Spectral Broadening due to Cross-Phase Modulation (XPM) in OOK WDM Transmission Systems

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Abstract

The signal broadening due to XPM in OOK-WDM systems is evaluated for asynchronous channels including walk-off and dispersion compensation, and it is shown it can be described by an equivalent laser linewidth[1].

I. Introduction

In Intensity-Modulated/Direct-Detection (IM/DD) Wavelength-Division Multiplexed (WDM) transmission systems, XPM induces a broadening of the signal spectrum so that a wider optical filter bandwidth is required at the receiver [1]. This degrades the system performance, because more spontaneous emission noise enters the receiver. Moreover, the deleterious effects of dispersion, which are larger on a wider spectrum, could become considerable in a multistage amplified system. We show that the effect of XPM on signal spectral broadening, in the case of an On-Off Keying (OOK) NonReturn-to-Zero (NRZ) system, is similar to the one induced by laser phase noise, so that it can be accounted for by an equivalent linewidth $\Delta\nu_{eq}$. Consider first the case of no walk-off [2] among the N channels. The complex envelope of the OOK modulated signal s (s=1 ... N) at the input of the fiber, is $A_s(0,t) = \sqrt{P_s(0)}B_s(0,t)e^{j\{\theta_{s0}+\theta_s(t)\}}$, where $P_s(0)$ is the input mark power and $B_s(0,t) = \sum_{k=-\infty}^{\infty} a_s(k)p(t-kT-\delta_s)$ is the OOK modulating signal; $p(t)$ is the NRZ transmission pulse of duration $T = 1/R$, where R is the bit rate; $\{a_s(k)\}$ is the sequence of modulation bits, IID random variables taking value 1 (mark) or 0 (space) with equal probabilities; δ_s is a time offset, uniformly distributed in $[-T/2;T/2]$; θ_{s0} is the initial phase term and $\theta_s(t)$ is the laser phase noise, modeled as a Wiener process producing a linewidth $\Delta\nu$ [3]. In case of negligible dispersion ($D_c \simeq 0$), the solution to the nonlinear Schrödinger propagation equation is $A_s(z,t) = A_s(0,t)e^{-\frac{\alpha z}{2}}e^{j\theta_{NL}(z,t)}$, where α is the fiber attenuation coefficient, and $\theta_{NL}(z,t)$ accounts for Self-Phase (SPM) and Cross-Phase Modulation [2]. The autocorrelation of the stationary process $A_s(z,t)$, is found to be $R_{A_s}(\tau) = P_s(z)\cdot R_s(\tau)\cdot R_\phi(\tau)\cdot R_{XPM}(\tau)$, where $P_s(z) = P_s(0)e^{-\alpha z}$, $R_s(\tau) = \frac{1}{4}\left[1+\Lambda\left(\frac{\tau}{T}\right)\right]$ is the contribution of the OOK/NRZ modulation [4], $R_\phi(\tau) = e^{-\pi\Delta\nu|\tau|}$ is the contribution of the laser phase noise and $R_{XPM}(\tau)$ is the XPM contribution. Note that in the case of a perfect OOK/NRZ transmission system, the SPM gives no contribution to the expression of the signal autocorrelation, that is SPM doesn't induce any spectral broadening in this case. In the case of a large number N of input channels, the distribution of the random phase process $\theta_{NL}(z,t)$ becomes approximately gaussian, so that in the interval $|\tau| \leq T$, we can approximate $R_{XPM}(\tau) = e^{-\pi\Delta\nu_{eq}|\tau|}$, where the equivalent linewidth is found to be:

$$\Delta\nu_{eq} = \frac{R}{\pi}\left[\gamma L_{eff}(z)\right]^2 \sum_{p\neq s} P_p^2(0)\ , \qquad (1)$$

where γ is the nonlinear coefficient, $L_{eff}(z)$ is the effective length, and the summation is extended over all the $N-1$ interfering p channels. In the region $|\tau| \geq T$, $R_{XPM}(\tau)$ takes the constant value $R_{XPM}(\tau) = e^{-\pi T\Delta\nu_{eq}}$; this is due to the limited correlation of the random input process $B_s(0,t)$. In the next paragraph we'll show by simulation that the broadening due to XPM is essentially the same as that due to an equivalent phase noise with linewidth as in (1). Note that the signal spectral broadening is proportional to the number N of channels and to the square of the signal input power $P_p(0)$. When relative channel walk-off is considered, it is still possible to define an equivalent linewidth:

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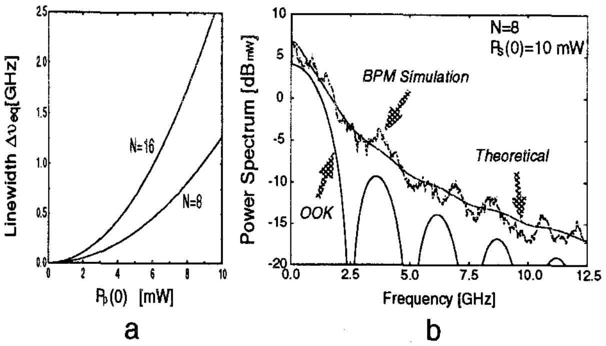

Figure 1: (a) Equivalent Linewidth and (b) Power Spectrum in a single-stage link. $D_c = 0$, z=85 km, $\alpha = 0.21$ dB/km, $\gamma = 2.35$ $W^{-1}km^{-1}$, $R = 2.5$ $Gbit/s$.

$$\Delta\nu_{eq} = \frac{R\gamma^2}{\pi}\sum_{p\neq s}[L^{sp}_{eff}(z,R)P_p(0)]^2, \qquad (2)$$

where now $L^{sp}_{eff}(z,R)$ is a complicated function that depends on the bit rate R and on the specific interfering channel p [5].

II. Results

In this section we show simulation results, obtained by the Beam-Propagation Method (BPM), of an OOK/NRZ transmission system in the case of single-stage and multi-stage links with ideal amplifiers, in order to verify equation (1). In Fig.1a we plot $\Delta\nu_{eq}$ in (1) vs. the input channel power, for a single span of $z = 85$ km and 8 and 16 channels. Note that the equivalent linewidth $\Delta\nu_{eq}$ can be much larger then the laser linewidth at high bit rates. In Fig.1b the power spectrum of the signal after propagation in a non-dispersive fiber is shown in the case $P_s(0) = 10$ mW and $N = 8$ channels both theoretically using (1) and by BPM simulation. The input OOK spectrum is also shown. There is a good match between the theory and the simulations. Fig.2 shows the same curves as in Fig.1, but applied to the multi-stage compensated system composed of $M = 5$ spans of NonZero Dispersion (NZD) fiber and standard Single Mode Fiber (SMF), shown in Fig.3. The effect of channel walk-off is now included in eq. (2), for which an analytical form is available [5]. Comparing Figs. 1a and 2a, we note the much reduced sensitivity to the number of channels in the presence of walk-off. Again, there is a very good match between the BPM simulation and the theoretical curve using (2). Such good match can be attribuited to the low dispersion of the transmission fiber (-2 $ps/nm/km$) and to the presence of dispersion compensation which drastically reduces the distortion due to chromatic dispersion.

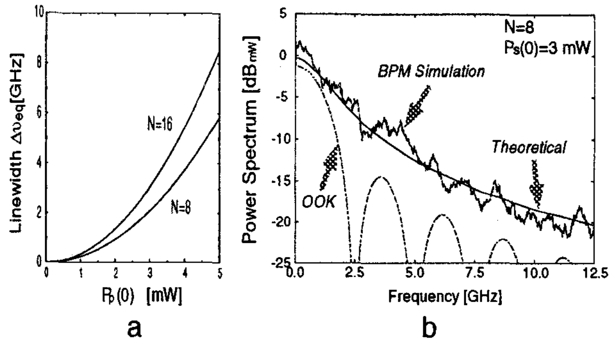

Figure 2: (a) Equivalent Linewidth and (b) Power Spectrum in a multi-stage link. $\alpha = 0.21$ dB/km, $\gamma = 2.35$ $W^{-1}km^{-1}$, $R = 2.5$ $Gbit/s$, wavelength separation $\Delta\lambda = 0.8$ nm. Fiber a: $D_c = -2$ $ps/km/nm$, $D'_c = 0.07$ $ps/km/nm^2$, $z = 85$ km. Fiber b: $D_c = 17$ $ps/km/nm$, $D'_c = 0.07$ $ps/km/nm^2$, $z = 10$ km.

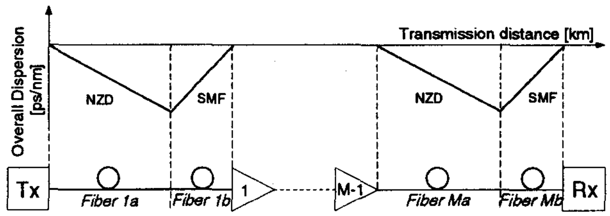

Figure 3: System.

III. Conclusions

We have found an analytical closed formula for the spectral broadening due to XPM in OOK/NRZ transmission systems in the presence of asyncronous input channels. We have proved by simulation that the XPM-induced signal broadening is similar to the one induced by laser phase noise, and an equivalent linewidth $\Delta\nu_{eq}$ is given. The effective laser linewidth can be usefully utilized to infer the system performance from well-know results derived for systems impaired by laser phase noise [1].

References

[1] O. K. Tonguz, and L. G. Kazovsky, *J. Lightwave Technol.*, vol. 9, no. 2, pp. 174-181, Feb. 1991.

[2] T. Chiang, N. Kagi, M. E. Marhic, and L. G. Kazovsky, *J. Lightwave Technol.*, vol. 14, no. 3, pp. 249-260, Mar. 1996.

[3] J. Salz, *IEEE Communications Magazine*, vol. 24, no. 6, pp. 38-49, June 1986.

[4] B. Carlson, *Communication Systems,* 3rd ed, McGraw-Hill, 1986, p. 668.

[5] G. Bellotti, Tesi di Laurea, Dip. Ing. dell'Informazione, Università di Parma, 1997.