Modeling of Signal-Noise Interactions in Nonlinear Fiber Transmission with Different Modulation Formats

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Abstract *We describe the physical principles behind nonlinear signal-noise interaction and its impact on dispersion-managed optical transmission system performance for both intensity and phase modulated signals.*

Introduction

Purpose of this paper is to highlight the principles of the nonlinear signal-noise interaction (NSNI) in dispersion-managed (DM) long-haul optical links, discuss the available analytical models for NSNI prediction, and provide a quantitative understanding of the system parameters for which NSNI sets the nonlinear performance of the most popular intensity and phase modulation formats, namely on-off keying (OOK), differential quadrature phase-shift keying (DQPSK) and coherent polarization-division-multiplexed quadrature phase-shift keying (PDM-QPSK).

Modeling of NSNI

Following the taxonomy of nonlinear effects presented in [1], Fig. 12, we will next consider both single-channel (or intra-channel) NSNI and wavelength division multiplexing (WDM) (or inter-channel) NSNI, and discuss the corresponding available analytical models.

A) Single-Channel NSNI Any optical signal of sufficiently large power propagating in a long-haul fiber link interacts with the additive white Gaussian amplified spontaneous emission (ASE) noise through a four-wave mixing (FWM) process. At the end of the link, the received ASE field probability density function (PDF) changes to a "bean-like" non-Gaussian shape [2], while the ASE power spectral density (PSD) changes from white to "colored", with a typical low-pass spectral enhancement around the signal optical carrier [3]. This NSNI is also known as parametric gain (PG) [3]. The so-called nonlinear phase noise (NLPN) [4] is the offset of the phase of the composite signal+ASE field with respect to the average nonlinear phase when only the low-pass nonlinearly enhanced ASE is considered.

As an illustrative example, we propagated with the split-step Fourier method (SSFM) [3] a continuous-wave (CW) signal along a singly-periodic 20x100 km DM terrestrial link with transmission fiber dispersion D_{tx} =4 ps/nm/km, full in-line compensation, pre-compensation of -85 ps/nm, and post-compensation adjusted so as to have zero total dispersion. The transmitted CW power was 4.6 dBm (0.6π rad of CW cumulated nonlinear phase). Gaussian ASE noise was added at each in-line amplifier, for a (linear) received optical signal-to-noise ratio (OSNR) of 12 dB over 0.1 nm (e.g. giving BER=10^{-5} in DQPSK at 10 Gbaud in linear regime). Fig. 1(left) shows the contours of the PDF of the normalized received field $\tilde{A} = A_r + jA_i = \mathrm{e}^{-j\Phi_{NL}}(1 + a_r + ja_i)$ (where Φ_{NL} is the phase of the average value of $\tilde{A}$) *after* an optical receiver filter of bandwidth 18 GHz, with the well-known "bean-like" contours down to probability 10^{-5}. For the same 10^7 time-samples used to obtain Fig 1(left), Fig 1(right) shows the estimated PSDs S_{ar}, S_{ai} of the in-phase and quadrature ASE components, respectively (i.e., as shown in Fig 1(left), the "radial" a_r and "tangent" a_i ASE components relative to the received CW), and the PSD S_ϕ of the phase of the received field, all measured *before* the optical filter. The dash-dotted horizontal line is the PSD of S_ϕ in absence of nonlinear Kerr effects (no-PG), which differs from the no-PG level of S_{ai} because the OSNR is rather low. All PSDs are normalized to the no-PG common PSD level of S_{ar}, S_{ai}. Dashed lines also show the ASE PSD predictions of a linear model of PG in DM links [5, 6], valid at large OSNR. From Fig. 1 we note that:

a) the PSD of the in-phase and quadrature ASE are correctly reproduced by the linear PG model [5, 6], except for an enhancement of the in-phase ASE near zero frequency. Such an enhancement – observable only at large-enough nonlinear phases and not captured by the linear PG model – is connected to the bean-like bending of the field PDF observed in Fig. 1(left) which induces both a variance increase and a non-zero negative mean value of the radial a_r component, as seen from the average field (circle marker) sinking slightly inside the unit circle in Fig. 1(left). Note that a simple formula is available ([7], eq. (11)) to predict at which power one should expect a significant increase of the zero-frequency PSD S_{ar} over its no-PG level for a given OSNR and fiber dispersion; such a formula is thus also useful to predict when PDF bending is significant.

b) the spectral shape of the received phase can be reasonably inferred from that of the quadrature ASE, except for an offset in the "white" high-frequency level, that would be present even in absence of PG when the OSNR is low and thus ASE ceases to be a small perturbation of the signal.

c) It was shown in [6] that the ASE PSDs (and thus also the NLPN PSD) in DM links with small in-line dispersion display a significant departure from the no-PG white shape over a one-sided bandwidth of about $2f_\Delta$, where $f_\Delta = \frac{1}{2\pi\sqrt{|\beta_2|/\alpha}}$, with β_2 the transmission fiber dispersion and α the fiber attenuation. In our simula-

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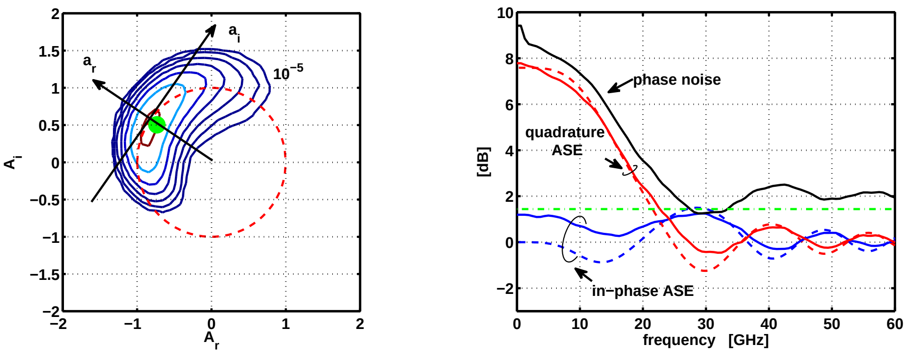

Fig. 1: (Left) simulated PDF contours (down to 10^{-5}) of received field (CW+ASE) after a 20x100 km DM link with fiber dispersion D_{tx}=4 ps/nm/km, zero in-line, -85 ps/nm pre-compensation, zero total dispersion, OSNR=12 dB/0.1 nm and CW power 4.6 dBm, after *optical filter of bandwidth 18 GHz. (Right) Solid lines: simulated power spectral density (PSD) of in-phase (S_{ar}) and quadrature (S_{ai}) ASE, and of the received field phase noise (S_{ϕ}) vs. frequency* before *optical filtering. Dashed lines: small-signal model [6] ASE PSD. Dash-dotted horizontal line: field phase PSD in absence of PG.*

tion, $f_{\Delta} \sim 16$ GHz, hence the optical filter of one-sided bandwidth of 9 GHz filters off not only the white high-frequency components, but also part of the colored spectrum. However, from the spectra in Fig. 1(right) it is clear that, as we increase the baud-rate and thus the optical filter bandwidth, the receiver will pass a larger and larger portion of the white noise, so that the "nonlinearly-enhanced" noise will contribute a smaller and smaller fraction of the total noise variance. As this happens, also the mix of white Gaussian linear noise and PG-correlated non-Gaussian noise will tend to become more Gaussian.

From the features observed in Fig. 1, we can make the following system considerations:

1) For intensity modulated signals such as OOK, it is the received field noise in the "radial" direction a_r that mostly affects performance. It was shown in [7] that single-channel OOK performance is markedly degraded by PG when the zero-frequency in-phase PSD level S_{ar} doubles with respect to its no-PG value (PG-doubling). The model in [7] is a pseudo-analytic BER prediction model for OOK signals in such a "strongly-nonlinear" PG. It postulates Gaussian elliptical PDF contours after the optical filter (i.e. neglects PDF bending) in order to use the Karhunen-Loeve (KL) theory of quadratic receivers in additive Gaussian noise. In [7], while the OOK modulated signal provided to the receiver came from a noiseless SSFM propagation, the ASE noise PSDs S_{ar}, S_{ai} were obtained from off-line, lengthy SSFM simulations with long time-sequences and an equivalent CW signal of appropriate level [8]. The effectiveness of the CW-equivalent trick for generating the ASE PSDs was later confirmed experimentally in [9]. Recently, Secondini *et al.* [10] came up with an analytical model capable of correctly reproducing the bean-like PDF shapes. Using such a model, one could thus also set-up a fast estimation by simulation of the correct S_{ar}, S_{ai} ASE spectra. Such "fast-generated" spectra could then be used in the KL model in [7] to considerably speed up computations.

2) For phase modulated signals, performance is instead set by the noise on the phase ϕ of the received field (i.e., the variations with respect to the average nonlinear phase Φ_{NL}) induced by the additive PG-distorted ASE, which can be thought of as the sum of a white linear phase noise ϕ_{lin} contributed by white Gaussian ASE (the one we would get in absence of nonlinearity), and of a colored nonlinear phase noise (NLPN) ϕ_{NL} [4]. Note that the statistics of NLPN and their impact on phase-modulated systems were studied analytically in the past only at zero group velocity dispersion (GVD) [4, 11, 2, 12]. In particular, the model by Ho [11, 2] for the derivation of the NLPN PDF at zero GVD considers ASE propagation only on the signal's bandwidth, and thus neglects the effect of the receiver optical filter on the ASE field statistics and thus on NLPN statistics. In absence of the tight optical filtering around the carrier along the DM line assumed by Ho (i.e. in the usual case), and with negligible GVD, the ASE PSD gets colored over a huge frequency band; when the ASE gets finally filtered at the receiver, its statistics get back to Gaussian [13]. Ho's BER predictions for 10 Gbaud DPSK and DQPSK in DM lines with standard single mode fiber (SMF) are close to real because the $2f_{\Delta}$ bandwidth over which ASE gets colored is close to 10 GHz, so that the "nonlinearly-inflated" ASE actually is present only over the signal's bandwidth, as in Ho's theory. This observation also hints to the fact that, for single-channel simulations with physically meaningful PG results, the SSFM fast-Fourier transform (FFT) frequency window should encompass at least the $\pm 2f_{\Delta}$ range, in order to correctly reproduce the "first-order FWM" products between signal and noise. Based on Ho's NLPN statistics model, the group of Kahn has developed very elegant BER prediction tools for coherent formats, including QPSK and quadrature amplitude modulation (QAM) [12].

While Ho's NLPN model has the above shortcomings, Secondini's recent method [10] can instead provide in principle the correct NLPN statistics for any DM system and CW signal, since it displays the correct ASE PDF in Cartesian coordinates, so that phase noise PDF can be obtained by switching to a polar coordinate system ([14], p. 146). It can also predict the "sinking" of the average field (eq. 26 in [10]). Even here, however, it is extremely challenging to obtain analytically the correct NLPN PDF after optical filtering.

Fortunately, it was shown in [8] that the performance of non-coherent differentially phase-modulated signals gets impaired by NLPN already at moderate nonlinear phases, so that the linear PG model [6] can be used to predict the NLPN PSD, and the (approximate) Gaussian nature of the ASE field allows one to use the KL method even for DPSK/DQPSK BER estimation, at least up to PG-induced penalties of 1-2 dBs [8].

In the previous discussion we concentrated on NSNI models based on the CW-signal assumption, which have been shown to be also applicable to modulated signals with both non-return-to-zero (NRZ) and return-to-zero (RZ) supporting pulses, provided that a correct equivalent CW-power is available [8]. However, a lot of research on NLPN has also been developed for phase modulated signals with RZ pulses, especially in the framework of DM-solitons. Most of these works are based on various kinds of perturbation analysis of the nonlinear Schroedinger equation (NLSE) around an RZ single-pulse ansatz. In this context we would like to mention the recent very general approach of Biondini and co-workers [15] on noise induced perturbations in the DM-NLSE, and the work of Kumar [16, 17] which is instead based on a first-order regular perturbation of the NLSE. Both approaches can provide the correct relevant single-channel NLPN statistics in presence of GVD in practical DM links.

B) WDM NSNI

Cross-phase modulation (XPM) is usually the dominant nonlinear effect in 10 Gb/s WDM OOK DM transmissions [1]. With OOK neighboring channels, the XPM is almost entirely due to the modulation-induced large power excursions of such channels. However, in WDM DQPSK and coherent QPSK transmissions, where the neighboring channels have a periodic envelope, then the XPM induced by the periodic power fluctuations gets completely suppressed by the phase difference operation performed by the interferometric DQPSK receiver [18] or the generalized phase difference operation [19] performed by the Viterbi and Viterbi (V&V) phase estimation algorithm commonly implemented in digital signal processing (DSP) based coherent receivers [20]. In such a case, what is left is the XPM due to non-periodic power fluctuations induced by ASE (XPM-NLPN [1]), which is a Gaussian process when XPM distortions can be modeled as small-signal perturbations. What in OOK is a second-order (and thus always neglected) effect, in phase modulated system emerges as a potentially dominant source of performance degradation [21]. A simple yet very effective model for BER evaluation with dominant XPM-NLPN was proposed by Ho in [18], where an approximate model of XPM in DM lines was used (for a discussion, see [19]) to get the variance of the Gaussian XPM-NLPN, and then known formulas for BER evaluation for phase modulated signals with Gaussian phase noise were applied [2]. In [18] a white ASE PSD on the "pump" channels was assumed. Hence another approximation in Ho's analysis is that it neglects the ASE PG on "pump" channels, i.e. the distributed evolution of the ASE PSD along the line while it generates XPM on the "probe" channel.

Nonlinear Threshold

In this section we illustrate when NSNI has to be taken into account in performance evaluation for three modulation formats, namely OOK, DQPSK and PDM-QPSK. Our target is to understand the dominant nonlinear effects, and the approach in this section is by Monte-Carlo (MC) simulation. For a 20x100 km DM system with D_{tx}=17 ps/nm/km, in-line dispersion 30 ps/nm/span, "optimized" pre-compensation -370 ps/nm for OOK and -270 ps/nm for phase modulations, optimized total dispersion equal to zero for phase modulations and non-zero for OOK, we evaluated the transmitted average power (called *nonlinear threshold*, NLT) that gives 1 dB of OSNR penalty at BER=10^{-3}. In each MC run, 256 new random symbols were transmitted, as well as random ASE noise samples, until counting 100 errors. This roughly corresponds to a relative BER estimation error of 10% with confidence 68%. This approach has been shown to be preferable over the use of pseudo-random sequences at large BER=10^{-3}, especially at large dispersion [22]. Classical direct-detection and interferometric receivers [1] were assumed for OOK and DQPSK, respectively, and standard DSP-based receivers for coherent PDM-QPSK [20]. NRZ pulse shaping was assumed for all formats. All receivers had a Butterworth-6 optical filter with bandwidth 1.8 times the symbol rate R, and (except for coherent) a Bessel-5 electrical filter of bandwidth 0.65R. For the coherent receiver we neglected laser phase noise, assumed a perfect compensation of the DM line Jones matrix so that perfect polarization demultiplexing was achieved in linear transmission, and the V&V phase estimation had 3+1+3 taps. In WDM simulations, we assumed all channels had the same modulation format and we fixed the fractional bandwidth utilization $\eta \triangleq \frac{R}{\Delta f} = 0.4$, where Δf is the channel spacing (e.g., at R=10 Gbaud, Δf=25 GHz). We scaled the number of channels N_{ch} with the symbol rate according to the law [23]: $N_{ch} = \lceil \frac{\eta}{S} \rceil$, where $S = \frac{\beta_2}{\alpha} R^2$ is the map strength (e.g., at R=10 Gbaud and D_{tx}=17 ps/nm/km, N_{ch}=11). The FFT window covered more than 3 times the WDM bandwidth, to correctly capture first order FWM. The maximum SSFM nonlinear phase per step was 0.003 rad. All fibers

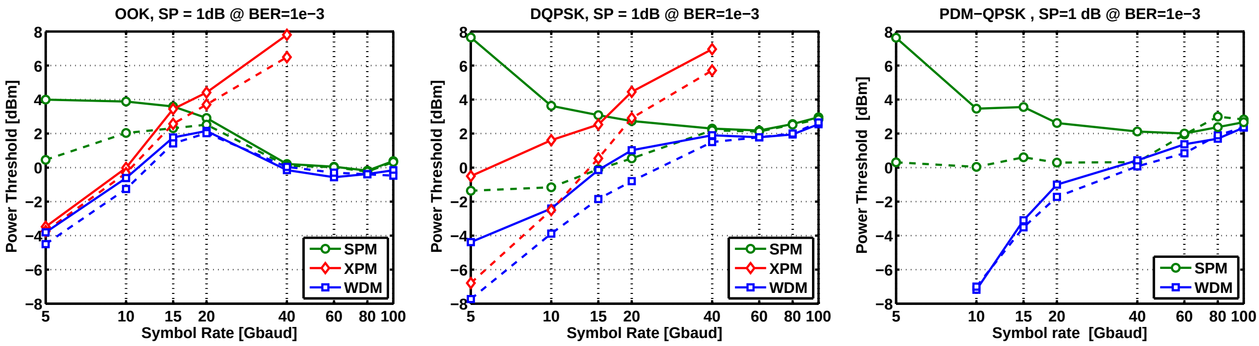

Fig. 2: Power threshold for SMF DM line for: (left) NRZ-OOK, (center) NRZ-DQPSK, (right) coherent PDM-NRZ-QPSK. Solid: no NSNI. Dashed: with NSNI. "SPM"= single-channel; "XPM"=no SPM, no FWM; "WDM"=all nonlinear effects.

had zero dispersion slope. For PDM simulations, the transmission in each fiber was emulated with the Manakov equation, polarization mode dispersion was absent, and WDM input states of polarization were random.

In Fig. 2 we show the NLT versus symbol rate R for: 1) single-channel transmission (label "SPM"); 2) WDM transmission with solution of individual NLSEs for all channels, with both self-phase modulation (SPM) and FWM switched OFF (label "XPM"); 3) WDM comb propagated as a single channel, hence with all nonlinearities ON (label "WDM"). In all three cases, we provide both the NLT obtained by noiseless signal SSFM propagation and white Gaussian noise loading at the receiver (solid lines, no NSNI case), and by noisy signal propagation with distributed ASE generation at each amplifier (dashed lines, case including NSNI).

Discussion In the single-channel OOK case we note that NSNI is significant at lower symbol rates, and ceases to be a problem at about 20 Gb/s for SMF fiber. The x-axis should more properly be the strength $S \propto D_{tx}R^2$ [23], so that e.g. for a fiber with $D_{tx} = 4$ ps/nm/km we expect NSNI to become negligible at 40 Gb/s. In the OOK WDM case we note that XPM sets the NLT at lower bit rates, as confirmed by the match of the "XPM" curve with the "WDM" curve. Only in the restricted range $R \in [15-20]$ Gbaud single-channel NSNI ("SPM" dotted) dominates over XPM ("XPM" dotted). In all cases NSNI changes the NLT by less than 0.5 dB.

A completely different scenario applies with phase modulated formats. In single-channel DQPSK we see that NSNI (i.e. NLPN) imposes much lower thresholds than in OOK, but the effect disappears around 40 Gbaud, since the linear component of phase noise passed by the receiver dominates over NLPN, as discussed at point c) in the previous section. In WDM DQPSK we see from the "WDM" curves that NSNI must be taken into account at lower symbol rates, while again after 40 Gbaud NSNI becomes negligible. Comparison of "WDM" and "XPM" curves reveals that, while signal-induced XPM is negligible compared with FWM ("XPM" vs. "WDM" solid), the XPM-NLPN ("XPM" dashed) is the dominant WDM nonlinear impairment at low R. In the range $R \in [15-40]$ Gbaud we see that single-channel NLPN ("SPM" dotted) dominates over XPM-NLPN ("XPM" dotted).

Finally for PDM-QPSK (for which the "XPM" curves were not theoretically available) we note that, while in the single-channel case NLPN is the dominant impairment up to 60 Gbaud, in the WDM case the impact of NSNI seems rather weak. This is attributed to the dominant role of another WDM nonlinearity, namely cross-polarization modulation, whose basic mechanisms are explained by some existing analytical models developed for OOK [24, 25].

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