## Nonlinear Limits in Single- and Dual-Polarization Transmission

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*Abstract*— The dominant nonlinear effects in single- and dualpolarization multichannel dispersion-managed optical transmissions are reviewed through an exhaustive simulation study of the nonlinear threshold with nonlinearity separation.

The long-haul performance of wavelength division multiplexed (WDM) optical transmissions is largely determined by the interplay of self- and cross-channel Kerr nonlinearities with chromatic dispersion. Dispersion management (DM) techniques have been developed to minimize Kerr nonlinear distortions, mostly for direct-detection demodulated formats [1]. However, most of the research interest in longhaul communications has today shifted towards digital signal processing (DSP) enhanced coherent detection with phase modulated formats, such as polarization-division multiplexing (PDM) quadrature phase shift keying (QPSK) [2]. Aim of this presentation is to review and complement a systematic study by simulation [3], [4] of the dominant nonlinearity in DM WDM homogeneous systems (i.e., where all channels have the same modulation format) for both single-polarization (SP) and PDM coherent transmissions over standard single-mode fiber (SMF) links, as a function of the per-channel baudrate.

## I. NONLINEAR THRESHOLD SIMULATIONS SET-UP

For any map and modulation format, the importance of nonlinear effects is well summarized by the nonlinear threshold (NLT), defined as the average transmitted power that produces 1 dB of optical signal-to-noise ratio (OSNR) penalty at a bit error rate (BER) of  $10^{-3}$  [5]. We have performed a systematic study by simulation of the NLT versus baudrate R when only one selected nonlinearity is active, while all others are turned off, and compared such individual NLTs to the true NLT obtained when all nonlinearities are simultaneously active. The result is a new kind of graphs that clearly reveal the dominant nonlinearity in each "map strength" regime, and show when two or more nonlinear effects strongly interact in setting performance [3], [4]. The considered single-channel nonlinear effects are: i) (noiseless) self-phase modulation (SPM), which we obtain in single-channel propagation with noise loading (i.e. all the amplified spontaneous emission (ASE) noise is loaded at the receiver); ii) nonlinear phase noise (NLPN), i.e., SPM in the real case when ASE is added at each amplifier. The cross channel effects are: iii) cross-phase modulation (XPM) [6], obtained in noiseless WDM propagation (with other nonlinearities off) and noise loading ; iv) cross nonlinear phase noise (X-NLPN) [7], obtained in WDM propagation (with other nonlinearities off) and distributed ASE; v) cross-polarization modulation (XPolM)

[6], both with noise loading and distributed noise (with all other nonlinearities off).

All the details of the NLT calculations are reported in [4]. Here we briefly summarize the main system assumptions. The reference DM line is a 20x100 km singly-periodic SMF terrestrial system with 30 ps/nm of residual dispersion per span (RDPS) and optimized pre- and post-compensating fibers.

In simulations, fiber polarization mode dispersion (PMD) was not considered. The number of WDM channels was selected large enough to well reproduce XPM. Correct simulations for XPolM would instead require a prohibitive number of channels in terms of simulation time [6]. The channel spacing  $\Delta f$  was scaled with baudrate so as to keep a constant bandwidth efficiency  $\eta = R/\Delta f$ =0.4. WDM channels were multiplexed without optical filtering, so that some residual linear crosstalk due to spectral overlap was present at  $\eta = 0.4$ .

The coherent optical receiver was a standard DSP-based receiver [2], with zero frequency offset between incoming signal and local oscillator, no laser phase noise, and the number of taps in the Viterbi and Viterbi (V&V) phase estimator was equal to 27, large enough to suppress linear crosstalk induced phase noise at lower baudrates [4]. No nonlinear phase noise electronic compensation was applied.

Each NLT was obtained by varying the noise figure of the optical amplifiers until BER= $10^{-3}$  is measured by Monte-Carlo error counting at 2000 km with a 1 dB penalty with respect to the back-to-back OSNR. About 100 errors were counted to estimate each BER value. In the SP case we used scalar propagation, i.e. the case of all copolarized WDM channels. In the PDM case, the input states of polarization were randomly and independently chosen at each run.

## **II. RESULTS**

Fig. 1 shows the NLTs versus baudrate R for (left) SP-QPSK and (right) PDM-QPSK. NLT in PDM is the total power of both polarizations. In all plots, solid and dashed lines denote the cases of noise loading and distributed noise, respectively. We distinguish the following effects:

**SPM** (circle markers): these are single-channel NLTs, both with unrealistic noise loading and with the realistic case of distributed noise that captures the nonlinear signal-noise interaction leading to NLPN;

**XPM** (diamond markers): here we solved the set of coupled nonlinear Schroedinger equations (NLSEs) for all channels (which neglect both four-wave mixing (FWM) and spectral overlap), with SPM (and XPolM in the vectorial case) turned OFF;



Fig. 1. Nonlinear threshold vs. baudrate for 20x100 km SMF system with 30 ps/nm RDPS. (Left) single polarization (SP); (right) dual polarization (PDM).

**XPolM** (triangle markers): here we solved the set of vector NLSEs and turned OFF the XPM and SPM operators, as defined in [6];

**WDM** (square markers): here we solved the single NLSE (scalar for SP and vectorial for PDM) for the entire WDM field. Hence all nonlinear effects, including FWM, are ON. The true total WDM NLT is the one obtained in the distributed noise case.

We now comment on the results, starting from single-channel performance. If the two polarizations did not interact, PDM would have a NLT 3 dB higher than SP. However, Fig 1 shows that the NLT difference is only  $\sim 2$  dB at R = 5 Gbaud, and fades to 0 dB at larger baudrates. Let's explain why. We note that single-channel performance is dominated by NLPN (circles, dotted) at lower baudrates in both SP and PDM.

In single-channel SP-QPSK at small R the sensitivity penalty is proportional to the NLPN variance [7], which in turn scales as the *square* of the average nonlinear phase [8]. In single-channel PDM-QPSK at small R, according to the Manakov equation [6], the SPM modifies instead the phase of each polarization (X,Y) by the same amount:

$$\phi_x = \phi_y = \gamma L_{eff} (|A_x|^2 + |A_y|^2) \tag{1}$$

where  $L_{eff}$  is the effective length over all spans and  $\gamma$  the nonlinear coefficient. At equal per-polarization power, the NLPN variance will thus be twice that in SP. Since penalty is again proportional to NLPN variance, we understand that at threshold we must have:

$$NLT_{SP}^{2} = 2(\frac{NLT_{PDM}}{2})^{2}$$
 (2)

where the 2 comes from NLPN variance doubling in PDM, and  $NLT_{PDM}/2$  is the power on each polarization at threshold. Hence we conclude that nonlinear coupling of X-Y gives a NLT  $10 \log_{10} \sqrt{2}$ =1.5 dB higher in PDM than in SP. Experiments at 10 Gbaud support such findings [9]. Another 0.5 dB difference of the observed 2 dB difference at low *R* is explained by the fact that  $\gamma$  in the vector Manakov equation is 8/9 than that in scalar propagation. From Fig. 1 we also see that NLPN is dominated by noiseless SPM above 40 Gbaud for both SP and PDM. When noiseless SPM dominates, penalty scales quadratically with average nonlinear phase, hence again from (1) we understand that we must have:

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$$VLT_{SP}^2 = NLT_{PDM}^2 \tag{3}$$

hence we have equal NLT for SP and PDM, as confirmed by Fig. 1 (circles, solid), except for a 0.5 dB offset due to vector propagation. Hence PDM is more penalized here with respect to SP than at lower baudrates.

Let's move to cross-channel effects. XPM depends on the total intensity (X+Y) of the field. In DM links and QPSK modulation, the modulated intensity is periodic, hence noiseless XPM (diamonds, solid) is almost completely removed by the differential phase detection implicit in the V&V estimator. Thus for the SP case, the X-NLPN (or noisy XPM, diamond dotted) is the dominant nonlinear mechanism at baudrates below 20 Gbaud; NLPN dominates between 20 and 40 Gbaud, and finally single channel SPM dominates (the 0.4 dB gap between WDM and SPM is here due to linear crosstalk: with optical filtering at the transmitter or at lower values of  $\eta$  such a gap disappears). The story is completely different for PDM. Here we note that XPolM is by far the dominant nonlinear impairment up to  $\sim 30$ Gbaud, then NLPN dominates (SPM dotted) and finally we have convergence to the single channel noiseless SPM performance above 40 Gbaud. Note that XPolM mostly depends on the modulation-induced random re-orientations of the Pivot in Stokes space [6], [10], hence the influence of ASE on XPolM is a second-order effect, as it is on XPM for OOK modulation [4]. Such observation justifies the closeness of the solid and dashed WDM NLT curves in the PDM case.

Acknowledgments: It is a pleasure to acknowledge stimulating discussions with O. Bertran-Pardo of Alcatel-Lucent Bell Labs France.

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