Gain Dynamics of Doped-Fiber Amplifiers for Added and Dropped Signals

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Abstract— Sun *et al.* [1] reduced the set of coupled first order nonlinear partial differential equations determining the wavelength-dependent, time-varying doped-fiber amplifier gain into a single ordinary differential equation (ODE). In this paper we further simplify the ODE, greatly enhancing its utility as an analysis and design tool. We find that the gain dynamics are completely specified by the total number of excited ions. We demonstrate that channel addition causes much faster transients than channel dropping in wavelength division multiplexing networks. We approximate the solution of the ODE by an exponential with time constant given as a function of amplifier parameters.

I. Introduction

Gain dynamics of erbium doped fiber amplifiers (EDFAs) are already of considerable interest in wavelength division multiplexed (WDM) networks, where network reconfigurations or network faults can lead to the adding or dropping of wavelength channels [1], [2], [3], [4]. Given the interest in EDFA gain dynamics, much research has been devoted to the solution of the set of coupled first order nonlinear partial differential equations determining the wavelength-dependent, time-varying amplifier gain. The complexity of the numerical solution of these equations has motivated efforts to reduce them or to study steady state solutions. For instance, Saleh *et al.* [5] eliminated the time dependence of the gain to arrive at a simple, single transcendental equation for the steady state gain. More recently, Sun *et al.* [1] at Bell Laboratories have succeeded in reducing the system of coupled differential equations into a single ordinary differential equation (ODE).

In this paper we further simplify the ODE identified by Sun *et al.*, [1] bringing into greater evidence the physical meaning of the amplification process, and greatly enhancing the utility of the ODE as an analysis and design tool. We show that the gain dynamics of a doped-fiber amplifier are completely specified by its total number of excited ions, which we call the *reservoir* $r(t)$, whose time behavior is described by a simple first-order ODE. The present analysis is based on the assumptions of the model in [1],[5], which neglects both excited state absorption and saturation induced by the amplified spontaneous emission (ASE) produced inside the amplifier. There are several methods to include such ASE contributions in the model [6], [7], and future work will address this issue.

As explained in [3], channel additions and drops in WDM systems lead to power disparities among channels and can compromise the quality of service. Schemes for dynamic control of the amplifer gain (and hence output powers) must react more quickly than the amplifier gain transients. We use the reduced ODE to examine the relative speed of transients in added versus dropped WDM channels.

We have found that the doped-fiber amplifier dynamics are connected to the depletion and the refill of the reservoir. While the refill process is mainly contributed by the pump, and is a process in which one pump photon can excite at most one ion, the depletion process is mainly caused by the signals, and is an avalanche process connected to stimulated emission: one signal input photon can consume a very large number of excited ions in the reservoir. Thus the time scales connected to the depletion process can be extremely fast, while those connected to the refill process are slow and depend on the pump power and the total number of dopant ions. In any case, the amplifier dynamics are essentially independent of the fluorescence time. As a consequence, channel addition causes much faster transients than channel dropping.

The paper is organized as follows. In Section II we derive the ODE describing the system. In Section III we develop the step response of the gain of a single amplifier, and an exponential approximation for the gain. In Section IV we find the step response for a chain of amplifiers. Finally, Section V concludes the paper.

II. Theory

We start from the rate and photon equations used in [1], derived assuming a two-level system for the dopant ions, homogeneously broadened gain spectrum, no excited state absorption, no background loss, and no self-saturation by ASE. The rate equation for the fraction of excited ions N_2, $0 \leq N_2 \leq 1$, is

$$\frac{\partial N_2(z,t)}{\partial t} = -\frac{N_2(z,t)}{\tau} - \frac{1}{\rho A}\sum_{j=0}^{N} u_j \frac{\partial Q_j(z,t)}{\partial z} \qquad (1)$$

and the equations describing the propagation along z of the photon fluxes Q_k [photons/s] of channel k, $k = 0, ..., N$, are

$$\frac{\partial Q_k(z,t)}{\partial z} = \rho u_k \Gamma_k [\sigma_k^T N_2(z,t) - \sigma_k^a] Q_k(z,t) \qquad (2)$$

where τ [s] is the fluorescence time, ρ [m^{-3}] is the ion density in the doped fiber core of effective area A [m^2]; Γ_k, σ_k^e [m^2], and σ_k^a [m^2] are the confinement factor, and the emission and absorption cross-sections of channel k, respectively, and $\sigma_k^T \triangleq \sigma_k^e + \sigma_k^a$. The length of the amplifier is L [m]. Channels entering at $z = 0$ have $u_k = 1$ while those entering at $z = L$ have $u_k = -1$. The pump is placed on channel 0.

Dividing both sides of (2) by $Q_k \neq 0$, multiplying by dz and integrating from $z = 0$ to L yields

$$G_k(t) = B_k\, r(t) - A_k, \quad k = 0, ..., N \tag{3}$$

where

$G_k(t) \triangleq \int_0^L \frac{u_k \partial Q_k}{Q_k} = ln(Q_k^{out}(t)/Q_k^{in}(t))$
is the logarithmic gain;

$r(t) \triangleq \rho A \int_0^L N_2(z,t)\, dz$
is the total number of excited ions in the amplifier, which we call the "reservoir". The reservoir is a number between 0 and $r_M \triangleq \rho A L$, the total number of ions in the doped fiber. The state variable represents the number of available ions ready to be converted into signal photons;

$A_k \triangleq \rho \Gamma_k \sigma_k^a L$ and $B_k \triangleq \Gamma_k \sigma_k^T / A$
are non-dimensional parameters.

The standard parameters used in [1]–[5] are the absorption coefficients $\alpha_k \triangleq \rho \Gamma_k \sigma_k^a = A_k / L$ and the intrinsic saturation powers $P_k^{IS} \triangleq h\nu \frac{A}{\Gamma_k \sigma_k^T \tau} = \frac{h\nu}{B_k \tau}$. We introduce parameters A_k, B_k, which are independent of τ, to stress that the gain depends on τ only through r. Note that the definitions of $Q_k^{in}(t)$ and $Q_k^{out}(t)$ include the directionality of the fluxes, therefore obviating the need for the parameter u_k [1].

Multiplying both sides of (1) by dz and integrating from 0 to L yields

$$\frac{\partial r(t)}{\partial t} = -\frac{r(t)}{\tau} - \sum_{j=0}^{N} (Q_j^{out}(t) - Q_j^{in}(t)). \tag{4}$$

Using (3) in (4) we arrive at a first-order ODE describing the dynamic time behavior of the system's state, *i.e.*, the reservoir $r(t)$

$$\boxed{\dot{r}(t) = -\frac{r(t)}{\tau} + \sum_{j=0}^{N} Q_j^{in}(t) \left(1 - e^{B_j r(t) - A_j}\right)} \tag{5}$$

Once the initial condition $r(0)$ is specified, it is easy to show that the solution of (5) is unique. $r(0)$ can be any number in the allowed range $[0, r_M]$, although the range spanned by a real amplifier can be narrower [10]. If at time $t = 0^-$, *i.e.*, one instant before the start of the observation period, the amplifier is at equilibrium, then $r(0)$ must satisfy (5) with $\dot{r}(0^-) = 0$

$$r(0) = \tau \sum_{j=0}^{N} Q_j^{in}(0^-) \left(1 - e^{B_j r(0) - A_j}\right) \tag{6}$$

which corresponds to the well-known Saleh steady state equation [5]. For a starting guess at its numerical solution, the upper bound $\tau \sum_{j=0}^{N} Q_j^{in}(0^-)$ can be used.

Note that, for given input fluxes $Q_j^{in}(t)$, the direction of their entering the amplifier has no effect on r. Hence co- and counter-propagating pumping is equivalent in this analysis. However we recall that ASE has been neglected in this analysis, and in fact a co-propagating pump always gives a larger optical SNR [8].

Equation (5) can be expressed equivalently in terms of the normalized reservoir $x(t) \triangleq \frac{1}{L} \int_0^L N_2(z,t)\, dz = \frac{r(t)}{r_M}$, also known as the *fraction of excited ions* [6],[8],

$$\boxed{\dot{x}(t) = -\frac{x(t)}{\tau} + \sum_{j=0}^{N} \frac{Q_j^{in}(t)}{\rho A L} \left(1 - e^{\rho L \Gamma_j (\sigma_j^T x(t) - \sigma_j^a)}\right)} \tag{7}$$

This form may be more useful in the study of an isolated amplifier, as the main amplifier parameters are clearly visible in the expression.

Note from (7) that input fluxes Q_j^{in} are meaningful in relation to the total reservoir capacity r_M. However, the normalized input photon fluxes $\frac{Q_j^{in}}{r_M}$ do not uniquely describe the system behavior, since amplifiers with identical $\sigma^T(\lambda)$, $\sigma^a(\lambda)$, but different r_M will behave differently even if driven by the same normalized photon fluxes. All other parameters being equal, however, one can trade ρ for L by keeping their product constant, *i.e.*, one can have shorter but otherwise identically behaving amplifiers by more strongly doping the core. Another implication of (7) is that if the core area is doubled the input fluxes must be doubled to obtain identical system dynamics.

III. Single Amplifier Step Response

Suppose we have a single amplifier, described by (5). At time $t = 0^-$ (*i.e.*, an instant before time 0) we are given the initial value $r(0)$ and the inputs $\{Q_j^{in}(0^-)\}$.[1] The system need not be at steady state at $t = 0^-$. At time $t = 0^+$ each input flux undergoes a discontinuity and then remains constant for all $t > 0$

$$Q_j^{in}(t) = Q_j^{in}(0^-) + \Delta Q_j^{in} \quad j = 0, .., N. \tag{8}$$

Then (5) at $t = 0^-$ gives

$$\dot{r}(0^-) = -\frac{r(0)}{\tau} + \sum_{j=0}^{N} Q_j^{in}(0^-) \left(1 - e^{G_j(0)}\right) \tag{9}$$

where $G_j(0) = B_j r(0) - A_j$, and at $t = 0^+$ gives

$$\dot{r}(0^+) = \dot{r}(0^-) + \sum_{j=0}^{N} \Delta Q_j^{in} (1 - e^{G_j(0)}) \tag{10}$$

For small values of t we can approximate the actual solution of (5) with a straight line

$$r(t) \cong r(0) + \dot{r}(0) t \tag{11}$$

More generally, we can differentiate (5) $n - 1$ times and find the derivatives at time zero $\frac{\partial^i}{\partial t^i} r(0^+)$, $i = 1, .., n$, and approximate $r(t)$ with its truncated Taylor series in $t = 0$

$$r(t) \cong \sum_{i=0}^{n} \frac{\partial^i}{\partial t^i} r(0^+) \frac{t^i}{i!} \tag{12}$$

[1] Note that $r(0^+) = r(0^-)$ since $\dot{r}(0)$ exists.

One problem with such polynomial approximations at $t = 0$ is that they fail to converge to the actual solution for $t \to \infty$. A reasonable compromise between accuracy near $t = 0$ and asymptotic convergence is obtained by the exponential approximation

$$r(t) \cong r^{ss}(\infty) + (r(0) - r^{ss}(\infty))e^{-t/\tau_e} \qquad (13)$$

where $r^{ss}(\infty)$ is the steady state value of r (approached asymptotically for $t \to \infty$) with the new input fluxes (8), and τ_e is the exponential time constant. To get accuracy near $t = 0$ we impose that $\dot{r}_e(0^+) = \dot{r}(0^+)$, *i.e.*, the derivative of the approximation matches that of the actual solution at time $t = 0^+$. We have from (13) $\dot{r}(0^+) \cong -\frac{(r(0)-r^{ss}(\infty))}{\tau_e}$, so that we find the time constant

$$\boxed{\tau_e = \frac{r^{ss}(\infty)-r(0)}{\dot{r}(0^+)}}. \qquad (14)$$

The exponential approximation gives a better model to describe the gain dynamics for longer time scales with respect to the polynomial approximations. It is good to describe transients in a circuit switching scenario, with channels being added/dropped dynamically. Such an approximation has been suggested in [3], but an explicit expression for the exponential time constant was missing. It has been shown in [3] that the exponential approximation is indeed closer to the experimentally measured value of $r(t)$. Bononi, *et al.* [10] give an analytical justification of the exponential approximation, corroborating the results in [4].

As a numerical example, consider a case very similar to the one presented in [1]. The amplifier has two input channels $\lambda_1 = 1552.4$ nm and $\lambda_2 = 1557.9$ nm, with initial input powers $P_1 = -2$ dBm and $P_2 = -2 + 10\,\mathrm{Log}_{10}(7)$ dBm, simulating the remaining 7 channels of an 8-channel system with -2 dBm/channel. The amplifier is pumped at 980 nm, with pump power 18.4 dBm, and has $L = 35$ m, $\tau = 10.5$ ms. The absorption coefficients are [0.257, 0.145, 0.125] m^{-1} and the intrinsic saturation powers are [0.440, 0.197, 0.214] mW at [980, 1552.4, 1557.9] nm respectively. The system is at equilibrium before $t = 0$. At time $t = 0$ part of the power on channel 2 is dropped, simulating the drop of a given number of channels.

Fig. 1(a) shows (top) the reservoir dynamics for the exact solution of (5) and for the exponential approximation (13), for power variations in channel 2 simulating the drop of 4 and 7 channels, and the addition of 7 channels, respectively. Fig. 1(a) (bottom) shows the corresponding output power excursion on channel 1, defined as $10\mathrm{Log}_{10}(Q_1^{out}(t)/Q_1^{out}(0+))$. This figure matches very well with Fig. 1 of [1]. The exponential approximation for the reservoir is always below the actual solution, and larger errors are obtained for larger power drops.

As another example of step response, consider in Fig. 1(b) the turn-on dynamics of the previous amplifier, in which no beam is present before $t = 0$ and the pump is turned on at $t = 0^+$. From (14) we have $\tau_e = \frac{r^{ss}(\infty)-r(0)}{Q_p(1-e^{G_p(0)})} = 595$ μs. Observing also that $r(0) \cong 0$ and $e^{G_p(0)} \cong 0$ (amplifier OFF), and that $\sigma_p^a/\sigma_p^T \cong 1$, we get $\tau_e \cong \frac{\rho_{M}}{Q_p}$. This means that the turn-on time is the time it takes the pump, providing Q_p photons/sec, to invert the whole ion population in a one-to-one process. This also justifies the observed nearly linear increase in the reservoir. Note that the exponential approximation is accurate for low values and high values of t, with poorest performance at the knee of the curve. This is not surprising, as the time constant was chosen to give exact results for $t = 0$ and $t \to \infty$.

This long time constant, as well as the ones relative to channel drops in the previous example, which are of the order of 100 μs, are connected to the long time required by the pump to refill the reservoir. One pump photon can at most excite one ion, and thus it takes a strong pump flux to have a fast refill. On the other hand, the fast dynamics observed in the signal add process are connected to the time it takes the added signal to deplete the reservoir. One signal photon can consume many excited ions in the stimulated avalanche process, and thus it is sufficient to have a relatively weaker signal to have much faster system dynamics.

IV. Amplifier Chain Step Response

In this section we apply our theory to the study of transient gain dynamics in a chain of amplifiers in response to channel dropping/adding in a circuit switching scenario, a case also studied in [2].

Consider a chain of m identical fiber amplifiers, identical inter-amplifier loss L_I, and $N+1$ beams at the chain input, with CW input fluxes $Q_1^{in}, .., Q_N^{in}$ for the signals, and $Q_0^{in} \triangleq Q_p$ for the pump. As in Section III, at time $t = 0^+$ the signal fluxes at the input of the chain have a discontinuity, so that for $t > 0^+$ equation (8) holds for $j = 1, .., N$.

Such discontinuity propagates instantly along the amplifier chain.[2] Equations (9) and (10) describe the derivative of reservoir $r_i(t)$ at each amplifier $i = 1, .., m$ along the chain.

Let Q_{ij}^{in} be the flux of channel j at the input of the i-th amplifier. By definition $Q_{1,j}^{in} = Q_j^{in}$ for all j, and $Q_{i0}^{in} = Q_p$ for all i since the pump is restored at each amplifier. Let $G_{ij}^{lin} \triangleq e^{B_j r_i - A_j}$ be the linear gain of channel j at the i-th amplifier. At the input of the m-th amplifier we can thus write for channel j:

$$Q_{mj}^{in} = Q_j^{in}\left[\prod_{i=1}^{m-1}\frac{G_{ij}^{lin}}{L_I}\right] = Q_j^{in}\left[e^{B_j\sum_{i=1}^{m-1} r_i-(m-1)(A_j+\ln L_I)}\right]. \qquad (15)$$

Equation (10) for the m-th amplifier then becomes

$$\dot{r}_m(0^+) = \dot{r}_m(0^-) + \sum_{j=1}^{N}\Delta Q_j^{in}\left[\prod_{i=1}^{m-1}\frac{G_{ij}^{lin}(0)}{L_I}\right](1 - G_{mj}^{lin}(0)). \qquad (16)$$

If the system is started at equilibrium ($\dot{r}_m(0^-) = 0$), and if the gains of each channel are almost equal along the chain, very close to the inter-amplifier loss and large ($\forall i$, $G_{ij}^{lin}(0) \triangleq G_j^{lin}(0) \cong L_I >> 1$), then

$$\dot{r}_m(0^+) \cong -\sum_{j=1}^{N}\Delta Q_j^{in} G_j^{lin}(0) \qquad (17)$$

[2] We are neglecting the propagation delay of light in the fiber chain.

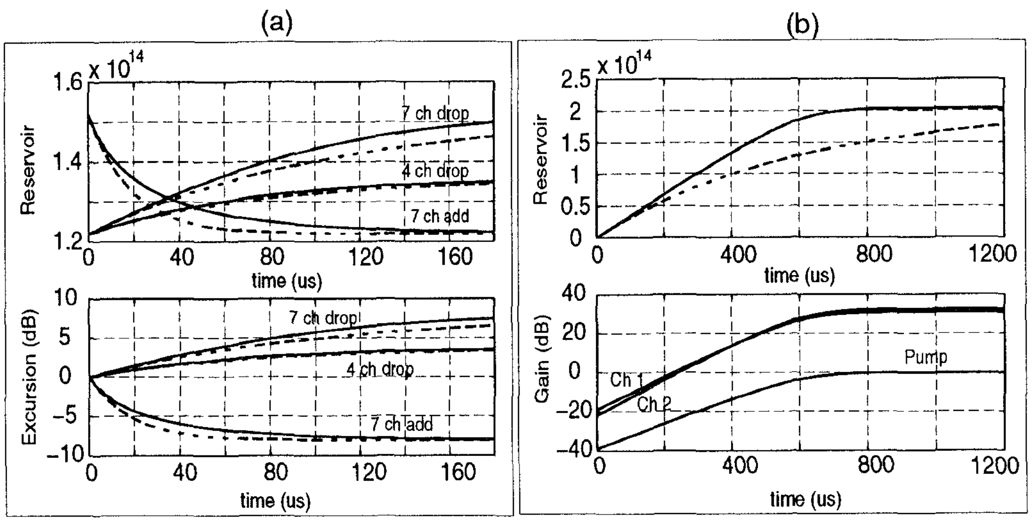

Fig. 1. (a): Time evolution of (top) Reservoir and (bottom) Output power excursion for surviving channel 1, whith channel 2 undergoing step variations at $t = 0$. Solid: exact solution of (5); Dashed: exponential approximation. (b): (top) Reservoir and (bottom) Gains at [980, 1552.4, 1557.9] nm, with pump turned on at $t = 0$.

which shows that the slope $\dot{r}_m(0^+)$ of every reservoir in an initially "balanced" chain is approximately equal to (17) for every amplifier.

As a numerical example, consider a case very similar to the one presented in [2]. The amplifier chain has 20 identical amplifiers and two input channels at 1552.1 and 1557.7 nm, with initial input power of 3 dBm/channel. The amplifiers are identical to that used in Section III. The inter-amplifier loss is $L_I = 10.32$ dB. The system is at equilibrium before $t = 0$. At time $t = 0$ the 1557.7 nm channel is dropped completely.

Fig. 2(left) shows the time evolution of the reservoirs $r_i(t)$, $i = 1, ..., 20$ along the chain, obtained by numerically solving (5) for each amplifier. It can be seen that, in this initially well-balanced chain, the initial slopes of all reservoirs are very close in value, confirming (17). The first amplifier's reservoir has a monotone increase, while all the following amplifiers have damped oscillations in their reservoirs. The reservoir values beyond the third amplifier ($i > 3$) converge in time to the asymptotic value $r^{ss}_{\infty} = \frac{A_1 + \ln L_I}{B_1} = 1.2039 \times 10^{14}$ determined by channel 1 [10].

Fig. 2(right) shows the output powers at channel 1 corresponding to the reservoirs in Fig. 2(left). A strong overshoot is visible as the amplifier index i increases. Fig. 3(left) is a zoom for small time t of Fig. 2(left), in order to show the initial slope of the curves. Fig. 3(right) shows the initial slopes calculated using (16).

The dB-power excursion at the m-th amplifier on channel s is [2] $\Delta P^{out}_{m,s}(t) \triangleq 10\mathrm{Log}_{10}(P^{out}_{m,s}(t)/P^{out}_{m,s}(0^+))$. We easily find: $\frac{P^{out}_{m,s}(t)}{P^{out}_{m,s}(0^+)} = \prod_{i=1}^{m} \frac{G^{lin}_{is}(t)}{G^{lin}_{is}(0)}$. Using the linear approximation $r_i(t) \cong r_i(0) + \dot{r}_i(0^+)t$ for each reservoir and (3), we finally get the dB-power excursion at the m-th amplifier as:

$$\Delta P^{out}_{m,s}(t) = 10\mathrm{Log}_{10}(e) B_s \sum_{i=1}^{m} \dot{r}_i(0^+)t \qquad (18)$$

which shows that the excursion grows (initially) linearly in time with slope proportional to the sum of the $t = 0^+$ slopes of the reservoirs along the chain. As noted in [2], if these are all equal, the slope of the excursion grows linearly with the amplifier index down the chain. Fig. 4 shows the power excursion on the surviving channel relative to the example above, (left) on a long time scale, and (right) on a short time scale.

V. Conclusions

In this paper we further simplify the ODE for the doped-fiber amplifier gain dynamics reported in [1], bringing into greater evidence the physical meaning of the amplification process, and greatly enhancing the utility of the ODE as an analysis and design tool. We find that the doped-fiber amplifier dynamics are connected to the depletion and the refill of the reservoir of excited ions in the amplifier. The time scales connected to the depletion process can be extremely fast, while those connected to the refill process are slow and depend on the pump power and the total number of dopant ions. In any case, the amplifier dynamics are essentially independent of the fluorescence time. This effect leads to faster transients in WDM channel additions than in channel drops. These results are important in determining the response time necessary for dynamic control of amplifier gain.

The present analysis is based on the assumptions of the model in [1],[5], which neglects both excited state absorption and saturation induced by the amplified spontaneous emission (ASE) produced inside the amplifier. There are several ways to include such ASE contributions in the model [6], [7], and future work will address this issue. However, the main findings of the present work will be valid as long as the signals give the main contribution to the amplifier saturation.

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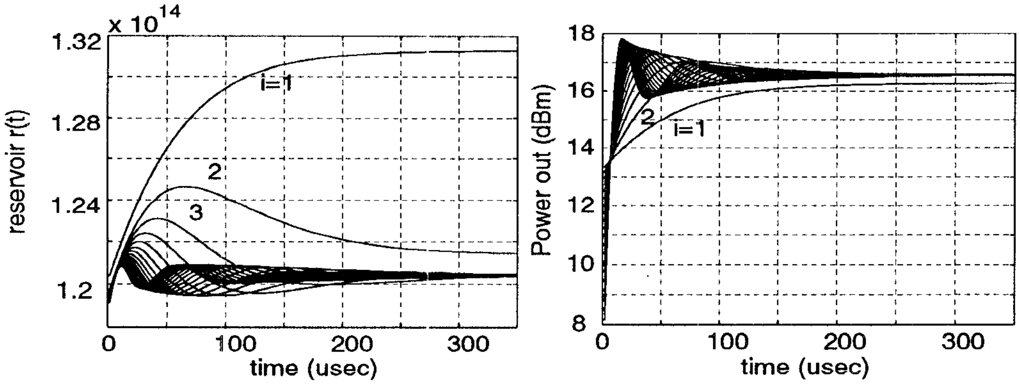

Fig. 2. Numerical solution of (5) giving the time evolution along a chain of 20 amplifiers, with one channel drop at $t = 0$: (left) Reservoir; (right) Output power of surviving channel. i = amplifier index along the chain.

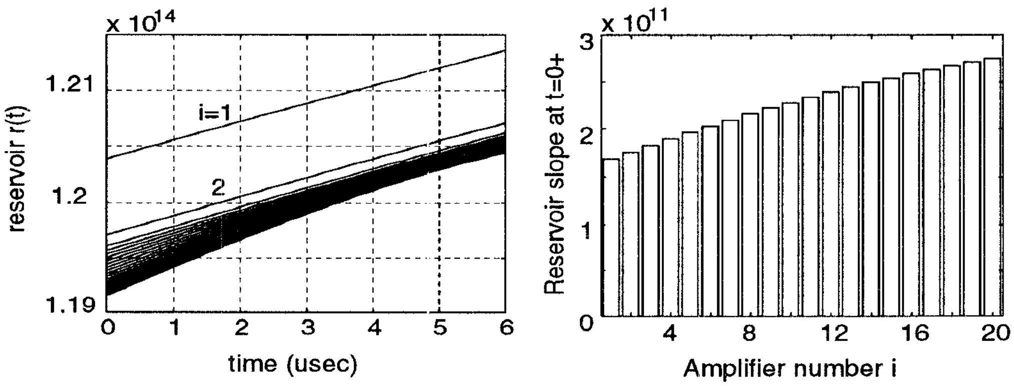

Fig. 3. (left) Zoom of Fig. 2 for small times t; (right) Initial slopes calculated as in 17.

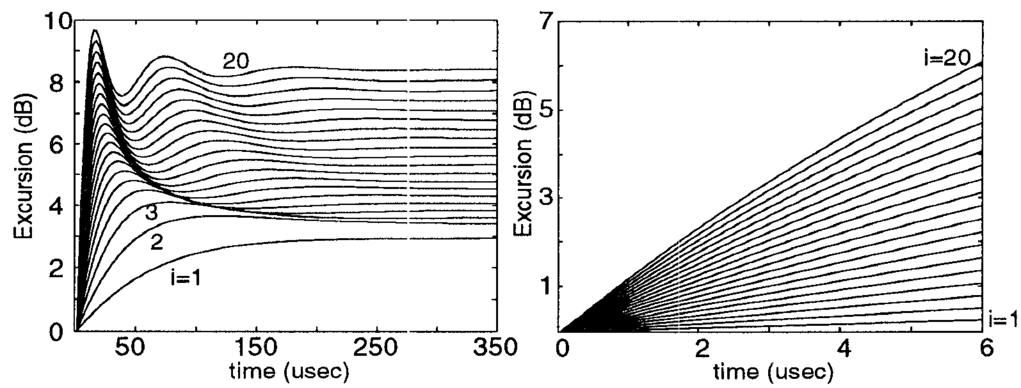

Fig. 4. Time evolution the dB-power excursion of surviving channel 1 along a chain of 20 amplifiers. (left) long time scale; (right) short time scale

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