoc and Wireless Networks*, Limassol (Cyprus), Apr. 2007.

[27] L. Rose, S. M. Perlaza and M. Debbah, "On the Nash equilibria in decentralized parallel interference channels," *Proc. of the 2011 IEEE International Conference on Communications ICC '11*, Kyoto (Japan), June 2011.

[28] F. Meshkati, M. Chiang, H. V. Poor and S. C. Schwartz, "A game-theoretic approach to energy-efficient power control in multicarrier CDMA systems," *IEEE J. Select. Ar. Commun.*, Vol. 24, pp. 1115 - 1129, Jun. 2006.

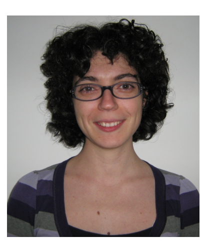
[29] C. U. Saraydar, N. B. Mandayam and D. J. Goodman, "Pricing and power control in a multicell wireless data network," *IEEE J. Sel. Areas Commun.*, Vol. 19, pp. 1883 - 1892, Oct. 2001.

Stefano Buzzi (M'98, SM '07) was born in Piano di Sorrento, Italy on December 10, 1970. He received with honors the Dr. Eng. degree in 1994, and the Ph.D. degree in Electronic Engineering and Computer Science in 1999, both from the University of Naples Federico II. In 1996 he spent six months at *CSELT* (Centro Studi e Laboratori Telecomunicazioni), Turin, Italy, while he has had short-term visiting appointments at the Dept. of Electrical Engineering, Princeton University, in 1999, 2000, 2001 and 2006. He is currently an Associate Professor at the University of Cassino, Italy. His current research and study interests lie in the area of statistical signal processing and resource allocation for wireless communications, satellite navigation, and radar applications. Dr. Buzzi was awarded by the AEI (Associazione Elettrotecnica ed Elettronica Italiana) the "G. Oglietti" scholarship in 1996, and was the recipient of a NATO/CNR advanced fellowship in 1999 and of three CNR short-term mobility grants. He has been author/coauthor of more than 40 journal papers and more than 70 conference papers, and is a former Associate Editor for the *IEEE Communications Letters*, the *IEEE Signal Processing Letters* and the *Journal of Communications and Networks*.

Giulio Colavolpe (S'96-M'00-SM'11) was born in Cosenza, Italy, in 1969. He received the Dr. Ing. degree in Telecommunications Engineering (cum laude) from the University of Pisa, in 1994 and the Ph.D. degree in Information Technologies from the University of Parma, Italy, in 1998. Since 1997, he has been at the University of Parma, Italy, where he is now an Associate Professor of Telecommunications at the Dipartimento di Ingegneria dell'Informazione (DII). In 2000, he was Visiting Scientist at the Institut Eurécom, Valbonne, France. His research interests include the design of digital communication systems, adaptive signal processing (with particular emphasis on iterative detection techniques for channels with memory), channel coding and information theory. His research activity has led to more than 150 papers in refereed journals and in leading international conferences, and 15 industrial patents.

He received the best paper award at the 13th International Conference on Software, Telecommunications and Computer Networks (SoftCOM'05), Split, Croatia, September 2005, the best paper award for Optical Networks and Systems at the IEEE International Conference on Communcations (ICC 2008), Beijing, China, May 2008, and the best paper award at the 5th Advanced Satellite Mobile Systems Conference and 11th International Workshop on Signal Processing for Space Communications (ASMS&SPSC 2010), Cagliari, Italy.

He is currently serving as an Editor for IEEE Transactions on Wireless Communications, and IEEE Wireless Communications Letters and as an Executive Editor for Transactions on Emerging Telecommunications Technologies (ETT).



Daniela Saturnino was born in Formia, Italy, on December 1st, 1981. She received the B. S. and M. S. degrees with honors in Telecommunications Engineering, in 2004 and 2006, respectively, both from the University of Cassino, Italy. She received the Ph.D. degree in Electrical and Information Engineering in 2010 from the University of Cassino, Italy. In the period June 2010- April 2011 she has been a post-doc researcher at *Scuola Superiore Sant'Anna*, Pisa, Italy, working on the application of game theory to resource allocation problems.

Alessio Zappone received the B.S. and M.S. degrees (with honors) in telecommunications engineering from the University of Cassino, Italy, in 2005 and 2007, respectively, and the Ph.D. degree in Electrical and Information Engineering from the University of Cassino, in 2011. He is currently with Consorzio Nazionale Interuniversitario per le Telecomunicazioni (CNIT), working as a Post-Doctoral Researcher and participating in the FP7 EU-funded project TREND on energy-efficient networking. His research interests lie in the areas of wireless communication systems and game theory applied to resource allocation problems. He has held research appointments with the Technical University of Dresden and with Politecnico di Torino.