Gain-shaped waterfilling is Quasi-optimal for Constant-pump Flattened-EDFA Submarine Links

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Abstract *For EDFA-amplified submarine links, we corroborate the results in[1] by analytically deriving an approximate capacity-achieving distribution where power is allocated inversely to the EDFA gain.*

Introduction

Multi-fiber Space division mutiplexing (SDM) for energy efficiency in submarine systems is considered a viable technology[2,3]. In this context, a recent fully-numerical search that solves the detailed erbium-doped fiber amplifier (EDFA) balance equations reported both the single-mode fiber capacity and the capacity-achieving wavelength division multiplexed (WDM) input distribution for gain-flattened links with fixed EDFA pump power[1]. At the small pumps per fiber planned for submarine SDM, only amplified spontaneous emission (ASE) noise matters[1] and we have a wavelength-parallel additive Gaussian noise channel. If we had a signal power constraint and the received noise power did not depend on the WDM allocation, the capacity-achieving distribution would be the well-known classical waterfilling (CW)[4]. Alas, EDFA gain and noise do depend on input power, and the constraint is here on the pump power.

In this paper we look inside the results in[1] by adopting a new procedure. For a single-mode EDFA-amplified link and for a given value of the EDFAs common pump, we first fix the inversion x_1 of the first EDFA (the state-variable of the link[5]) and we scan the space of *feasible* input signals achieving that inversion. We find an x_1-dependent quasi-optimal distribution that we call gain-shaped waterfilling (GW) since the sum of signal and noise is not flat as in CW, but shaped as the inverse EDFA gain. We then optimize the inversion x_1 for maximum achievable information rate (AIR) with GW. We find that GW-AIR is close to capacity at all inversions, and confirm that at typical submarine span loss any reasonable input power allocation has AIR close to capacity at the optimal inversion[6].

EDFA physical model

We will use the Saleh EDFA model[7] with ASE self-saturation[8,9]. In brief, the EDFA gain at frequency ν_j is $G_j(x) = e^{\ell((\alpha_j+g_j^*)x-\alpha_j)}$ where x is the inversion, ℓ is the doped-fiber length, and $g^*(\nu), \alpha(\nu)$ (m^{-1}) are the Erbium gain and attenuation coefficients [1, Fig.7]. ASE flux amplified inside the EDFA (forward and backward) is[8]: $Q_{ASE}^{F+B} \cong 2\sum_{l=1}^{L} 2n_{sp,l}(x)(G_l(x)-1)\Delta\nu$ (ph/s) and is calculated over the wavelength range $1470-1670$nm in L ASE frequency bins of width $\Delta\nu = 50$GHz; $n_{sp}(x)$ is the spontaneous emission factor, and the noise figure at ν_l is $F_l = 2n_{sp,l}\frac{G_l-1}{G_l}$. The equivalent-input forward ASE flux at ν_l over band $\Delta\nu$ is $F_l\Delta\nu$. If we use input WDM fluxes Q_j^{in}, $j = 1,..,N_{ch}$, the steady-state photon flux balance at the EDFA is given by the extended Saleh equation (ESE)[7,8]:

$$\sum_{j=1}^{N_{ch}} Q_j^{in}(G_j(x)-1) = K(x,Q_p) \qquad (1)$$

where the parameter

$$K(x,Q_p) \triangleq Q_p(1-G_p(x)) - \frac{r_M}{\tau}x - Q_{ASE}^{F+B}(x) \qquad (2)$$

is the pump flux that converts into EDFA output signal flux; here Q_p is the pump flux, $G_p < 1$ the pump gain, τ the fluorescence time, and r_M the total number of Erbium ions in the EDFA. The ESE therefore includes self-gain saturation by ASE[8,9].

Bandwidth: the EDFA is flattened by a gain flattening filter (GFF). The flattened bandwidth B is quantized in multiples of the bin size $\Delta\nu$. Let $A > 1$ be a reference span attenuation. As in[1], we define B (THz) at the reference A as $B = N_{ch}\Delta\nu$, where N_{ch} is the number of frequency bins at ν_j such that $G_j(x) \geq A$.

Fig. 1 shows in the inset the gain versus wavelength for EDFA data as in [1, Fig.7] where we also show a horizontal line at level A =9.5dB, i.e., the span attenuation considered in[1]. When we use an ideal GFF after the EDFA, we clip the EDFA gain at level A over the bandwidth B. The EDFA+GFF bandwidth B (THz) is plotted in the

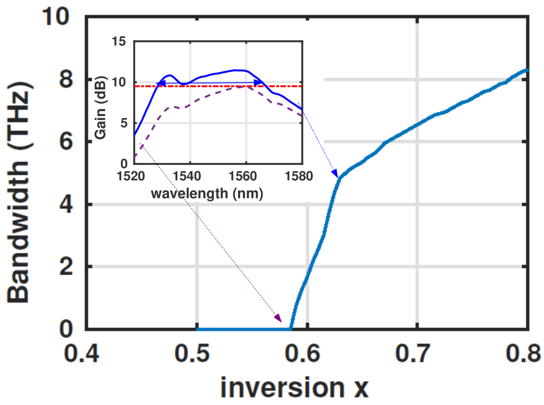

Fig. 1: Bandwidth vs inversion. Attenuation $A = 9.5$dB, EDFA length ℓ=6.27 m.

main Fig. 1 versus inversion x for A =9.5dB. B is zero at all inversions below the cutoff value 0.585 which is the largest x at which the gain curve is fully below the attenuation (dashed gain in inset). Above cutoff, B increases initially quite fast, and then more slowly after a "knee" (at $x = 0.63$), which (solid gain curve in inset) corresponds to the smallest x for which the gain trough at 1538 nm fully belongs to B, so that bandwidth comes from a compact wavelength set, instead of disjoint segments as at lower inversions.

Constant-gain chain

We now derive the AIR of a WDM link of M end-amplified single-mode fiber spans as a function of EDFA-1 inversion, pump and WDM allocation. The span attenuation is $A > 1$. Each end-span amplifier consists of EDFA+GFF, such that each amplifier has constant gain (CG) equal to A over its bandwidth, and zero else. EDFA length ℓ, absorption α and emission g^* parameters, and the power P_p are the same at all EDFAs. Amplifier+fiber is thus a unit-gain block over B, so that the transmitted WDM power flows unchanged down the line, while ASE accumulates. Hence the last amplifier will have the smallest inversion x_M and smallest bandwidth $B(x_M) = N_{ch}(x_M)\Delta\nu$, and this will be the overall bandwidth of our N_{ch}-channel WDM system.

Let $\underline{Q} \equiv \{Q_j\}$ be the set of transmitted (TX) WDM signal fluxes, $j = 1, .., N_{ch}$ (the *flux vector*). The pump flux Q_p and EDFA-1 inversion x_1 are fixed. TX flux vector $\underline{Q}$ must be *feasible* for the given pump flux Q_p and inversion x_1, i.e., it must satisfy the ESE (1) at EDFA-1:

$$\sum_{j=1}^{N_{ch}(x_M)} \frac{Q_j}{A}(G_j(x_1) - 1) = K(Q_p, x_1). \qquad (3)$$

The set of feasible input vectors represents a closed convex simplex, and is thus power-bounded. The inversions x_k of the subsequent EDFAs are recursively found from the known x_1, Q_p and the $\underline{Q}$ by solving the ESE at the k-th EDFA, for all $k = 2, ..., M$. Because of the unit span gain at every ν_j, Q_j is also the received (RX) flux after M spans, while the RX ASE flux is the sum of the span-input equivalent ASE fluxes:

$$N_j \triangleq \frac{A}{\Gamma} \sum_{k=1}^{M} F_j(x_k)\Delta\nu \qquad (4)$$

where we included an SNR penalty ($\Gamma = 0.79$ in all calculations[1]). Thus the AIR (per 2-polarization spatial mode) at fixed $Q_p, x_1, \underline{Q}$ is[1]:

$$AIR(Q_p, x_1, \underline{Q}) = \sum_{j=1}^{N_{ch}} 2\Delta\nu \log_2(1 + \frac{Q_j}{N_j}). \qquad (5)$$

Flux allocations

Let's consider a few interesting flux allocations $\underline{Q}$:

Constant SNR (CSNR): this is typical of most subsea systems. To equalize the SNR, the feasible fluxes must be proportional to noise (4), hence for $j = 1, .., N_{ch}$

$$Q_j = \frac{AK(Q_p, x_1)\sum_{k=1}^{M} F_j(x_k)}{\sum_{c=1}^{N_{ch}(x_M)}(\sum_{k=1}^{M} F_c(x_k))(G_c(x_1) - 1)}. \qquad (6)$$

Gain-shaped waterfilling: The optimal $\underline{Q}$ maximizes AIR (5) subject to the ESE constraint (3) at the first EDFA. *If we consider N_{ch} and N_j as independent of $\underline{Q}$* (the crucial assumption that makes GW sub-optimal), as with CW[4] we can set to zero the derivatives of the Lagrangian and get explicitly the GW solution for all $j = 1, ..., N_{ch}$ as:

$$Q_j = (\frac{\theta}{G_j(x_1) - 1} - N_j)^+ \qquad (7)$$

where $(.)^+ \triangleq \max(., 0)$. This flux allocation corresponds to waterfilling with a non-flat *inverse-gain-shaped* water-level $\theta_j \triangleq \frac{\theta}{G_j - 1}$ at all bins j. The water-level parameter θ is found by forcing compliance with the ESE at EDFA 1.

Classical waterfilling: we here choose for all $j = 1, ..., N_{ch}$[4]: $Q_j = (\theta - N_j)^+$, and the flat water-level θ is found by forcing compliance with the ESE constraint at EDFA 1.

Results

We analyzed the same CG chain with $M = 287$ spans with attenuation $A = 9.5$dB as in[1].

To highlight the bandwidth-SNR tradeoff at play

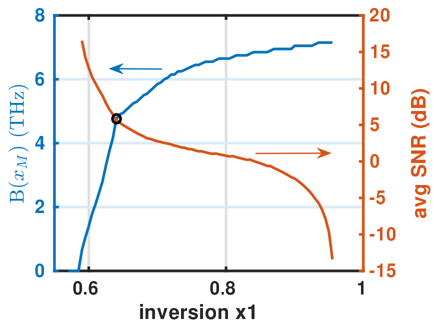

Fig. 2: WDM bandwidth $B(x_M)$ (THz) and average SNR (dB) versus inversion x_1 at $P_p = 60$mW, EDFA length $\ell = 6.27$m. Optimal inversion at $x_1 = 0.64$ is circled.

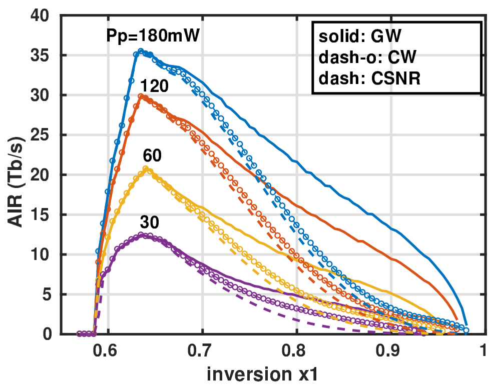

Fig. 3: AIR versus inversion x_1 at various pump powers. $M = 287$ spans with loss $A = 9.5$dB, EDFA $\ell = 6.27$m.

in AIR (5), Fig. 2 shows WDM bandwidth $B(x_M)$ (left axis) and average SNR (right axis) versus inversion x_1 for the GW allocation at $P_p = 60$mW and EDFA $\ell = 6.27$m. An optimum exists at $x_1 = 0.64$ (circle), i.e. at the "knee" of the corresponding $B(x_M)$ vs. x_1 curve.

Fig. 3 shows the AIR (Tb/s) versus inversion x_1 at various pump powers for $\ell = 6.27$m and allocations: GW (solid); CW (dash-o); CSNR (dash). At all pumps the AIR maximum is achieved at x_1 very close to the "knee". By searching over the feasible simplex, we verified that GW AIR is close to optimum at all inversions. However, GW is sharply superior to the other allocations only at the largest inversions. Reason is as follows: As in Fig. 2, at fixed pump, increasing x_1 implies decreasing WDM powers and thus SNR. Hence when increasing x_1 right above cutoff, SNR is largest and AIR is dominated by bandwidth expansion. In this situation any reasonable power allocation has similar AIR. This holds almost up to the top AIR. Then at larger x_1 and smaller SNR the quasi-optimal allocation becomes markedly better than the others, a known fact for scenarios where CW is optimal[10].

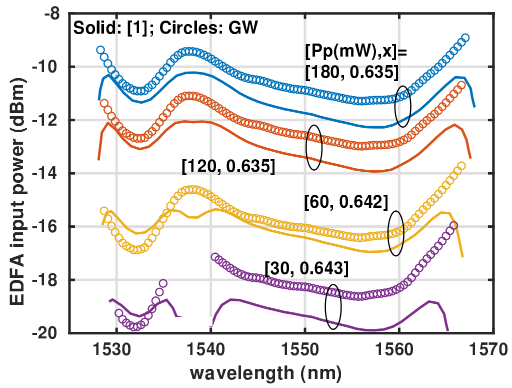

Fig. 4: Power at EDFA input vs wavelength at various P_p and optimal inversions, at $\ell = 6.27$m. Solid:[1]Fig.2a; Circles:GW.

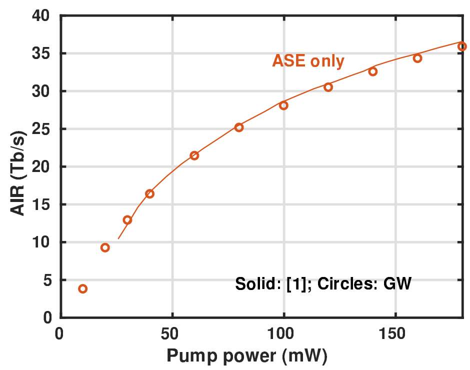

Fig. 5: AIR vs. P_p at optimized ℓ; Solid: [1,Fig.4a]; Circles: GW. $M = 287$ spans with loss $A = 9.5$dB.

Fig. 4 shows the WDM powers at every EDFA input at $\ell = 6.27$m and various pumps and at the optimal inversions, where symbols are our GW values and solid curves are extracted from [1, Fig.2a]; the GW power profile follows the inverse gain and reasonably matches the capacity-achieving profile in[1].

When EDFA length ℓ is optimized at all pumps, Fig. 5 reports the found top AIR values versus pump power for the GW allocation (circles), and compares with the published capacity curve in [1, Fig.4a]. We see that GW has just slightly inferior AIR than the capacity results in[1].

Conclusions

We confirmed the capacity results of submarine links with fixed EDFA pump in[1] and provided an analytical expression of the observed optimal power allocation. We called it the gain-shaped waterfilling, since power is allocated in inverse proportion to the EDFA gain. We also found the optimal EDFA inversion at capacity and provided a justification for the fact that, at typical submarine span loss, any reasonable power allocation performs close to capacity if operated at the optimal inversion[6].

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