

On Sparse Gain Flattening in Pump-Constrained Submarine Links

Alberto Bononi, *Senior Member, IEEE*, Paolo Serena, *Member, IEEE*, Chiara Lasagni, *Student Member, IEEE*, Juliana Tiburcio de Araujo, and Jean-Christophe Antona, *Member, IEEE*

Abstract—Having in mind the capacity optimization of power-constrained submarine links, by following the work in [1] we first compare the achievable information rate (AIR) of gain-flattened and un-flattened short links (called here blocks) of $N_b \leq 12$ spans with span loss 16.5dB and with end-span single-stage co-pumped erbium-doped fiber amplifiers (EDFA) when the transmitted wavelength division multiplexed (WDM) channels all have the same transmitted power. All EDFAs have the same pump power and the same physical parameters. In the flattened case, each EDFA is followed by an ideal gain-flattening filter (GFF) that chops off the EDFA gain exceeding the span loss. No GFFs are used in the un-flattened case. We show that, for block length $N_b > 7$, at large-enough input power the AIR of the GFF block exceeds that of the no-GFF block, while for $N_b \leq 7$ at large input power the AIR is about the same. We next build a long submarine link by concatenating many N_b -span no-GFF blocks, and placing a GFF at the last EDFA of each block in order to flatten the block gain down to the N_b -span loss, and calculate the AIR of the resulting sparse-GFF submarine link, accounting also for nonlinear interference. For a 287-span case-study link with span loss 9.5dB, we show that the best power efficiency is achieved by blocks of size $N_b = 6$ (i.e., one GFF every 6 spans) when the pump is around 11 mW. When the GFF excess loss is 0.3dB the top-AIR gain over the standard all-GFF system is 9.5%, a value that decreases to 4% when the excess loss is zero. Considering that modern submarine-grade GFFs have almost zero excess loss, and that the most efficient pump power is likely too low to operate with, we conclude that sparse-GFF links offer little advantage in practice over the current design.

Index Terms—Optical Communications, Optical amplifiers, Submarine transmission, Signal Droop.

I. INTRODUCTION

Gain-flattening filters (GFF) for Erbium-doped fiber amplifiers (EDFA) were proposed at the dawn of wavelength division multiplexing (WDM) [2]. Submarine system design with GFFs at every EDFA was introduced more than 20 years ago, at a time where only one technology/setting of transponders was possible, with a single fixed modulation format, forward error correcting code, and bit-rate. The goal at that time was to ensure that all WDM channels could have a minimum received signal to noise ratio (SNR), and therefore simple

solutions were sought: a homogeneous line, with equalization at each amplifier to ensure the spectrum remains flat or with a predefined tilt. More recently, the advent of rate-adaptive transponders able to operate close to the Shannon limit at any practical SNR, and the arriving FlexEthernet technology [3], have led the submarine industry to revisit the design optimization for this new scenario. Now each wavelength channel can carry data at a different information rate depending on its generalized SNR, which includes amplified spontaneous emission (ASE) noise, Kerr nonlinear interference (NLI), and transmitter (TX) and receiver (RX) distortions.

An optimized future system design will therefore no longer seek to ensure a minimum SNR / Q^2 factor for all channels, but to ensure that the sum of the information rates of all channels is maximized or reaches a customer target. In this new context, it is necessary to assess whether the traditional system design has become obsolete or not. A priori, the answer is not obvious. This motivated some recent reports based on machine-learning, where better power efficiency (i.e., achievable information rate (AIR) at fixed EDFA pump power) was achieved by partially removing the usual GFFs [1], [4], [5]. Recent experimental work also used a partial removal of GFFs [6].

In this paper we study the relevance of such new research direction and assess the non-obsolescence of the good old design in this new context.

For our analysis, we explore the settings where the link with partial removal of GFFs is more power efficient than the classical submarine link with GFFs at all amplifiers. For simplicity, we compare the two kinds of link when the transmitted WDM channels all have the same transmitted power, since the performance of optimized submarine links with such a flat input power is known to be quite close to that of the optimal input power profile [7], [8]. We accurately numerically model the transmission line by using the extended Saleh EDFA physical model with ASE noise self-saturation [7], [9], [10].

By following the work in [1], we first compare the AIR of gain-flattened and un-flattened short links of $N_b = 12$ spans with end-span single-stage co-pumped EDFAs at a span loss of 16.5 dB. All EDFAs have the same pump power and the same physical parameters. In the flattened case, each EDFA is followed by an ideal GFF that chops off the EDFA gain above the span loss. We show that, when the WDM input power is large-enough, the AIR of the unfiltered 12-span link is smaller than that of the link with gain-flattened EDFAs. We also show that, when the flattening filter is imperfect, with a

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A. Bononi, P. Serena, C. Lasagni are with the Dipartimento di Ingegneria e Architettura, Università di Parma, Parma 43124, Italy (corresponding author e-mail: alberto.bononi@unipr.it). J.-C. Antona and J. Tiburcio de Araujo are with Alcatel Submarine Networks, Villarcoux, France.

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significant residual frequency tilt as in [1], then the unfiltered link is superior at all powers, thus confirming the findings in [1]. Finally, we show that for shorter links with $N_b \leq 7$ there is no clear superiority of the GFF link even at large powers.

We next concatenate many un-flattened short links of N_b spans (which we call *blocks*) to build a long submarine link. We place an ideal GFF at the last EDFA of each block in order to flatten the block gain down to the N_b -spans attenuation. The obtained link is referred to as sparse-GFF. For various block sizes N_b , we calculate the AIR of the sparse-GFF link for the case-study 287-span submarine link with span loss 9.5dB considered in [7], [11]. EDFA-induced droop [12] is accurately included by the Saleh EDFA model. For such long sparse-GFF links we do consider also the nonlinear interference due to fiber propagation, analytically calculated at each span by an extension of the Gaussian Noise model [13] that accounts for the non-uniform WDM channel power in the line fibers [14]. Since the WDM powers used to calculate the generated NLI are those launched into each fiber, including signal, ASE and NLI at each frequency bin, the nonlinear signal-noise and signal-NLI interactions are automatically accounted for.

We show that, for the case-study link, blocks of size 6 and 7 achieve the largest possible power efficiency at an optimal pump power around 11mW, with a theoretical maximum top-AIR gain over the all-GFF reference link of 9.5% at 0.3dB GFF excess loss (i.e., background and splicing loss), and below 4% at zero excess loss. Such an AIR gain quickly disappears at larger pumps. Since today's submarine-grade thin-film GFFs can have excess losses well below 0.1dB, and the most efficient pump power 11 mW is likely too low to operate with, we conclude that sparse-GFF links provide negligible or no gain at all in top-AIR.

The paper is organized as follows. Section II tackles the AIR of short blocks of N_b spans in isolation, both with and without GFFs. Section III shows AIR results of 287-span sparse-GFF submarine links of various block sizes. Section IV concludes the paper.

This paper is an extension of the work presented in the short conference paper [15]. All numerical results except part of Fig. 2 are new with respect to [15].

II. GFF VERSUS NO-GFF SHORT LINKS

By following [1], we first compare the AIR of a single short link (a block) with N_b gain flattened or un-flattened end-amplified spans. The tacit assumption of this approach taken in [1] is that if the AIR of a single block is optimized, then also the AIR of a long concatenation of such blocks will be optimum.

The structure of the block is sketched in Fig. 1(top). It consists of N_b single-mode fiber spans with span loss $A > 1$ followed by a single-stage co-pumped EDFA. All EDFAs in the block have the same physical parameters and same optical pump power P_p , but possibly different inversions and thus gain G . As in [1], we consider a transmitted WDM signal composed of $N_c = 40$ channels with bandwidth $B_c = 50$ GHz, spaced by 100GHz, with carrier wavelengths from 1532.64 to 1563.80 nm, covering about 4THz in the C band. In the

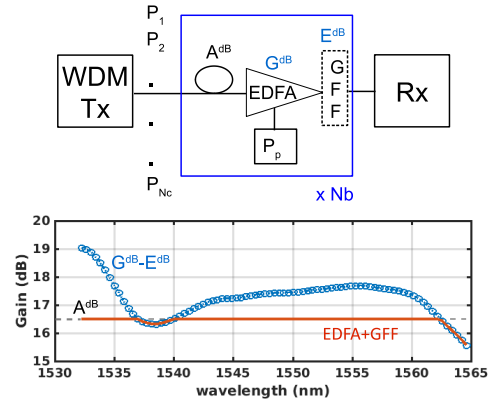


Figure 1. (Top) Single-mode WDM short link with $N_c = 40$ channels and N_b spans, span attenuation A and co-pumped single-stage EDFAs with optical pump power P_p . In the un-flattened case, no GFF is present. In the gain-flattened case, every EDFA is followed by a GFF with excess loss $E > 1$. (Bottom) Sketch of the EDFA unfiltered Gain G at channel locations (circles), decreased by the GFF excess loss, and amplifier gain after the GFF (solid line). The dashed horizontal line indicates the span loss A .

gain-flattened case, each EDFA is followed by an ideal GFF with excess loss $E > 1$, that chops off all the EDFA gain in excess of A on all channels such that $G/E = A$, while all remaining channels have loss E (see Fig. 1(bottom)). Since EDFA inversions in general differ from EDFA to EDFA in the block, the GFFs we consider are ideally tailored to each EDFA in the line, and to each pump and signal level, differently from experimental works where the GFF is tailored to a specific EDFA, pump, and signal power, and applied to all line EDFAs, also at different pump and signal powers [1], [6], [8]. As in [7], the EDFAs are numerically simulated by the homogeneously-broadened Saleh gain model [9], enriched with ASE noise self-saturation [10], i.e., forward and backward ASE generated inside each EDFA over a broad bandwidth from 1470 to 1670 nm is considered for calculating each EDFA inversion. Then only the WDM signal range is propagated down the line. This way, we realistically model both the correct EDFA gain and noise-figure frequency profiles. Having in mind transmission in a space division multiplexed (SDM) submarine link [1], [11], of which our single-mode link represents one spatial mode, we initially assume that only ASE impairs transmission, so that for a given input WDM power distribution $[P_1, \dots, P_{N_c}]$ the AIR is

$$AIR = 2B_c \sum_{j=1}^{N_c} \log_2(1 + SNR_j) \quad (1)$$

where SNR_j is the received SNR at channel j , i.e., the ratio of received signal power and cumulated ASE power, which depend on the inversions of the EDFAs in the block and their non-flat noise figures. For some selected values of the EDFAs common pump, we wish to compare the AIR of this block without and with GFFs as we vary the inversion x_1 of the first EDFA, which induces that of the remaining line EDFAs [7]. Throughout this paper, for the purpose of simple AIR comparisons among different systems, we assume a constant input power (CIP) transmission, where all 40 WDM channels have the same TX power P_c . Although the works in [1], [5]

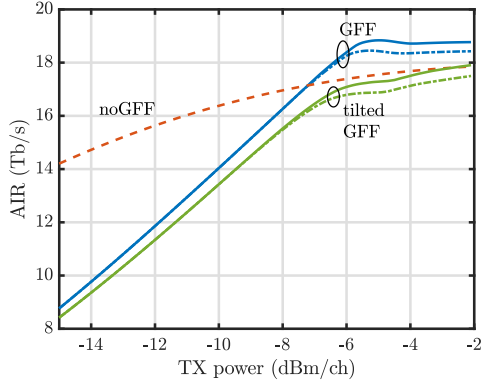


Figure 2. AIR (Tb/s) versus TX power per channel P_c (dBm) for a block of $N_b = 12$ spans, span loss $A = 16.5$ dB, pump power $P_p = 25$ mW, doped fiber length $\ell = 8.3$ m, 40 WDM equal-power input channels as in [1]. No GFF: dashed. True GFF: solid (no loss), dash-dot (0.3 dB excess loss). Tilted GFF as in [1]: solid (no loss), dash-dot (0.3 dB excess loss). At this low pump, we do not include NLI.

consider also an optimized, non-flat WDM distribution, it was shown in [7] that for GFF submarine links the CIP distribution (around the optimal EDFAs inversion) has AIR very close to Capacity, i.e., the AIR maximum over all possible input WDM distributions subject to the constraint on x_1 . Thus the CIP distribution is more than appropriate for comparing the no-GFF to the GFF systems.

A. Short link Results

For the above 40 WDM CIP signal, we analyze the AIR of a short 12-span link (i.e. a single block) with span loss 16.5 dB similar to the one in [1]. The 980 nm co-pumped single-stage EDFAs we use have the same absorption and emission profiles as in [11, Fig. 7]. In order to reasonably match the WDM power values after $N_b = 12$ spans in the unfiltered block in [1, Fig. 4b], with input WDM CIP total power 5 dBm ($P_c = -11$ dBm) into the first EDFA, we selected an EDFA length $\ell = 8.3$ m, with optical pump levels $P_p = [25, 80, 170]$ mW roughly corresponding to the [75, 150, 450] mA reference pump currents in [1].

For the lowest pump $P_p = 25$ mW, Fig. 2 shows the AIR versus TX power per channel P_c for the 40 channel WDM CIP input. The dashed curve shows the AIR in absence of GFF. The GFF label shows the AIR when using true flattening filters, both without loss (solid) and with 0.3 dB GFF excess loss (dash-dot). The tilted GFF label shows the AIR for an imperfect GFF such that the flattened gain equals the span loss at the center WDM channel, with a 2 dB linear tilt across the C band, as in [1]. Even here, solid line is for zero excess loss, and dash-dotted for 0.3 dB loss. Note that, in all plots in this paper, at the maximum value of the TX power P_c axis the total WDM TX power equals the pump power. From the figure we note that:

- 1) the tilted GFF AIR is always inferior to the no-GFF case, consistently with the findings in [1];
- 2) the no-GFF block has larger AIR than the GFF block at all powers $0 \leq P_c \leq P_c^*$ up to a cross-point P_c^* above which the GFF link is superior. This is true even considering

a GFF excess loss of 0.3 dB. The top AIR in the GFF case is reached at $P_c = -5$ dBm at zero excess loss, and 0.3 dB below at 0.3 dB loss. Although we show here only the $P_p = 25$ mW case, the qualitative behavior of Fig. 2 remains unchanged if pump power is increased by Δ dB, with an increase by about Δ dB of the TX power at top AIR in the GFF case.

The reasons of the GFF block superiority at point 2) are explored in Fig. 3, where at both a low TX power $P_c = -15$ dBm and at the GFF-optimal power $P_c = -5$ dBm we show for both the no-GFF and the (truly flat) GFF cases at zero excess loss the EDFAs Gain (EDFA 1, EDFA 12 and the link-average EDFA gain (dashed)) and the RX power after 12 spans, all plotted versus wavelength.

At low power ($P_c = -15$ dBm, left box), in the no-GFF case EDFA 1 works in the unsaturated small-signal regime with largest gain, but all remaining EDFAs work in deep saturation, with large output power and thus large RX SNR (the SNR can be deduced from the RX power on the ASE-only channels). In the GFF case instead the WDM signal does not saturate the EDFAs since the GFFs chop off all the EDFA gain in excess of the span loss A (horizontal dashed red line). Hence the EDFAs all work in the small-signal regime, with largest gain and a minimum noise figure. However the RX power is small and equals the TX power, and thus also the RX SNR is small. This is the regime where GFFs uselessly “waste power in the over-performing channels” [16], so that the no-GFF case has larger AIR.

As TX power grows ($P_c = -5$ dBm, right box), in the no-GFF block also EDFA 1 works now in saturation, and the RX SNR slightly increases, mostly because of the improved SNR at the first span. Also the GFF block now has all EDFAs working in saturation and thus large RX power, but the gain-chopping effect of the GFF spreads more evenly the RX SNR among channels compared to the no-GFF case, so that there is a larger number of WDM channels with a “significant” SNR, and the overall GFF AIR is larger than the one without GFFs. In fact, the AIR in eq. (1) is seen to linearly increase with the number of significant-SNR channels, while the SNR gives only a logarithmic contribution to the AIR. The top AIR in the GFF case is reached at the power such that some channels (in the region around 1538 nm) have EDFA gain that starts sinking below the span loss A , and the number of significant-SNR channels starts decreasing. If we push the input power above the optimum, more channels have sinking gain below A in the 1538 nm region even in the GFF block, but the GFF AIR still remains larger than the no-GFF AIR, at least for reasonably small GFF excess losses such as those employed in modern submarine links. While for blocks longer than 12 spans the AIR advantage of the GFF block over the no-GFF block becomes more striking and extends to a larger TX power range [15], Fig. 4 shows the AIR vs. P_c when considering short blocks of sizes $N_b = [12, 9, 7, 5, 3]$ at 25 mW pump, both for the no-GFF block (dashed) and the GFF block with 0.3 dB excess loss (solid). We note that the no-GFF AIR dominates the GFF AIR at all powers when $N_b \leq 7$. Does this dominance persist when concatenating no-GFF blocks to form a long submarine link? We explore the answer in the next section.

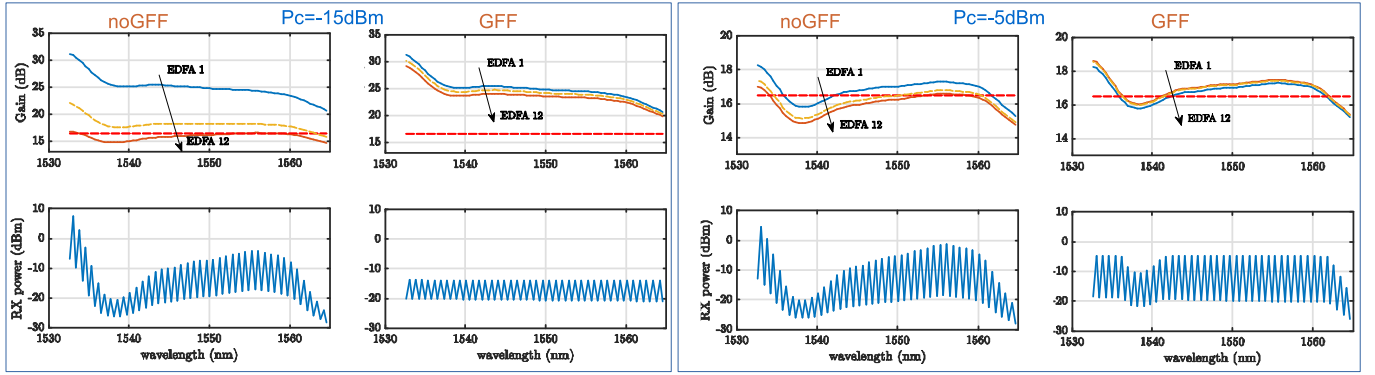


Figure 3. (top row) Gain at EDFAs 1, 12 (solid) and block-average gain (dashed) and (bottom row) RX power vs. wavelength, for a single block of $N_b = 12$ spans, span loss $A = 16.5\text{dB}$, doped fiber length $\ell = 8.3\text{m}$, 40 WDM channels (bandwidth 50GHz, spacing 100GHz), at pump powers $P_p = 25\text{mW}$. Both the GFF (zero excess loss) and the no-GFF cases are shown, at both a small TX power ($P_c = -15\text{dBm}$) and the GFF-top-AIR power ($P_c = -5\text{dBm}$).

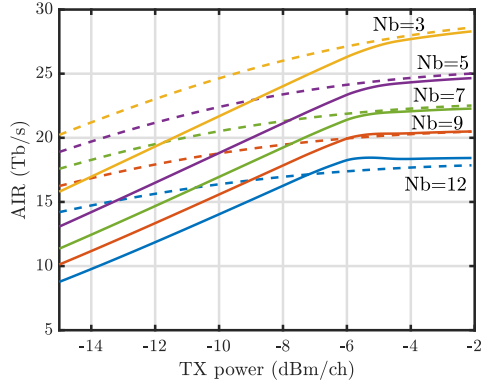


Figure 4. AIR (Tb/s) vs. P_c (dBm) for blocks of length [3, 5, 7, 9, 12] spans, 40 WDM CIP channels, pump $P_p = 25\text{mW}$, span loss $A = 16.5\text{dB}$, EDFA length $\ell = 8.3\text{m}$. no NLI. Solid: GFF with excess loss 0.3dB; dashed: no-GFF.

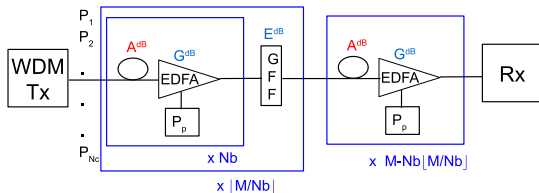


Figure 5. Block diagram of a sparse-GFF link, obtained by concatenating no-GFF blocks of size N_b , with GFFs in between them, up to a total M span link. All fiber spans have the same loss A . All EDFAs in the link have the same physical parameters and pump.

III. SPARSE-GFF LINKS

In this section, using the same 40 WDM CIP channels across the C band as before, we wish to compare the transmission over a long link with GFFs at all EDFAs, i.e., the reference case in deployed submarine links, against a sparse-GFF link of the same length composed of (see Fig. 5) a concatenation of no-GFF blocks, having a GFF at the last EDFA of each block that flattens the block gain down to the block loss at all channels for which the block gain (minus the GFF excess loss) exceeds the block loss, and just attenuates by its excess loss the remaining channels. The EDFA at the GFF

has the same pump and physical parameters as all remaining EDFAs. Similarly to [7], [11], we consider an $M = 287$ span submarine link, formed by a concatenation of $\lfloor \frac{M}{N_b} \rfloor$ no-GFF blocks of size N_b with end-block GFF having an excess loss of 0.3dB, plus the remaining $M - N_b \lfloor \frac{M}{N_b} \rfloor$ unflattened spans. The reference case with a GFF at every EDFA indeed corresponds to $N_b = 1$. The span loss is $A = 9.5\text{dB}$. EDFAs have a doped-fiber length $\ell = 5.3\text{m}$, optimized to the new span loss at a pump $P_p = 25\text{mW}$.

We now include NLI in the AIR computation as an extra Gaussian additive noise using the Gaussian Noise model [13]. For NLI calculations, we assume a pure silica core fiber (PSCF) with attenuation 0.162 dB/km, nonlinear index $n_2 = 2.5 \cdot 10^{-20} \text{ m}^2/\text{W}$, effective area $130 \mu\text{m}^2$ (yielding a nonlinear coefficient $\gamma = 0.78\text{W}^{-1}\text{km}^{-1}$) and dispersion 21 ps/nm/km. The span loss $A = 9.5\text{dB}$ includes 1.25dB of margin (as in [11]), hence the fiber span length is $L = 50.9\text{km}$. In this highly-dispersive line we calculate the NLI variance by including only single- and cross-channel interference, using [14, eqs. (124)-(126)], as also done in [17].

For the reference submarine link with GFFs at every EDFA ($N_b = 1$), Fig. 6 shows (a) AIR versus TX power per channel and (b) AIR versus EDFA 1 inversion x_1 , at the three pump values 25, 80 and 170 mW considered in [1]. Dashed lines are without NLI, solid lines include NLI.

When NLI is neglected, we see that the top AIR occurs at an optimal inversion around $x_1 \cong 0.68$ at all pumps [7], which corresponds to a common optimal gain-versus-wavelength profile, such that the gain at 1538nm is just slightly smaller than the span loss A , similarly to the $P_c = -5\text{dBm}$ box (GFF case) in Fig. 3. The corresponding optimal TX CIP power can be found in closed form as a function of x_1 by solving the Saleh equation [7, eq. (14)].

Note also that the dashed no-NLI AIR curves in Fig. 6(a) coincide at low power at all pump values, which indicates the presence of a signal-independent noise figure, i.e., the GFF line has EDFAs working in their small-signal regime, up to a little before the top AIR. The AIR maximum comes from the best compromise between the EDFA saturation-induced decrease

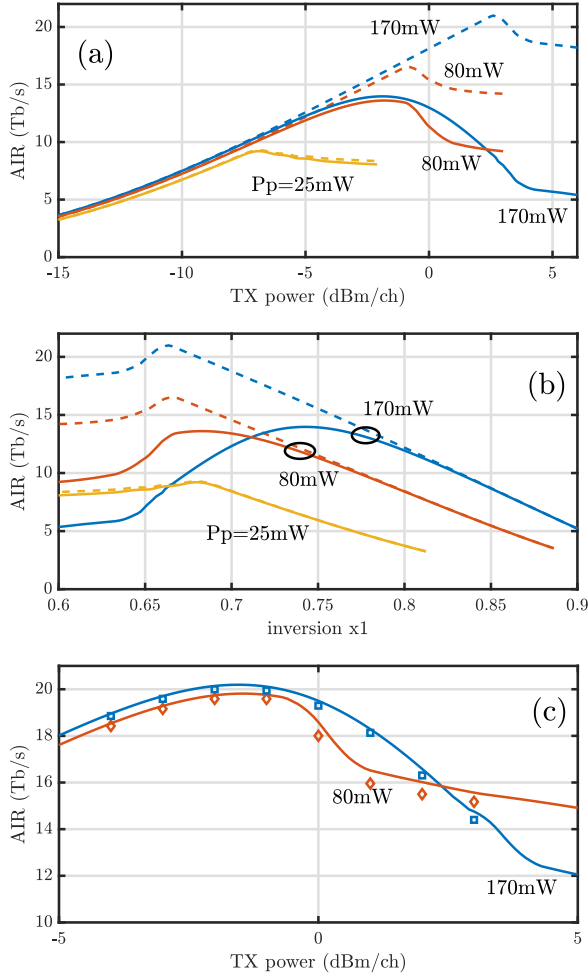


Figure 6. (a) AIR vs. P_c and (b) AIR vs. inversion x_1 of the first EDFA, for a reference GFF link ($N_b = 1$) with 287 spans, span loss $A = 9.5$ dB, span length 50.9 km, EDFA length $\ell = 5.3$ m, 40 WDM channels, at pump powers $P_p = [25, 80, 170]$ mW. Dashed: no NLI; Solid: with NLI on a PSCF fiber; span loss margin 1.25 dB. GFF excess loss 0.3 dB. (c) AIR vs. P_c at 100 spans and same data as above, at $P_p = [80, 170]$ mW, with NLI. Solid: theory. Circles: split-step simulations.

of the number of WDM channels with a significant SNR and their SNR increase with TX power. The AIR decrease after the maximum is mostly due to the power fading of several channels with a gain below the attenuation A .

If instead NLI is included in the calculations (solid lines), we see from Fig. 6(a) that the AIR curve at pump 25 mW is little affected, while the top AIR for pumps 80 and 170 mW is set by nonlinearity, which clamps the optimum TX power P_c to the NLI optimal power predicted by the GN model, here around -1 dBm/ch. We also note from Fig. 6(b) that the larger the pump, the larger the inversion of the EDFAs at the top-AIR working point. As a sanity check of the above results with NLI, Fig. 6(c) shows a split-step Fourier method (SSFM) estimation of the AIR versus TX power at the largest pumps 170 and 80 mW, for a shorter $M = 100$ span link with GFFs at all EDFAs, where SSFM simulations were feasible in a reasonable time. We used the same 40 WDM channel allocation as in the theory, modulated at 50 Gbaud per channel with root-raised-cosine

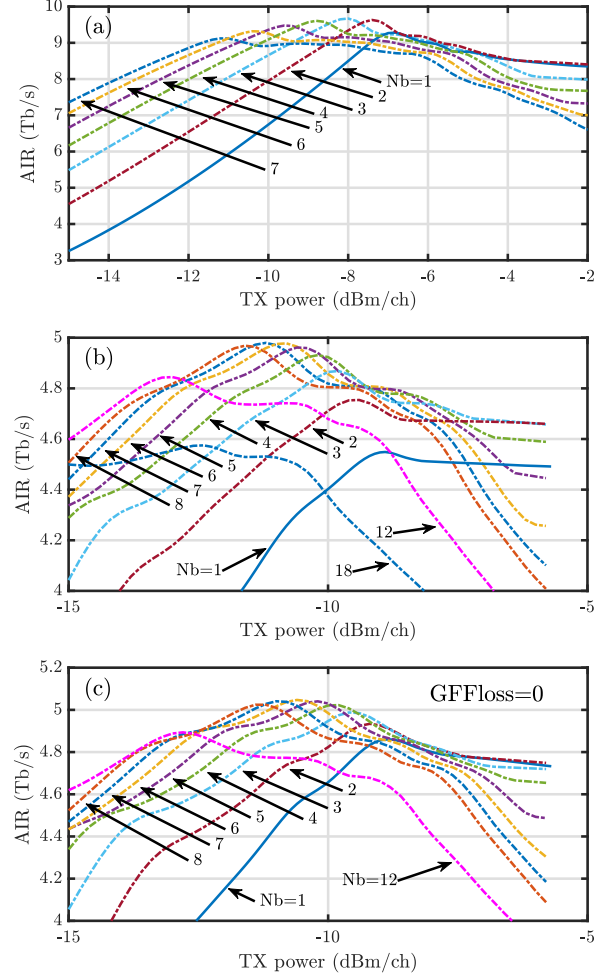


Figure 7. AIR vs. P_c for transmission of 40 WDM CIP channels over 287 spans, span loss $A = 9.5$ dB, EDFA length $\ell = 5.3$ m, at various sparse-GFF block sizes N_b ($N_b = 1$ is the reference all-GFF link). (a) $P_p = 25$ mW, GFF loss 0.3 dB (b) $P_p = 11$ mW, GFF loss 0.3 dB; (c) $P_p = 11$ mW, zero GFF loss. NLI included.

supporting pulses with roll-off 0.01 and complex Gaussian symbols (to be consistent with the used GN model [14]). On each channel we used a modulating sequence of 2^{17} symbols. The SSFM step size was chosen as in [18]. The inversion of each EDFA was obtained by solving the Saleh equation [7, eq. (14)] in which the input fluxes were derived from the Fast-Fourier Transform spectral lines of the EDFA input field. We see from Fig. 6(c) that our theory is in reasonable agreement with the simulated AIR. The slight AIR over-estimation by theory at total WDM signal power approaching the pump power is due to the theoretical assumption of a flat amplifier gain over each signal bandwidth, which ceases to be true on those channels with EDFA gain below the span loss in the final link blocks, which cannot be equalized by the GFF.

At the 25 mW pump, in a range of interest for SDM submarine links [6], [8], Fig. 7(a) shows for the above 287 span link the AIR versus TX power P_c for various sparse-GFF block sizes $N_b = [1 : 7]$, with block-connecting GFF excess loss 0.3 dB. We included NLI in the simulations, although its effect is sizable only at the largest powers. We observe that the

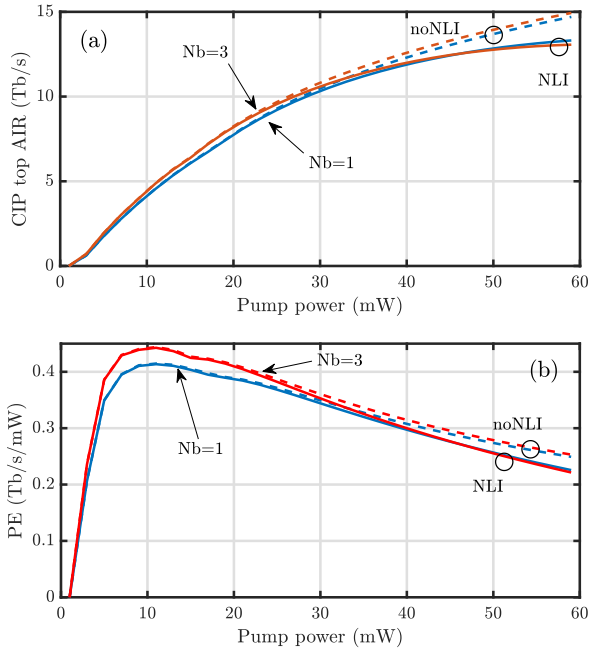


Figure 8. (a) top-AIR for our 40 WDM CIP channels vs. EDFA pump power P_p at optimal EDFA length 5.3m, both for reference all-GFF system ($N_b = 1$) and for sparse-GFF at optimal block size $N_b = 3$. Dashed: no NLI, solid: NLI. 287 spans, span loss $A = 9.5$ dB, span length 50.9km, PSCF fiber. (b) Power efficiency $PE = \text{top-AIR}/P_p$ versus P_p . GFF loss 0.3dB.

top-AIR for block sizes smaller than 7 exceeds the reference $N_b = 1$ top-AIR, with decreasing optimal power as the block size increases. The fact that the $N_b = 7$ case has top-AIR below that of the reference $N_b = 1$ all-GFF line shows that comparing the AIR of the blocks in isolation as done in Fig. 4 is a reasonable, but not perfect practice. The most important message is that, at 0.3dB GFF excess loss, the $N_b = 3$ block size at $P_b = 25$ mW is the optimal one, and it offers a 4% top-AIR increase w.r.t. the reference $N_b = 1$ case. We verified that at zero GFF excess loss, $N_b = 3$ is still the best size, but its top-AIR gain reduces to a negligible 0.6%.

Fig. 7(b) shows the AIR vs. TX power curves when the pump is reduced down to $P_p = 11$ mW, i.e., the most power-efficient pump value as we will see later. We observe that now the block size values that maximize the top-AIR are $N_b = 6$ and 7 with a gain over the reference $N_b = 1$ case that reaches its theoretical maximum of 9.5%, and for block sizes up to 18 the top AIR is larger than that of the reference $N_b = 1$ case. Hence the best block size depends on the pump value. Fig. 7(c) shows that, at $P_p = 11$ mW, $N_b = 6$ and 7 are still optimal at zero GFF excess loss, but their theoretical gain over the reference all-GFF ($N_b = 1$) case reduces to 4%.

For our 287 span system and block sizes $N_b = [1, 3]$, Fig. 8 shows (a) top-AIR and (b) power efficiency $PE \triangleq (\text{top-AIR})/P_p$ versus EDFA pump power P_p for EDFA length 5.3m. Dashed lines are without NLI, solid lines include NLI. From Fig. 8(a) we see that for our 40 WDM CIP signal the effect of

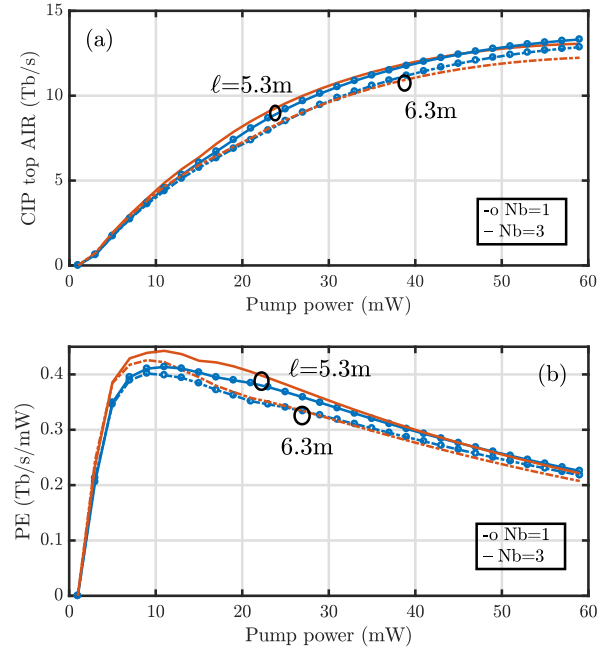


Figure 9. (a) top-AIR for our 40 WDM CIP channels vs. EDFA pump power P_p at both optimal EDFA length 5.3m (solid) and sub-optimal length 6.3m (dash-dot), both for all-GFF reference system ($N_b = 1$, circles) and sparse-GFF optimal block size $N_b = 3$. $M=287$ spans, span loss $A = 9.5$ dB, span length 50.9km, including NLI from PSCF propagation. (b) corresponding power efficiency $PE = \text{top-AIR}/P_p$ versus pump power.

NLI becomes visible above $P_p = 30$ mW¹. We also note that in the real case with NLI the $N_b = 3$ block ceases to have the largest PE at pumps above 45mW. The reason is that the larger output powers in the $N_b = 3$ block cause more nonlinear effects. Therefore it is *only at the very low pump powers envisaged for SDM submarine links that removing some of the GFFs may become advantageous*, as recently demonstrated in [6]. Fig. 8(b) reveals that the pump power at largest power efficiency is 11mW for both block sizes, and in line with the value extrapolated from the top-AIR versus pump power in [7, Fig. 6]. We also verified that at full bandwidth occupancy (80 channels at spacing 50GHz), the top power efficiency is obtained at 13mW. The larger pump values at top power-efficiency reported in recent system experiments [6], [8] are probably mostly due to the fixed GFFs used at all pump power values in the experiments [8].

To highlight the importance of selecting the correct EDFA length, Fig. 9 shows, for the case including NLI, and for block sizes $N_b = [1, 3]$, the top-AIR and PE versus P_p curves at both the optimized $\ell = 5.3$ m (same as in Fig. 8) and those for a sub-optimal length $\ell = 6.3$ m. We see that at 6.3m the $N_b = 3$ block is practically never superior to the reference all-GFF system. Also, the top power efficiency occurs at even lower pumps when the length is sub-optimal.

Fig. 10 finally shows the same top-AIR and PE curves as in Fig. 8 for $N_b = [1, 3]$ (only NLI case) but now also including

¹This is about half of the value found in [11, Fig.4a], [17] and is due to the 50% spectral occupancy of our WDM system. We verified that when populating also the ASE-only bins, for a total of 80 WDM channels with spacing 50GHz, the effect of NLI becomes visible above $P_p = 60$ mW.

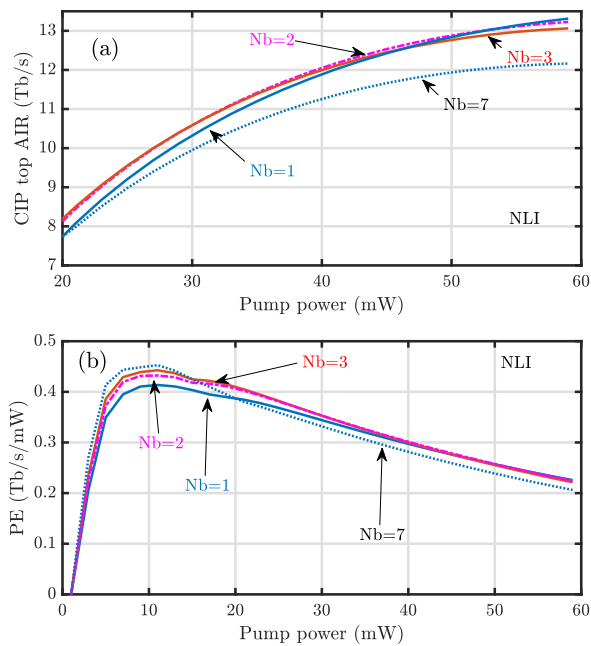


Figure 10. Same as Fig. 8 (only NLI case) but we added the $N_b = [2, 7]$ cases. GFF excess loss 0.3dB.

the block sizes $N_b = [2, 7]$. We see that in our case-study link the AIR of the $N_b = 2$ system is always very close but slightly inferior to the $N_b = 3$ system up a pump of 33mW. From 33 to 53mW, $N_b = 2$ is the best system. Above 53mW the $N_b = 1$ reference system is the best one, which confirms that for traditional submarine systems, which are operated at larger pumps, the all-GFF system is the best possible. We also see that at pumps above 20mW the $N_b = 7$ case is worse than the reference case since more affected by NLI, while it provides the best power efficiency at the best pump power 11mW.

IV. CONCLUSIONS

In this paper we critically verified the comparison presented in [1] between a short link (a block) of $N_b = 12$ EDFA amplified spans without GFF, and 12 spans of gain-flattened EDFAs, when the span loss is 16.5dB and the pump power is 25mW. We confirmed that the no-GFF block has superior AIR at all input powers when the flattening filters are imperfect, such as those in [1]. However, for truly flattening filters, the 12-span GFF block has superior AIR at large-enough input power, and this remains true for block lengths N_b down to 7. For shorter blocks there is no clear advantage of using the flattening filters. However, the comparison of such short blocks in isolation is not sufficient to prove superiority of long sparse-GFF submarine links, i.e., having one GFF every N_b spans. We thus studied such sparse-GFF links and found that in the 287 span, 9.5dB span loss case-study link tackled in [7], [11], for a 25mW pump power the most power-efficient choice is having one GFF every 3 spans, for GFF excess loss below 0.3dB. However, at the most power-efficient pump power around 11mW, the top-AIR and PE are achieved by block sizes $N_b = 6$ and 7, i.e., one GFF every 6-7 amplifiers. The top-AIR gain with respect to the standard all-GFF case is

9.5% at 0.3dB GFF excess loss, and decreases to 4% at zero excess loss.

The final system conclusion we draw from this study is the following. Since today’s submarine-grade thin-film GFFs can have excess losses well below 0.1dB, sparse-GFF links offer in practice less than 4% potential top-AIR gain over the traditional all-GFF links at the most efficient pump power 11 mW, which is likely too low to operate with. With higher, more practical pump values, sparse-GFF links have negligible or no gain at all in top-AIR. If one also considers the construction simplicity of the traditional all-identical amplifier design, then sparse-GFF links seem to offer very minor advantages over traditional ones.

We close with some notes on the limits of our investigation:

- 1) the no-GFF block could have been better optimized by allowing the last EDFA of the block which precedes the GFF to have length and pump power possibly larger than the remaining EDFAs, akin to the extra EDFA at the GFF considered in [5]. However, we find that preceding the GFF with an EDFA identical to the others is always a very reasonable choice, almost close to optimal;
- 2) although the comparisons are presented for a power-flat WDM input signal, the qualitative conclusions are not expected to change if optimized input allocations are used;
- 3) although our investigations have been restricted to a half-populated 1532-1564nm band for consistency with the reference work [1], we verified that the conclusions do not change when considering a fully-populated C band 1528-1565nm;
- 4) the used EDFA model disregards spectral hole burning (SHB). We expect SHB to partly smooth out the power excursions in the noGFF blocks [19], and thus further reduce the possible AIR advantage of sparse GFF links with respect to the traditional design.

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