

# Experimental measurement of signal-to-FWM ratio in non-zero dispersion fibers

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## ABSTRACT

Experimental measurement of in-band Four-Wave-Mixing (FWM) power in non-zero dispersion fiber is presented. A comparison with known methods shows how the proposed procedure is more accurate in frequently interesting cases.

**Keywords:** Four-Wave-Mixing, measurements, optical fiber

## 1. INTRODUCTION

The characterization of optical fiber nonlinear Kerr effects is a delicate issue in high power Wavelength Division Multiplexed (WDM) transmission systems, because Self-Phase-Modulation, Cross-Phase-Modulation and Four-Wave Mixing (FWM) act together to degrade system performance. In this section, the tackled problem is the correct evaluation of the in-band FWM terms, i.e. crosstalk terms which are difficult to discriminate from the WDM signal since they are spectrally overlapped to it. The known measurement methods introduce a non negligible error in this evaluation. In the channel suppression method, the suppression of the channel in the band where we want to measure the FWM power is not sufficient because some FWM contributions are also suppressed. In the channel detuning method it is not accurate to visualize the FWM power by detuning the channel from its nominal wavelength, because the efficiency of the FWM contributions is altered and the optical spectrum analyzer (OSA) may not allow a precise measure.

We present a novel method for accurate in-band FWM evaluation in equally spaced WDM systems on non-zero dispersion fibers. By using only three channels of the WDM comb, we measure the out-of-band FWM terms at the frequencies where only one FWM contribution is present and obtain the efficiency of such terms. The in-band FWM power on every channel in the WDM system is then inferred from such efficiencies. A comparison with the above-mentioned traditional methods shows the better precision of the novel method for WDM systems with less than 64 channels. The method is experimentally verified by measures on a four channel system.

## 2. THEORY

The FWM power  $P_{ijk}$  at frequency  $\omega_{ijk}$ , generated by three continuous wave (CW) channels at frequencies  $\omega_i$ ,  $\omega_j$  and  $\omega_k$ , which satisfy the relation  $\omega_{ijk} = \omega_i + \omega_j - \omega_k$ , is [1]:

$$P_{ijk} = d^2 k P_i P_j P_k \eta_{ijk} \quad , \quad k = \left( \frac{2\pi n_2}{A_{eff} \lambda_0} \right)^2 e^{-\alpha z} L_{eff}^2 \quad , \quad (1)$$

where  $P_i$ ,  $P_j$ ,  $P_k$  are the input powers of channels  $i$ ,  $j$  and  $k$ ,  $d$  is the degeneracy factor, which takes value 1 or 2 for degenerate and non degenerate terms, respectively,  $n_2$  is the non linear fiber coefficient,  $\lambda_0$  is the central wavelength,  $c$  is the light speed,  $A_{eff}$  is the core effective area,  $L_{eff} = (1 - e^{-\alpha z})/z$

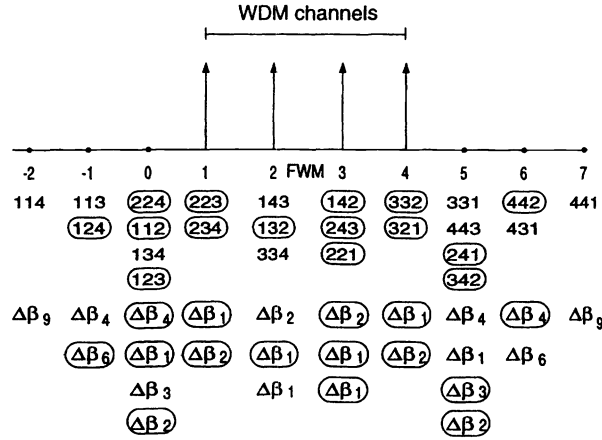


Figure 1: FWM contributions and corresponding  $\Delta\beta_n$  for a four channel system.

is the effective length, with  $\alpha$  the fiber attenuation and  $z$  the fiber length, and the efficiency  $\eta_{ijk}$  takes expressions:

$$\eta_{ijk} = \frac{\alpha^2}{\alpha^2 + \Delta\beta_{ijk}^2} \left[ 1 + \frac{4e^{-\alpha z} \sin^2(\Delta\beta_{ijk}z/2)}{(1 - e^{-\alpha z})^2} \right]. \quad (2)$$

Away from the zero dispersion region, the phase matching coefficient is given by  $\Delta\beta_{ijk} = \frac{2\pi c}{\lambda_0^2} D \Delta\lambda_{ik} \Delta\lambda_{jk}$ , where  $\Delta\lambda_{ik}$  and  $\Delta\lambda_{jk}$  are the wavelength spacings between channels  $i$  and  $k$ , and  $j$  and  $k$ , respectively. For equally spaced channels,  $\Delta\beta_{ijk}$  takes the discrete values

$$\Delta\beta_n = n \left( \frac{2\pi c}{\lambda_0^2} \right) D \Delta\lambda^2, \quad n = |i - k| |j - k| \quad (3)$$

where  $\Delta\lambda$  is the minimum channel spacing. In Fig. 1 we summarize all FWM terms falling on each frequency band in an equally-spaced four-channel system. Each term is represented by the indices  $ijk$  of the three channels involved in the combination. For instance, the term 132 labels the FWM contribution jointly generated by channels 1, 3 and 2. Note that since such term falls on channel 2, its power cannot be measured by simply suppressing channel 2. For each FWM term, the corresponding phase matching coefficient  $\Delta\beta_n$  is also shown in Fig. 1. Note that FWM terms arising from different channel combinations can have the same  $\Delta\beta_n$ , and thus the same efficiency  $\eta(\Delta\beta_n)$ , which rapidly decreases for increasing  $n$ .

We now present a novel method that uses three CW channels to estimate the  $\eta_n$  values up to  $n = 4$ , so that the in-band FWM powers in an N-channel system, up to order  $n = 4$ , can be calculated.

As an example, consider the four-channel system in Fig. 1. When channel 2 is switched off the circled FWM terms disappear, so that the power of the individual FWM terms  $P_{113}$ ,  $P_{134}$  falling on channels  $-1$ ,  $0$ , respectively, can be directly measured.

Define the coefficients  $c_n \triangleq k \eta(\Delta\beta_n)$ . From (1) and the directly measured FWM terms we can estimate:

$$\begin{aligned}
c_3 &= k\eta_{134} = \frac{P_{134}}{4P_1P_3P_4} \\
c_4 &= k\eta_{113} = \frac{P_{113}}{P_1^2P_3}
\end{aligned}
\tag{4}$$

Now the FWM power on channel 2, in the assumption of incorrelation between the FWM terms, is (see Fig. 1) the sum of the power of term 143 (with phase matching  $\Delta\beta_2$ ) and the term 334 (with phase matching  $\Delta\beta_1$ ). Similarly, on channel 5 we have the sum of the power of the term 331 (with phase matching  $\Delta\beta_4$ ) and 443 (with phase matching  $\Delta\beta_1$ ). Thus from (1) and (4):

$$\begin{aligned}
P_2 &= 4P_1P_4P_3c_2 + P_3^2P_4c_1 \\
P_5 &= P_3^2P_1c_4 + P_4^2P_3c_1
\end{aligned}
\tag{5}$$

Utilizing the second of (4) and (5), we obtain an indirect measure of the most important coefficients:

$$\begin{aligned}
c_1 &= \frac{P_5 - P_3^2P_1c_4}{P_3P_4^2} = \frac{P_5 - P_{113}P_3/P_1}{P_3P_4^2} \\
c_2 &= \frac{P_2 - P_3^2P_4c_1}{4P_1P_3P_4}
\end{aligned}
\tag{6}$$

A better precision on the  $c_1$  evaluation can be obtained by measuring the FWM power at channel 5 using an input configuration with only channels 3 and 4 active, so that from the FWM term  $P_{443}$  on channel 5 one gets:  $c_1 = \frac{P_{443}}{P_3P_4^2}$ . The difference between the direct and indirect measurements has been found to be very small.

As seen in Fig. 1, having the coefficients  $c_1, c_2$ , the total in-band FWM terms in the four-channel system can be obtained using (6):

$$\begin{aligned}
P_{1FWM} &= P_2^2P_3c_1 + 4P_2P_3P_4c_2 \\
P_{2FWM} &= (4P_1P_3P_2 + P_3^2P_4)c_1 + 4P_1P_4P_3c_2 \\
P_{3FWM} &= (4P_2P_4P_3 + P_2^2P_1)c_1 + 4P_1P_4P_2c_2 \\
P_{4FWM} &= P_3^2P_2c_1 + 4P_3P_2P_1c_2
\end{aligned}
\tag{7}$$

In the general N-channel case, neglecting FWM terms with phase matching  $\Delta\beta_n, n > 4$ , the total FWM power on a generic channel  $i$  with more than 5 channels to its right and to its left is:

$$\begin{aligned}
P_{iFWM} &= (4P_{i-1}P_iP_{i+1} + P_{i-2}P_{i-1}^2 + P_{i+1}^2P_{i+2})c_1 + \\
&4(P_{i-2}P_{i-1}P_{i+1} + P_{i-1}P_{i+1}P_{i+2} + P_{i+1}P_{i+2}P_{i+3} + P_{i-3}P_{i-2}P_{i-1})c_2 + \\
&4(P_{i-1}P_{i+2}P_{i+3} + P_{i-3}P_{i-2}P_{i+1} + P_{i+1}P_{i+3}P_{i+4} + P_{i-4}P_{i-3}P_{i-1})c_3 + \\
&[4(P_{i-2}P_iP_{i+2} + P_{i-1}P_{i+3}P_{i+4} + P_{i-4}P_{i-3}P_{i+1} + P_{i+1}P_{i+4}P_{i+5} + \\
&P_{i-5}P_{i-4}P_{i-1}) + P_{i-4}P_{i-2}^2 + P_{i+2}^2P_{i+4}]c_4
\end{aligned}
\tag{8}$$

For equal channel power  $P$  we get:

$$P_{iFWM} = P^3(6c_1 + 16c_2 + 16c_3 + 22c_4)
\tag{9}$$

The coefficients  $c_1$  through  $c_4$  can be obtained as outlined above.

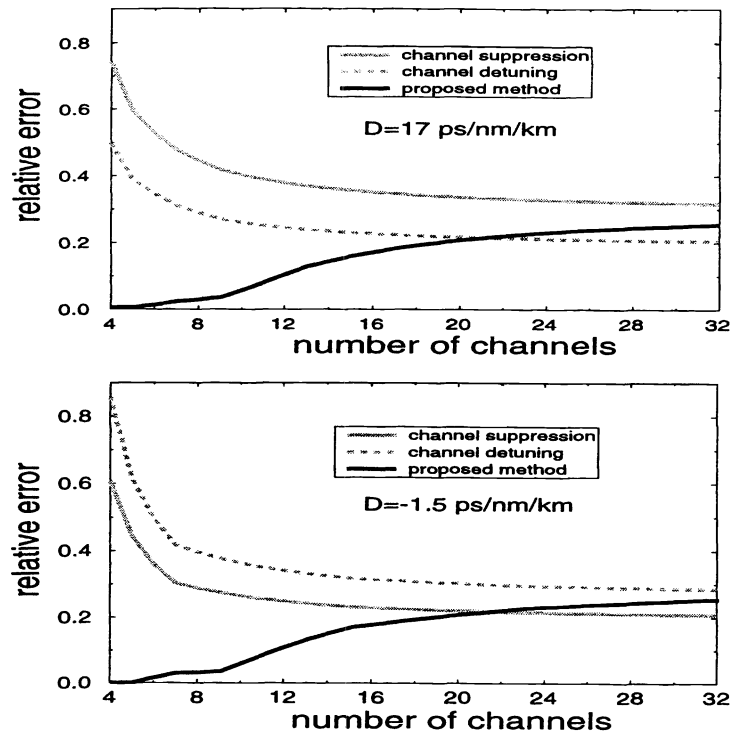


Figure 2: Analytical relative error vs number of WDM channels, for the three methods of in-band FWM power measurement, for  $D=+17$  ps/nm/km (top) and  $D=-1.5$  ps/nm/km (bottom). In the detuning method the detuning was 0.2 nm due to the measurement precision of the OSA. WDM equal channel spacing 0.4 nm.

### 3. COMPARISON WITH OTHER MEASUREMENT METHODS

In this section we compare our novel method with two well-known measurement techniques. In the channel suppression method, the total FWM power falling on channel  $i$  is measured by switching channel  $i$  off. The total FWM power is underestimated, as all terms involving channel  $i$  itself are not present in the measure.

In the channel detuning method, channel  $i$  is detuned from its nominal wavelength by an amount large enough to allow a direct measurement (often with an optical spectrum analyzer (OSA)) of the in-band FWM terms not involving channel  $i$ , as in the suppression method, *and* of the FWM terms involving channel  $i$ , which are also detuned and can be separately measured. In the detuning method the efficiency of the terms involving channel  $i$  is altered, either increased or decreased according to the detuning direction. Therefore such method can either over- or under-estimate the total FWM power. The error is larger for larger detunings, often needed for a clear reading on an OSA.

Our novel method based on (8) suppresses all channels in the  $N$ -channel system except three, and then measures the coefficients  $c_1$  through  $c_4$ . Being truncated to  $c_4$ , it is very precise when  $N$  is small, or when the dominant FWM contributions come from the nearest neighbors, as for instance when the dispersion is larger than a few ps/nm/km. Therefore our method is accurate exactly where the other two methods are most inaccurate.

In order to quantitatively compare the three methods, we define a relative error in the measurement

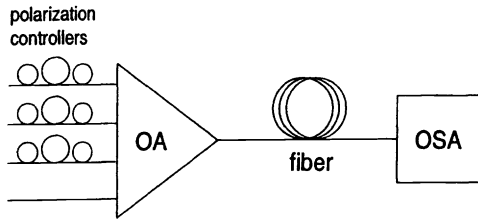


Figure 3: Measure set up.

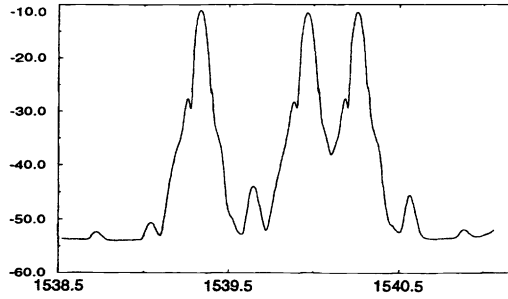


Figure 4: Optical power spectrum at the fiber output.

of the total FWM power on channel  $i$  as:

$$\mathcal{E} = \left| \frac{\sum P_{FWM} - \sum P_{FWM}^M}{\sum P_{FWM}} \right| \quad (10)$$

where  $\sum P_{FWM}$  is the sum of the power of all FWM terms falling on channel  $i$ , and  $\sum P_{FWM}^M$  is the sum of the power of the FWM terms on channel  $i$  actually measured by the method.

Fig. 2 shows the relative error  $\mathcal{E}$  for the central channel versus the number of channels  $N$ , for dispersion  $D=17$  ps/nm/km (top) and  $D=-1.5$  ps/nm/km (bottom). The FWM terms have been exhaustively found and their power calculated using the theoretical formulae (1) and (2), with equal power per channel  $P = 10$  dBm, attenuation  $\alpha = 0.24$  dB/km, channel spacing  $\Delta\lambda=0.4$  nm,  $A_{eff} = 55\mu m^2$ ,  $\lambda_0 = 1550$  nm,  $N_2 = 2.7 \cdot 10^{-20} m^2/W$ , and fiber length  $z = 25$  km. The effect of a typical dispersion slope  $S = 0.07$  ps/(nm<sup>2</sup> km) on formula (3) has been verified to be very small for the considered values of  $D$ .

As seen from the figures, at the central channel the error of both the suppression and the detuning methods decreases with channel number  $N$  as expected, while that of our proposed method increases with  $N$ . For  $N$  equal to 4, 8, 16 channels the novel method is more precise of the other two, while for 32 and up to 64 channels the precision is essentially the same as the other methods. On edge channels the suppression method is obviously the most accurate as there are no self-induced FWM terms. The curious inversion of error curves between the suppression and detuning methods when  $D$  is changed from 17 to -1.5 ps/nm/km is due to the change in the efficiency connected with the detuning, which was 0.2 nm in this case.

#### 4. EXPERIMENTAL RESULT

Fig. 3 shows the set up used for the measure: four CW channels, amplified by the optical amplifiers OA, enter a non-zero dispersion shifted fiber, whose parameters are summarized in Tab. 1. Fig. 4 shows the measure output power spectrum. Channel polarizations are aligned at the input by polarization controllers to maximize the FWM interaction during the propagation. With channel 2 off, and with an amplifier output power of 2.3 dBm per channel, we measure the FWM powers at the frequencies -1, 0, 2, 5 and 6; from Eq. (4)-(6) we calculate  $\eta_1$ ,  $\eta_2$ ,  $\eta_3$ ,  $\eta_4$  and  $\eta_6$ . Finally, FWM powers at each channel are calculated using Eq. (7). The amplifier noise (ASE noise) level is calculated through the best fit method [4], and subtracted from the FWM power measures. Tab. 2 shows the comparison between the experimental results obtained with the proposed measurement method and the analytical

results obtained from eq. 1-2-7, where the fiber parameters in Tab. 1 are used. The difference between analytical and experimental values is shown to be less than 0.5 dB, over all the signal spectrum.

dispersion coef. $D_c$	-1.5 ps/nmkm
slope	0.07 ps/nm <sup>2</sup> km
nonlinear coef. $n_2$	$2.7E - 20$ m <sup>2</sup>
attenuation $\alpha$	0.24 dB/km
fiber lenght L	25.259 km
effect. area $A_{eff}$	$55E - 12$ m <sup>2</sup>
effect. lenght $L_{eff}$	$1.36E4$ m

Table 1: fiber parameters

	Analytical(dBm)	Experimental
$P_{1_{FWM}}$	-45.1	-45.0
$P_{1_{FWM}}$	-40.2	-39.8
$P_{3_{FWM}}$	-39.9	-39.4
$P_{4_{FWM}}$	-44.6	-44.8

Table 2: Comparison between analytical and experimental values.

## 6. CONCLUSIONS

A new method for in-band FWM power measurement in non-zero dispersion fibers is presented. Measuring out-of-band FWM terms power only, all the in-band FWM power contributions not negligible can be easily evaluated. The obtained precision is definitely superior to other methods precision for just 16 channel's systems measuring the FWM power in the central channel's band. The novel method precision increase if we measure the FWM power on the side channels except the extreme channels.

## 7. REFERENCES

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