

A MODEL FOR THE GENERALIZED DROOP FORMULA

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Abstract

We provide a new analytical model that fully justifies the recently disclosed Generalized Droop Formula of the nonlinear signal-to-noise (SNR) ratio in very-long submarine links with power-mode amplifiers, and show its relation with the Gaussian-Noise model SNR.

1 Introduction

It is well known that amplified spontaneous emission (ASE) induces a droop of the desired signal power at the end of a very long submarine link with end-span power-mode amplifiers, thus reducing the received signal to noise ratio (SNR) [1, 3, 4]. The generalized droop formula (GDF) for the received SNR has recently been heuristically introduced to also account for the effect of nonlinear interference (NLI) on droop [1, 2]. It is the purpose of this paper to provide a solid mathematical model that fully justifies the GDF and allows a direct comparison with the SNR provided by the Gaussian Noise (GN) model [5, 6].

2 Droop in power-mode amplifiers

We begin with the droop induced by ASE only. Consider the transmission of a single optical channel (normally a wavelength division multiplexed (WDM) comb: amplifier gain is assumed to be flat over the comb) of power P along a chain of N identical fiber spans. The k -th span has fiber span loss (in linear units) $\mathcal{L} < 1$. The span is followed by an optical amplifier operated in power mode that makes the line transparent, i.e., its output power is again P . Since $P\mathcal{L}$ is the power entering the amplifier, the gain G is such that $P = (P\mathcal{L} + \mu_a)G$, where $\mu_a = h\nu F_N B_{amp}$ is the equivalent input power of the ASE noise generated within the amplifier bandwidth B_{amp} ($h\nu$ is the photon energy at the center frequency ν ; F_N is the amplifier noise figure, which for large-enough gain G is independent of the gain itself). Hence $G = \frac{P}{P\mathcal{L} + \mu_a} = \mathcal{L}^{-1}\chi_a$ where

$$\chi_a \triangleq \left(1 + \frac{\beta}{P}\right)^{-1} \quad (1)$$

is the gain droop due to ASE, with $\beta = \mu_a\mathcal{L}^{-1}$ the per-span output ASE that would be generated in absence of droop.

What is happening is re-interpreted graphically in Fig. 1(top). The droop term $\chi_a < 1$ pops out to attenuate the signal power P entering the k -th line+amplifier span (sum of

desired signal $P_s(k-1)$ and cumulated ASE $P_a(k-1)$) down to $\chi_a P$ in order to make room for the locally-generated ASE $\delta P_a = \mu_a G \equiv \beta\chi_a$ and meet the output power constraint P , so that $P\chi_a + \delta P_a = P$. This is an equation in the variable χ_a whose solution is (1).

Note that the expressions of gain and droop remain the same if we interpret P as the per-channel power in a C -channel single-mode fiber WDM system and μ_a is calculated on the per-channel bandwidth B_{amp}/C . Similarly, if propagation is on multi-mode fibers where modal loss and amplification are identical for all modes, then again P is interpreted as the per-mode per-channel power. Thus for every channel and mode, the SNR at the output of the N -span chain is obtained by noting that $P\chi_a^N$ is the received signal power, and thus $P(1 - \chi_a^N)$ must be the ‘‘droop aware’’ received ASE power, so that one gets [1, 2]: $\text{SNR} = 1/(\chi_a^{-N} - 1)$.

2.1 Droop induced by NLI

NLI-induced signal depletion, or droop, was already empirically tackled in [7] with gain-mode amplifiers. We show next that with power-mode amplifiers the treatment of NLI-induced gain droop can be nicely integrated with the ASE-induced gain droop, if we assume that the NLI contributions of the various spans are uncorrelated, as in the incoherent GN model.

The logical reasoning goes as depicted graphically in Fig. 1(bottom). The initial powers at output of amplifier $k-1$ (input of k -th fiber span) are $P_s(k-1) + P_a(k-1) + P_n(k-1) = P$. Nonlinearity operates a redistribution of the power P : it generates NLI as $\delta P_n = \alpha_{\text{NL}}P^3$ [5, 6], where α_{NL} is the per-span NLI coefficient, and rescales all power components entering the fiber $P_s(k-1), P_a(k-1), P_n(k-1)$ by the same multiplicative scaling factor $\chi_n < 1$ such that power

*It is tacitly assumed that WDM signal and ASE occupy the same bandwidth.

is redistributed but otherwise conserved: $P\chi_n + \delta P_n = P$, i.e.,

$$\chi_n = 1 - \frac{\delta P_n}{P} = 1 - \alpha_{\text{NL}} P^2. \quad (2)$$

Note that *NLI here includes nonlinear interaction of signal with itself, ASE with itself, NLI with itself, signal with ASE, signal with NLI, and ASE with NLI.*

Now the story goes as in the ASE-only previous case: the compound signal (desired plus ASE plus NLI) entering span k sees a gain of $G \triangleq \mathcal{L}^{-1}\chi_a$ (which defines the ASE-induced droop $\chi_a \leq 1$), hence at output of k th amplifier has power $\chi_a P$, which is the ‘‘attenuated output signal component’’ at amplifier k , to which the amplifier-generated output ASE δP_a is added in order to form the amplifier total output power P : $\chi_a P + \delta P_a = P$, leading to the usual expression of the ASE droop (1).

The SNR at the output of the N -span chain is obtained by noting that $P\chi^N$ is the received signal power, where $\chi = \chi_a\chi_n$ is the total signal power droop, and $P(1 - \chi^N)$ is the ‘‘droop aware’’ received ASE+NLI power, so that one gets the generalized droop formula:

$$\text{SNR}_{\text{GDF}} = \frac{1}{\chi^N - 1} = \frac{1}{\left[\left(\frac{1}{1 - \alpha_{\text{NL}} P^2}\right) \left(1 + \frac{\beta}{P}\right)\right]^N - 1} \quad (3)$$

which coincides with that reported in [1, 2] for small NLI to signal power ratio $\alpha_{\text{NL}} P^2$, i.e., within the first-order perturbative limits of validity of the GN model.

We note that the NLI droop derivation extends verbatim to any power-redistributing effect in the fiber, such as for instance the guided-acoustic wave Brillouin scattering (GAWBS) [2, 9]. Considering both NLI and GAWBS, we find that the *redistribution* droop factor (2) to be used in the GDF-SNR (3) becomes $\chi_n = 1 - \alpha_{\text{NL}} P^2 - \gamma_{\text{GAWBS}} \ell$, where γ_{GAWBS} (km^{-1}) is the GAWBS coefficient and ℓ (km) the span length.

The GDF-SNR (3) should be checked against the GN-model SNR. The GN model is traditionally derived using gain-mode amplifiers, whose gain is $G = \mathcal{L}^{-1}$ and neglects droop. The SNR, assuming incoherent accumulation of NLI, is calculated as

$$\text{SNR}_{\text{GN}} = \frac{P}{\mu_A G N + \alpha_{\text{NL}} P^3 N} = \frac{1}{N} \left(\frac{1}{\frac{\beta}{P} + \alpha_{\text{NL}} P^2} \right) \quad (4)$$

Fig. 2(top) shows both the GDF-SNR eq. (3) (solid line) and the classical GN-SNR (dashed line) plotted versus power P for the QPSK case reported in [1, Fig. 1]. We note that the top SNR values (circles) for both GDF and GN models occur at the same optimal power P_o . However, the tails of the GDF ‘‘bell curves’’ have steeper slopes than their GN counterparts, as well evidenced by their linear and nonlinear asymptotes. The SNR gap from GN to GDF in the top figure is plotted as a solid line against the GN-SNR in Fig. 2(bottom). We see that a significant difference above 0.5dB occurs at all SNRs below 7dB.

Now approximate the total **inverse** droop as

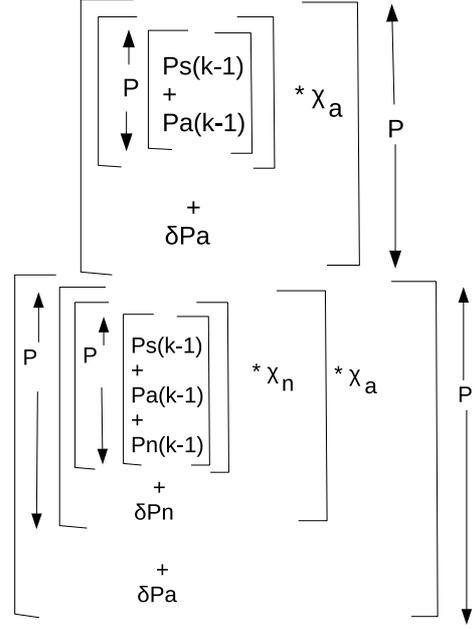


Fig. 1 Origin of droop: **(top)** ASE only. **(bottom)** NLI and ASE. (s,a,n) \rightarrow (signal, ASE, NLI).

$$\chi(P)^{-1} \cong (1 + \alpha_{\text{NL}} P^2) \left(1 + \frac{\beta}{P}\right) \cong 1 + \frac{\beta}{P} + \alpha_{\text{NL}} P^2$$

which is always a reasonable approximation at large enough single-span linear and nonlinear SNR, namely, $\text{SNR}_{1a} \equiv \frac{P}{\beta}$ and $\text{SNR}_{1n} \equiv \frac{1}{\alpha_{\text{NL}} P^2}$. Now let $x \triangleq \frac{\beta}{P} + \alpha_{\text{NL}} P^2$, and expand in Taylor to 2nd order $(1+x)^N - 1 \geq Nx(1 + \frac{1}{2}(N-1)x)$. Then $1/(Nx) \equiv \text{SNR}_{\text{GN}}$ and we get the upper-bound

$$\text{SNR}_{\text{GDF}} \leq \frac{\text{SNR}_{\text{GN}}}{1 + \frac{(1 - \frac{1}{N})}{2\text{SNR}_{\text{GN}}}} \triangleq \text{SNR}_{\text{GDF-UB}} \quad (5)$$

valid at any power P and essentially independent of N . The upper-bound $\text{SNR}_{\text{GDF-UB}}$ in (5) is shown in Fig. 2(top) in red dash-dotted line and is a very good approximation to within 0.5 dB to the true GDF-SNR, as shown by the blue dashed line SNR difference in Fig. 2(bottom).

Also, from (5) we see that the gap from GN to GDF in dB is approximately $10 \log_{10}(1 + (1 - \frac{1}{N})/(2\text{SNR}_{\text{GN}}))$, so that it can be well approximated as

$$\text{SNR}_{\text{GN}}(\text{dB}) - \text{SNR}_{\text{GDF}}(\text{dB}) \cong \frac{5 \log_{10}(e)(1 - \frac{1}{N})}{\text{SNR}_{\text{GN}}} \quad (6)$$

as shown in dashed red line in Fig. 2(bottom).

2.2 Optimal power at max SNR

The optimal power P_o at maximum SNR is obtained in the GN model by setting the derivative of SNR_{GN} w.r.t. P to zero, yielding the condition $\beta = 2\alpha_{\text{NL}} P_o^3$ (i.e., ASE is twice the NLI at

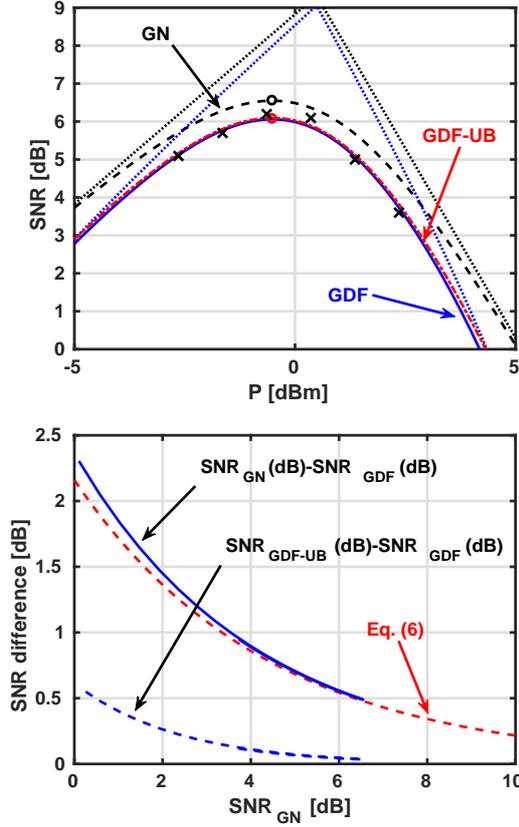


Fig. 2 **(top)** nonlinear SNR(dB) versus power per channel P (dBm) for QPSK link in Fig1.a of [1], with: span length 78 km, fiber loss 0.171 dB/km, amplifier noise figure 8 dB, receiver bandwidth 33 GHz, $N = 228$ spans, $\alpha_{NL} = 4.1 \times 10^{-4} \text{ mW}^{-2}$. Blue solid: GDF-SNR, eq. (3). Black dashed: GN-SNR, eq. (4). Linear and nonlinear asymptotes also shown. Red dash-dotted: GDF-SNR upper-bound (5). **(bottom)** Blue solid line: SNR difference from GN to GDF measured in top figure, plotted versus GN-SNR. Blue dashed line: GDF-UB to GDF-SNR difference in top figure, as per eq. (5). Red dashed line: SNR difference from GN to GDF as per eq. (6).

P_o) and the explicit optimal GN power $P_{oGN} = (\beta/2/\alpha_{NL})^{1/3}$. Maximum GN-SNR is thus

$$\text{SNR}_{oGN} = \frac{1}{N} \frac{1}{3\alpha_{NL}P_{oGN}^2}. \quad (7)$$

Similarly, the GDF-SNR is maximum at the power P_o that maximizes the total droop $\chi(P_o)$, leading to the condition $\beta = \frac{2}{\chi(P_o)}\alpha_{NL}P_o^3$, ie, ASE is *slightly more than twice* the NLI at P_o . This leads to $P_o = P_{oGN}\chi^{1/3} \lesssim P_{oGN}$, thus *the optimal P_o is in practice the same as in the GN case*. Using the bound (6), starting from the top GN-SNR we computed the red circle in Fig. 2, which falls right on top of the maximum of the GDF-SNR curve. Fig. 2(bottom, red-dashed line) shows that predicting the GDF-SNR using the upper-bound (5) based on the GN-SNR is accurate down to SNR values as low as 0 dB.

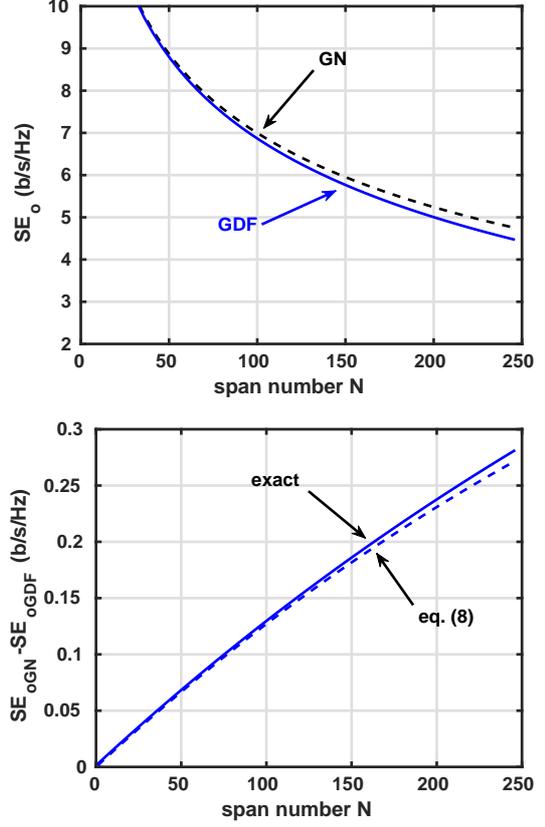


Fig. 3 **(top)** Maximum Spectral efficiency per mode versus span number N . Dashed: GN model. Solid: GDF. Same parameters as in Fig. 2(top). **(bottom)** top SE difference between GN and GDF. Solid: exact; dashed: eq (8).

2.3 Spectral efficiency

A lower bound on the capacity per mode of the nonlinear optical channel for dual-polarization transmissions is obtained from the equivalent white Gaussian noise channel (AWGN) Shannon capacity, i.e., by considering the NLI as an additive white Gaussian process independent of the signal. Hence a lower bound on spectral efficiency per mode is [8]: $\text{SE} = 2 \log_2(1 + \text{SNR})$. Its top value SE_o is achieved at P_o .

Fig. 3(top) reports SE_o versus number of spans N for both GN and GDF using the same data as in Fig. 2. We note that a significant departure from the GN formula occurs only at spectral efficiencies smaller than 5 b/s/Hz. We find that the SE_o gap from GN to GDF is well approximated by the formula

$$\text{SE}_{oGN} - \text{SE}_{oGDF} \cong \frac{2}{\ln(2)} \frac{\text{SNR}_{oGN}}{1 + 2\text{SNR}_{oGN} + 2\text{SNR}_{oGN}^2} \quad (8)$$

which is plotted in Fig. 3(bottom) together with the exact gap. It is seen that the GN model over-estimates SE_o by less than 0.3 b/s/Hz at the longest distance.

3 Conclusions

We have presented an analytical model that fully justifies the generalized droop formula for SNR. We quantified its difference from the GN model SNR, which, for the analyzed link, becomes larger than 0.5 dB only at SNR values below about 7dB, typical of submarine links with hundreds of amplifiers. We note that the NLI droop derivation extends verbatim to any power-redistributing effect in the fiber, such as for instance the GAWBS.

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