The Generalized Droop Model for Optical Long-Haul Transmission Systems

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Abstract *We review the fundamentals of the recently disclosed Generalized Droop model and highlight its use in optimization of long-haul low-SNR space-division multiplexed submarine links.*

Introduction

Optical amplifiers in submarine links are normally operated in constant output power (COP) mode. The link capacity is limited by the available amplifier COP, which has fundamental physical limits due to the line maximum end-to-end electrical voltage. This is the major reason why submarine links will use space division multiplexing (SDM) to make the best use of the available amplifiers power, and thus in turn of the available signal power. The reason is easily understood from the Shannon capacity of the link

$$C = N_m N_c B_c 2\log_2(1+\Gamma\, SNR) \qquad (b/s) \qquad (1)$$

(N_m is the number of spatial modes (i.e., parallel 2-polarization single-mode fibers in a first implementation[1,2], or multicore fibers (MCF) in a second phase[3]), N_c the number of wavelength division multiplexed (WDM) channels of bandwidth B_c on each mode, SNR their received signal to noise ratio, and $\Gamma < 1$ is an SNR penalty often used in design): Instead of logarithmically increasing C by increasing signal power, it is more advantageous to increase C by linearly increasing the number of parallel channels $N_m N_c$[4]. Note that the capacity formula (1) assumes the Gaussian mode/wavelength channels are identical and independent, i.e., there is no multi-input multi-output processing[5] at the receiver.

Therefore in submarine SDM we work at low power and low SNR, such that fiber nonlinear interference (NLI) is of minor concern[6], and amplified spontaneous emission (ASE) noise is the dominant impairment, with the modal crosstalk (XT) in MCF[7] and the guided acoustic-wave Brillouin scattering (GAWBS) noise[8] in each spatial mode being other relevant impairments[9].

It was recently shown that in the low-SNR regime envisaged for long-haul submarine SDM the standard inverse-of-sum-of-inverses SNR accumulation rule ceases to be accurate and a new accumulation rule, known as the generalized-droop (GD) formula, applies[9,10]. The physical reason is the existence of a signal droop due to the COP amplifiers[11] whose study was recently reviewed[12]. The GD formula has been extensively tested and compared to experimental results with proved good accuracy[13–15]. This paper will review the analytical model underlying the GD formula[15–17] and provide hints to its use in the optimization of SDM submarine links.

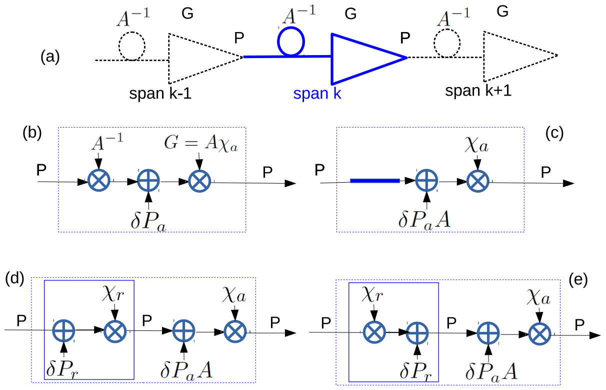

Fig. 1: (a) Generic span k of submarine link with end-span COP amplifier; (b) Span power diagram; (c) Equivalent diagram with attenuation factored out, and ideal fiber (blue thick line); (d) ideal fiber in (c) replaced by add-and-attenuate block; (e) ideal fiber in (c) replaced by attenuate-and-add block. P: saturation output power, $A > 1$: span attenuation, χ_a: addition droop, χ_r: redistribution droop.

GD model

Fig. 1(a) shows the considered physical link made of N_s identical spans with span attenuation $A > 1$ and end-span amplifier with gain G and COP P. The generic span power flow diagram is shown in Fig. 1(b). Here δP_a is the amplifier-input equivalent ASE power, and $\chi_a < 1$ is the net span gain, also called the ASE(-induced) droop. Fig. 1(c) shows an equivalent block diagram where span attenuation is “factored-out”. $\delta P_a A$ is now the span-input equivalent ASE power. Droop exists because of the COP constraint, which from Fig. 1 (c) reads as: $(P + \delta P_a A)\chi_a = P$ and yields the ASE droop expression $\chi_a = (1 + \frac{\delta P_a A}{P})^{-1}$. We define $SNR_{1a} \triangleq \frac{P}{\delta P_a A}$ the SNR degraded by a single amplifier, hence $\chi_a = (1 + SNR_{1a}^{-1})^{-1}$. Once fiber loss is factored out, fiber is an ideal block in Fig. 1(c), indicated by a thick blue line. Indeed several power-conserving noise processes (such as NLI, XT and GAWBS) may take place during fiber propagation causing a rearrangement of power, at constant total power P. To account for them, Fig. 1(d) expands the ideal fiber into an input sub-block where first a rearrangement noise δP_r is added to P and then multiplication by a rearrangement droop $\chi_r < 1$ re-scales the sum to P, thus conserving power at each span. From diagram (d) the power-conservation constraint is: $(P + \delta P_r)\chi_r = P$, which yields[17]: $\chi_r = (1 + \frac{\delta P_r}{P})^{-1}$. An alternative choice is shown in diagram (e) where we first attenuate P by χ_r and then add the perturbation δP_r, getting instead[15,16]: $\chi_r = (1 - \frac{\delta P_r}{P})$. Since the SNR degraded by a single fiber pass $SNR_{1r} \triangleq \frac{P}{\delta P_r}$ is normally several tens of dB, in practice diagrams (d) and (e) yield almost identical results. In diagram (d) the rearrangement block is identical to the amplifier block, which leads to a more elegant SNR expression. In both diagrams (d) and (e) the net span gain (or droop) is $\chi = \chi_r \chi_a$ (in[15] the approximation $\chi \cong 1 - SNR_{1a}^{-1} - SNR_{1r}^{-1}$ is made).

The received (RX) signal after N_s spans is $P\chi^{N_s}$, and by the COP constraint the RX noise is $P(1 - \chi^{N_s})$, hence the GD formula for the RX SNR is (from now on we use diagram (d)):

$$SNR = \frac{1}{[(1 + SNR_{1a}^{-1})(1 + SNR_{1r}^{-1})]^{N_s} - 1}. \quad (2)$$

As long as power and gain are frequency- and mode-flat and all amplifier modes/channels are populated by signals, these are also per-channel SNRs. The GD formula can be generalized to non-homogeneous links by the following cascading formula[10,17] :

$$1 + SNR^{-1} = \prod_{k=1}^{N_s} \left(1 + SNR_{1ak}^{-1}\right)\left(1 + SNR_{1rk}^{-1}\right)$$

and in the "large SNR" regime where we can approximate the above product as $1 + \sum_{k=1}^{N}(SNR_{1ak}^{-1} + SNR_{1rk}^{-1})$ we retrieve the standard SNR cascading formula.

GD formula for general amplifier fill-in factor

Assuming that only signal-carrying modes are amplified, the amplifier fill-in factor is defined as $\eta_A \triangleq (N_c B_c)/B_a \leq 1$, where B_a is the amplifier bandwidth. When this factor is not unity, the standard GD formula (2) must be modified. The corrected formula, called the COP-GD [17, eq. (23)], is complicated because of a span-dependent rearrangement droop due to a span-dependent effective signal power [17, eq. (25)]. By using instead its link-average value

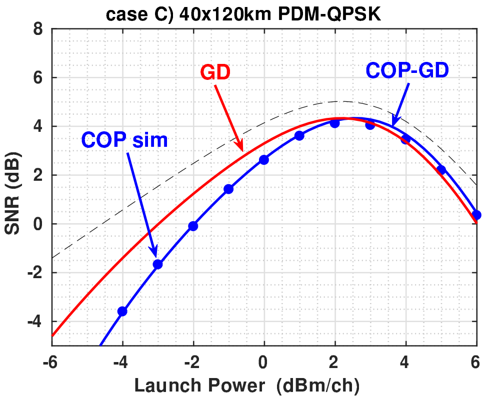

Fig. 2: SNR versus launch power per channel P_c. Data (Case C in[17]): N_c =15 channels, B_c =49GHz, PDM-QPSK, 40x120km NZDSF link. ASE filtered on WDM bandwidth, channel spacing $\Delta f = 100$GHz, yielding $\eta_A = 0.49$. Symbols: simulations with COP saturation power $N_c P_c$. We show: the basic GD (2), and the COP-GD, eq. (4). Dashed line: standard SNR cascading formula.

$$\overline{P}_e = P_c - \frac{\beta(\eta_A^{-1} - 1)}{1 - \chi_a}\left(1 - \frac{1 - \chi_a^{N_s}}{N_s(1 - \chi_a)}\right) \quad (3)$$

we simplify the COP-GD to

$$SNR = \frac{1}{\left[\eta_A + (1 - \eta_A)\frac{\chi_r^{-1} - 1}{\chi^{-1} - 1}\right](\chi^{-N_s} - 1)} \quad (4)$$

where $\chi = \chi_a \chi_r$ and:

1) $\chi_a^{-1} = 1 + \beta/(\eta_A P_c)$ with P_c the launch power per channel, and $\beta \triangleq h f_0 F B_c A$ ($h f_0$ photon energy, F amplifier noise figure, B_c channel band);

2) $\chi_r^{-1} = 1 + SNR_{1r}^{-1}$, where we use $SNR_{1r} = \frac{P_c}{\alpha_{NL}\overline{P}_e^3}$ for NLI (α_{NL} is the link-average per-span NLI coefficient) and $SNR_{1r} = \frac{P_c}{\gamma_x \ell \overline{P}_e}$ for XT/-GAWBS (γ_x is the XT/GAWBS coefficient per km and ℓ(km) the span length). Eq. (4) is essentially coinciding with the complete COP-GD [17, eq. (23)].

An example with a single-mode system with ASE and NLI only is provided in Fig. 2 (this figure corrects the erroneous GD curve reported in[17] Fig. 9). The figure shows the RX SNR vs. launch power per channel P_c in a $N_s = 40$ span non-zero-dispersion shifted (NZDSF) single-mode link with 120km per span with N_c =15 PDM-QPSK

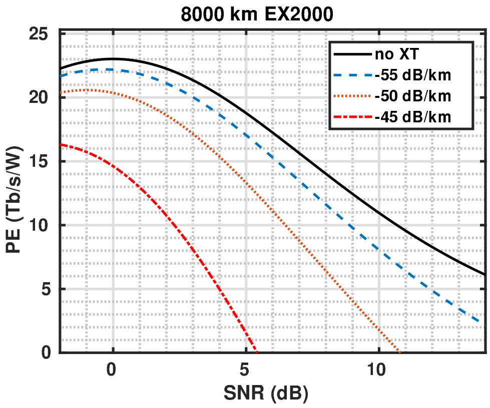

Fig. 3: Power efficiency PE (Tb/s/W) eq. (5) versus received SNR (dB) for 8000km EX2000 link[15] Fig. 7. Solid black: no XT; Dash blue: XT=-55 dB/km; dotted orange: XT=-50 dB/km; dash-dot red: XT=-45 dB/km. Γ=1.

channels at B_c =49GHz (Case C in[17]). The GD formula (2) (same as (4) at $\eta_A = 1$) over-estimates the SNR at low powers (the regime of interest in submarine SDM) and under-estimates the high-power SNR because for NLI evaluation it uses P_c instead of the lower $\overline{P}_e$. We also report in dashed line the standard SNR cascading formula, to show how far it is from reality at low SNR.

Power efficiency optimization

One key parameter for the optimization of submarine SDM systems is power efficiency, which may be defined as[12]: $PE = C/P_{tot}$ where $P_{tot} = N_m N_s P_{sat}$ is the total amplifiers optical output power, with single-mode-amplifier saturation power $P_{sat} = N_c P_c$. We elaborate here on the results in[2,12,13,15] and report a couple of original extensions derived with our GD/COP-GD formulas. When $N_s \to \infty$, (in practice for $N_s \gtrsim 30$ at all relevant SNR), by following the derivation in[12,15], we find that:

1) with ASE and XT at unity fill-in, using the GD SNR:

$$PE \to K \log_2(1+\Gamma SNR) \frac{\ln(1+SNR^{-1}) - N_s\gamma_x\ell}{1+\gamma_x\ell} \tag{5}$$

with $K \triangleq 2/(hf_0\, F\, A\, N_s^2)$. Analysis of the derivative w.r.t. SNR shows that the maximum without XT is reached exactly at[13]: $SNR^{dB} = \Gamma^{dB}/2$ (here $\Gamma^{dB} \triangleq -10\log_{10}(\Gamma)$) and that the more the crosstalk or the span count, the more the SNR maximum is reached before $\frac{\Gamma^{dB}}{2}$.

This is confirmed by Fig. 3 which shows the PE eq. (5) vs. SNR for the 8000km EX2000 link described in[15] Fig. 7, and very well matches the cited figure.

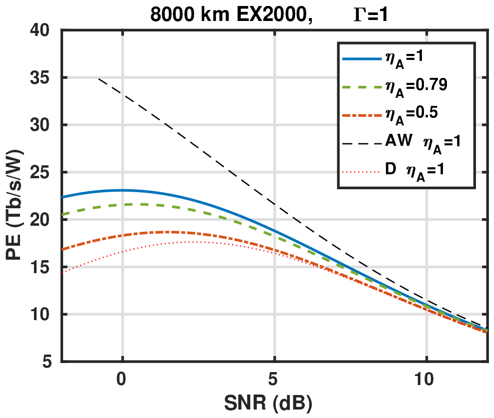

Fig. 4: PE (Tb/s/W) versus received SNR (dB) [eq. (6), thick lines] for 8000km EX2000 link[15] at various fill-in factors $\eta_A = [1, 0.79, 0.5]$ (or equivalently $\eta_A^{dB} = 10\log_{10}(1/\eta_A) = [0, 1, 3]$ dB). Γ=1. Thin dashed (AW) and dotted (D) curves are the AWGN and Original Droop PE curves in [15, Fig. 4b].

2) with ASE only at any fill-in η_A, using the COP-GD SNR:

$$PE \to \eta_A K \log_2(1+\Gamma SNR)\ln(1+(\eta_A SNR)^{-1}) \tag{6}$$

whose maximum occurs exactly at $SNR^{dB} = (\eta_A^{dB} + \Gamma^{dB})/2$, with $\eta_A^{dB} \triangleq -10\log_{10}(\eta_A)$.

Fig. 4 shows PE eq. (6) vs. SNR for the same 8000km EX2000 link[15] at various values of η_A (thick lines), and also reports (thin lines) the $\eta_A = 1$ PE curves in [15, Fig. 4b] for the AWGN and the Original Droop models. The maximum is confirmed to be *exactly* at $\frac{\eta_A^{dB}}{2}$ (here $\Gamma^{dB} = 0$). The whole PE curve is seen to decrease when decreasing η_A. Finally note that when SNR is large, curves for all η_A tend to converge to the $\eta_A = 1$ standard case because ASE saturation is less and less important.

Conclusions

We reviewed and discussed the similarities and differences of two published almost-identical GD models[15,17]. We reviewed (and simplified) the extension of the GD formula (the COP-GD formula[17]) to the case where the WDM signals do no entirely occupy the COP amplifier bandwidth. We finally showed the use of the GD formula in optimizing the “optical” power efficiency in SDM submarine links, and provided two new closed-form expressions for the PE when including modal crosstalk/GAWBS and amplifier fill-in factor. To optimize “electrical” PE, more complex models may be needed to account for the optical amplifier power conversion efficiency, especially at short to medium reach.

Acknowledgment: Support from project PRIN 2017 FIRST is acknowledged.

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