Analysis of ShuffleNets with limited number of wavelength converters employing deflection routing

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Optical node performance analysis in terms of number of wavelength converters for a multihop optical ShuffleNet under deflection routing is presented. From the computed results for a given number of wavelengths, it is found that in order to achieve the minimum deflection probability at full load, the number of wavelength converters required is at most 60% of the number of wavelengths. Any additional wavelength converters would not be necessary in reducing the overall deflection probability. These upper-bound findings are indeed helpful for network engineers designing a cost-effective network node. © 2004 Optical Society of America

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1. Introduction

The multihop optical network based on wavelength-division multiplexing (WDM) technology has received extensive study since 1987 [1, 2]. Various types of multihop regular network topologies have been investigated. Two common topologies that are still being used for performance analysis are ShuffleNet and Manhattan Street networks [3]. A simple deflection routing scheme, so-called hot-potato routing, has been proposed to reduce the packet contention probability. This scheme allows packets to be deflected to other nodes when a contention occurs between packets at a particular node. Consequently, deflected packets need to take more hops to reach their destinations. The performance of the hot-potato routing scheme based on ShuffleNet topologies has also been studied and compared with the conventional store-and-forward routing scheme [4]. Furthermore, Forghieri *et al.* have done and presented an extensive study comparing the hot-potato routing networks with single-buffer routing networks [5]. Results show that single-buffer deflection routing recovers more than 60% of the lost throughput of hot potato with respect to store-and-forward when uniform traffic is assumed. Bononi and Prucnal have investigated various access techniques, to improve the performance of multihop deflection routing optical networks. Results show that bypass queuing access technique outperforms others [6]. Following that, Bononi *et al.* have extended that research to wavelength-convertible multihop optical networks with deflection routing. Three new access schemes were proposed and analyzed in Ref. [7]. Computed results indicate that with a small number of wavelengths, when transmission is feasible, it may be preferable to use optical buffers than to employ wavelength converters. From the model mentioned previously, Chien *et al.* evaluated the bit-error-rate performance both with and without wavelength-conversion multihop optical networks by employing convolutional coding [8, 9]. Also, Chien *et al.* proposed hot-potato routing with buffers to enhance the throughput/delay performance and reduce the number of wavelength converters needed, compared with the hot-potato routing multihop wavelength-convertible networks [10].

In this paper, we further investigate the performance of multihop wavelength-convertible ShuffleNet with hot-potato routing that employs a limited number of wavelength converters. The probability model presented in Ref. [7] is extended in order to have a more accurate analytical model. All formulations are addressed in Section 3. We consider the main contribution of this paper to be as important as the research previously presented, such as the issues of sparse wavelength conversion and wavelength-converter placement in optical networks [11, 12], because from this research, we expect that the number of wavelength converters needed to achieve the highest throughput in multihop ShuffleNet for each node can be optimized. The characteristics of multihop ShuffleNet with limited number of converters in each node can be further explored.

2. Description of Logical Node Operation

Figure 1 illustrates the logical structure of a node. There are two input and output fibers in each node. All the n_w wavelengths from each input fiber are demultiplexed and sent to a stack of n_w modules. It is assumed that all functions in each module—such as packet absorption, injection, wavelength conversion (λ conversion), and routing—are sequentially and independently performed. In the final stage, packets are remultiplexed onto the output fiber to be sent to the next nodes according to the shortest-path algorithm.

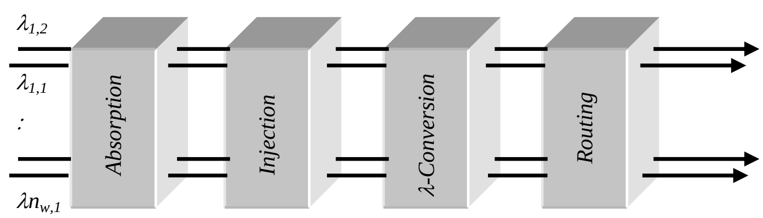


Fig. 1. Logical structure of node.

The absorption block is assumed to have one receiver per input wavelength to ensure that all packets that are destined to the node can be dropped. On the other hand, the injection block transmits the locally generated packets and the process can take place only when there is at least one empty slot. The function of the wavelength-conversion block is to solve packet contentions by rearranging the packets on the various wavelengths to minimize the wavelength conflicts before sending them to the routing block. There are a total of n_c wavelength converters in the wavelength-conversion block, where n_c is in the range of $1 \leq n_c \leq n_w$. The routing is a simple unbuffered 2×2 switch. In cases of output contention, packets are selected randomly, and the selected packets are deflected to the undesired port.

Four categories of packets are defined. "Don't Care" (DC) is a packet that can take either output, whereas "Care 1" (C1) is a packet that cares to exit on output 1. Similarly, "Care 2" (C2) is a packet that cares to exit on output 2. "For Node" (FN) means that a packet is destined to a node. Slots on each wavelength can be empty (E), or carry a FN packet, or carry a C1 packet, or carry a C2 packet, or carry a DC packet. At every clock, we label the time slots from the two input fibers (after the absorption block) as i_j, $j = 1, 2, \ldots, 2n_w$. Here we make the usual key assumption that i_j's are independent random variables with identical probability distribution $f_i = \{P(i_j = s), s \in \{\mathrm{E}, \mathrm{DC}, \mathrm{C2}, \mathrm{C1}\}\}$. This assumption leads to accurate results only when the topology is regular and the input traffic is uniform, as in our case. Also, in this paper we assume uniform traffic, which allows simple comparisons of node structures and control algorithms, and the conclusions usually hold true in most nonpathological nonuniform traffic scenarios [5]. Also, since injection of packets is operated at random on the available empty slots, the slots (empty or with packets) at the input of the node remain independent random variables as they were before injection.

3. Wavelength-Conversion Algorithm and Traffic Analysis

Before a packet is sent to the routing block, it is directed to the wavelength-conversion block to solve contention and to avoid deflections. The wavelength-conversion algorithm is as follows:

Modules with contending input packets are grouped in two sets: set A (C1, C1) and set B (C2, C2). Let a be the number of elements in set A, and let b be the number of elements in set B. Subsequently, modules without input contention are grouped in other three sets: (1) set C, modules that do not contain any C1 packets and empty slots [(C2, DC), (DC, C2), (DC, DC)] ; (2) set F, modules that do not contain any C1 packets and contain at least one empty slot [(C2, E), (E, C2), (DC, E), (E, DC), (E, E)]; (3) and set G, modules that contain a C1 packet [(C1, E), (E, C1), (C1, DC), (DC, C1), (C1, C2), (C2, C1)]. Let c be the number of elements in C, and let f the number of elements in F. In the following algorithm we assume that $a \geq b$ and conversion priority will be given to C1 packets otherwise if $b \geq a$ reverse the reasoning.

3.A. Algorithm

To solve contentions and avoid deflections at the routing block, the node controller uses the following algorithm presented in pseudocode (Algorithm 1).

When swapping of packets is performed, the system needs at least two wavelength converters, which is why we use the floor function [$\mathrm{floor}(n_{cl}/2) \geq 1$] to ensure that the system has two converters available. Note that contentions are never created by swapping packets between A and B and between A and C or wavelength conversion from A to F. Also, note that variable a_l is the number of remaining contentions at the end of the algorithm. If $a_l > 0$, then a_l contentions are left in A, which will cause a_l deflections at the routing block.

3.B. Analysis

Let u be the input slot utilization, i.e., the probability that a slot from the input carries a packet. Define P_{dc} as the probability that a packet is DC ("Don't Care"), i.e., that the packet can take either outputs, and r be the probability that the packet is destined for the node. Let g be the packet-generating probability and P_{dc0} be the probability of DC when a new packet is injected into the network. At every clock cycle the input slots are assumed to be the independent random variables with the same probability distribution

$f_i = \{Pi_j = s), s \in \{\mathrm{E}, \mathrm{DC}, \mathrm{C2}, \mathrm{C1}\}\}$, where C1 (C2) is the packet that cares to exit on output 1 (2), $j = 1, 2, \ldots, 2n_w$. At the moment when packets reach the absorption block, f_i

Algorithm 1

/* BEGIN */

Get a, b, n_c, f, and c variables

$a_l = a$; *% initialize a_l*

$b_l = b$; *% initialize b_l*

$c_l = c$; *% initialize c_l*

$f_l = f$; *% initialize f_l*

$n_{cl} = n_c$; *% initialize n_{cl}*

If ($b_l \geq 1$) & (floor(n_{cl}/2) $\geq$ 1) *% swap packets between modules A and B*

{ *% Swapping is achieved by interchanging the packets and also the wavelength of the two packets.*

from the a_l (b_l) modules with conflict in A(B) select at random one packet from each set and swap them, this will remove contentions in modules of both sets A and B;

$b_l = b_l - 1$; $a_l = a_l - 1$; $n_{cl} = n_{cl} - 2$; *% update variables*

}

while ($a_l \geq 1$) & ($f_l \geq 1$) & ($n_{cl} \geq 1$) *% continue with the translation process from A to F*

{

from the remaining a_l modules with conflict in A select at random one packet and translate it to the empty slot of one of the available f_l modules in F; *% update variables*

$f_l = f_l - 1$; $a_l = a_l - 1$; $n_{cl} = n_{cl} - 1$;

}

while ($a_l \geq 1$) & ($c_l \geq 1$) & (floor(n_{cl}/2) $\geq$ 1) *% swap packets between modules A and C*

{

from the remaining a_l modules with conflict in A and from the available c_l modules in C select at random one packet from each set and swap them, this will remove contentions in modules of set A; *% update variables*

$c_l = c_l - 1$; $a_l = a_l - 1$; $n_{cl} = n_{cl} - 2$;

}

/* END */

can be rewritten as

$$f_i = \{ f_i(\mathrm{E}), f_i(\mathrm{DC}), f_i(\mathrm{C}) \} = \{ 1 - u(1-r), uP_{\mathrm{dc}}, u(P_{\mathrm{dc}} - r) \} . \tag{1}$$

It is assumed that among the care packets, both output 1 and output 2 are equally likely. At steady state and under uniform traffic assumption, at each node and clock time, the average number of absorbed packets per wavelength S_{abs} must be equal to the average number of injected packets S_{inj}, and hence their common value T can be termed as throughput per node per wavelength S. Since there are on average ru packets destined to the arriving node per wavelength per input, and all of them are absorbed, we have $S_{\mathrm{abs}} = ru$. By Little's law, throughput T for two-connected networks is obtained as $2u/H$, where H is the average number of hops. Following that, r can be obtained immediately as $r = 1/H$. According to the mentioned access scheme, the average number of packets injected per wavelength can be written as $S_{\mathrm{inj}} = g\left\{1 - [1 - f_i(\mathrm{E})]^2\right\}$ and the closed expression for u is [7]

$$u = \frac{\sqrt{r^2 + g^2(1-r)^2} - r}{g(1-r)^2} . \tag{2}$$

3.C. Deflection Probability

Owing to the properties of the regular network topology and uniform traffic assumptions, the deflection probability d of a care test packet (TP) at an intermediate node and deflection probability d_0 of a care TP at its injection block can be derived. A deflection happens on TP if TP enters the conversion block in a module with another competing packet and the contention is not removed in the conversion block. Let P_{cont} be the probability that TP belongs to a particular module in which contention is not solved and P_c is the combination of contention probabilities of all modules; according to the wavelength-conversion algorithm mentioned previously, both P_{cont} and P_c could be derived as

$$P_c = \sum_S P_{\text{cont}} P(a,b,c,f), \tag{3}$$

where $S \in$: $\{(a,b,c,f): n_w \geq (a+b+c+f) \geq 1; a > 0; b \geq 0; c \geq 0; f \geq 0\}$ is the set of feasible states of the four variables (a,b,c,f) where contentions remain for the TP. For programming purposes s can be found as follows. Fix $1 \leq a \leq n_w$ (it must be larger than 0, since the TP is in A). Then select the number of modules in B: $0 \leq b \leq n_w - a$. Also select the number of modules in F: $0 \leq f \leq n_w - a - b$. Finally, we select the number of modules in C: $0 \leq c \leq n_w - a - b - f$. This guarantees that $a+b+f+c \leq n_w$. Now to obtain the probability P_{cont} we can use the following equation:

$$P_{\text{cont}} = a_l / a, \tag{4}$$

where a is the number of contentions in set A and a_l is the number of remaining contentions. Therefore since the selection of packets was done randomly, the probability that TP is in contention is $P_{\text{cont}} = a_l/a$. The parameter a_l can be computed using the algorithm already mentioned in Subsection 3.A.

Now $P(a,b,c,f)$ is derived using the following equation, which is the probability that the four variables (a,b,c,f) may occur:

$$P(a,b,c,f) = P(TP)\left[\frac{(n_w-1)!P(A)^{(a-1)}P(B)^b P(C)^c P(F)^f P(G)^{(n_w-a-b-c-f)}}{(a-1)!b!c!f!(n_w-a-b-c-f)!}\right]. \tag{5}$$

Note that the events TP, A, B, C, F, and G are defined as follows:

$$\begin{aligned}
\text{TP} &= \{\text{the packet conflicting with flow-through test packet TP is C1}\},\\
\text{A} &= \{\text{a submodule with a conflict (C1, C1)}\},\\
\text{B} &= \{\text{a submodule with a conflict (C2, C2)}\},\\
\text{C} &= \{\text{a submodule without conflicts nor C1s nor E}\},\\
\text{F} &= \{\text{a submodule without conflicts nor C1s with at least one E}\},\\
\text{G} &= \{\text{a submodule with 1 and only one C1}\}.
\end{aligned}$$

The probability for each event can be written as

$$P(\mathrm{TP}) = \frac{u(1-P_{\mathrm{dc}}-r)+g(1-u+ur)(1-P_{\mathrm{dc0}})}{2}, \tag{6}$$

$$P(\mathrm{A}) = P(\mathrm{B}) = \left[\frac{u(1-P_{\mathrm{dc}}-r)}{2}\right]^2 + \left[\frac{u(1-P_{\mathrm{dc}}-r)}{2}\right](1-u+ur)g\left(\frac{1-P_{\mathrm{dc}}}{2}\right), \tag{7}$$

$$\begin{aligned} P(\mathrm{C}) = &\left\{\left[1-(1-u+ur)-\frac{u(1-P_{\mathrm{dc}}-r)}{2}\right]^2 - \left[\frac{u(1-P_{\mathrm{dc}}-r)}{2}\right]^2\right\} \\ &+ \left\{2\left[1-(1-u+ur)-\frac{u(1-P_{\mathrm{dc}}-r)}{2}\right](1-u+ur)\right\}\left\{1-\left[\frac{g(1-P_{\mathrm{dc0}})}{2}\right]\right\}, \end{aligned} \tag{8}$$

$$P(F) = (1-u+ur)^2\left\{1-\left[\frac{g(1-P_{dc0})}{2}\right]\right\}, \tag{9}$$

$$P(\mathrm{G}) = 1-P(\mathrm{A})-P(\mathrm{B})-P(\mathrm{C})-P(\mathrm{F}). \tag{10}$$

Hence, the deflection probability of TP after the conversion block can be written as

$$d = P_c/2. \tag{11}$$

The initial deflection probability of a TP at the injection, d_0 is identical to that of d, except $P\{\mathrm{TP}\}$ must be changed to

$$P(\mathrm{TP}) = u(1-P_{\mathrm{dc}}-r)/2. \tag{12}$$

4. Results and Discussion

This paper studies the deflection probability at care nodes with reference to the link utilization in the analysis of the 64-node ShuffleNet. A simulation was carried out to validate the accuracy of the analytical model. Both simulation and theoretical results of average number of hops in the 64-node ShuffleNet for $n_w = 15$ against network throughput have been examined as shown in Fig. 2. The discrepancies in the results between theory and simulation result mostly from traffic inhomogeneities for the case of ShuffleNets [7]. Although the network is regular and the traffic is uniform, the number of C1 and C2 packets received from the two input links of a module will be imbalanced. However, the discrepancies are reasonability low for $n_c < 6$ and negligible for $n_c \geq 6$. Furthermore, an interesting bistable characteristic is found to happen when $n_c = 1$ and $n_c = 3$ for both simulation and theoretical results. This bistable characteristic is due to the incapacity of the system to solve packet contentions (because there are not enough converters) and traffic imbalance. In the case of $n_c = 1$, only one (C1, C1) contention can be solved when $f > 0$ and all the remaining (C1, C1) and (C2, C2) contentions cannot be solved, creating additional traffic imbalance in the network. Also, there is a point in the throughput when the use of the converter saturates and the network operates as if wavelengths were independent, i.e., no converters, and because of this, deflections increase and therefore average number of hops increases and throughput decreases. Also, in Fig. 2, it is found that the average number of hops decreases with the increment of the number of converters. However, it will reach a level whereby any additional increment in the number of wavelength converters will cause insignificant improvement in the average number of hops required.

A plot of the probability mass function (PMF) of the number of hops is shown in Fig. 3. It is shown that for a given number of hops, the PMF is narrower with a greater number of wavelengths ($n_w = 10$, $n_c = 8$–10) than with ($n_w = 5$, $n_c = 4$–5). Also, the PMF reaches a plateau of improvement when more converters are used. In addition, we found that for

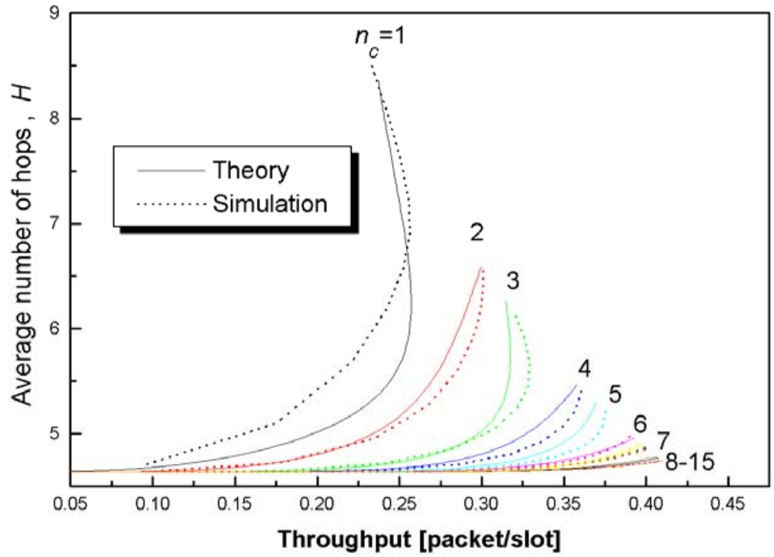


Fig. 2. Average number of hops H versus throughput per wavelength T for case $n_w = 15$ wavelengths.

a given number of hops, when number of converters $n_c < 4$, the PMF is wider when the number of wavelengths is greater. Nevertheless, the phenomenon happens only when n_c is small; when $n_c \geq 4$, the PMF is narrower for a given number of hops if n_w is greater.

Figure 4 depicts the results of deflection probability versus link utilization for number of wavelengths = 10 and 20 with different number of wavelength converters. It is found that as the number of converters in a node increases, the deflection probability will decrease, which is true in practice. When the number of wavelengths in the ShuffleNet increases (from $n_w = 10$ to $n_w = 20$), it is indeed interesting to note that the deflection probability will reach a saturated level of improvement regardless of any additional use of wavelength converters. Subsequently, when number of converters is small ($n_c < 5$), it is more effective to decrease the number of wavelengths in the network for all the values of link utilization, in order to have lower deflection probability. For $n_c \geq 5$, there will be a trade-off of either increasing the number of wavelengths or using additional converters in order to minimize the network cost. By comparing the deflection probability for the case of $n_w = 10$ with $n_c = 3$ and $n_w = 20$ with $n_c = 4$, we found that even though both n_w and n_c are increased, the deflection probability has only a slight difference for link utilization $u < 0.4$ and is almost identical for link utilization $u \geq 0.4$. This shows that in order to obtain a lower deflection probability, increasing n_w and n_c is not sufficient; the ratio of n_c/n_w is also an important aspect to be considered. However, the number of wavelengths is still a main factor for further improvement on the deflection probability of a network with high load as can be observed in Fig. 4. Figure 4 shows that the deflection probability improves depending on the number of wavelengths used and the number of wavelength converters n_c.

Figure 5(a) shows the deflection probability at full load versus the ratio of number of converters to the number of wavelengths (n_c/n_w) for both 64- and 324-node ShuffleNet. It is found that the deflection probability at full load will reach a saturation level when the number of converters in each node is approximately 60% of the number of wavelengths in the network and is independent of the network size. It is also evident that when n_w becomes larger (e.g., 30) the deflection probability for the ratio $n_c/n_w > 0.6$ has no difference; again,

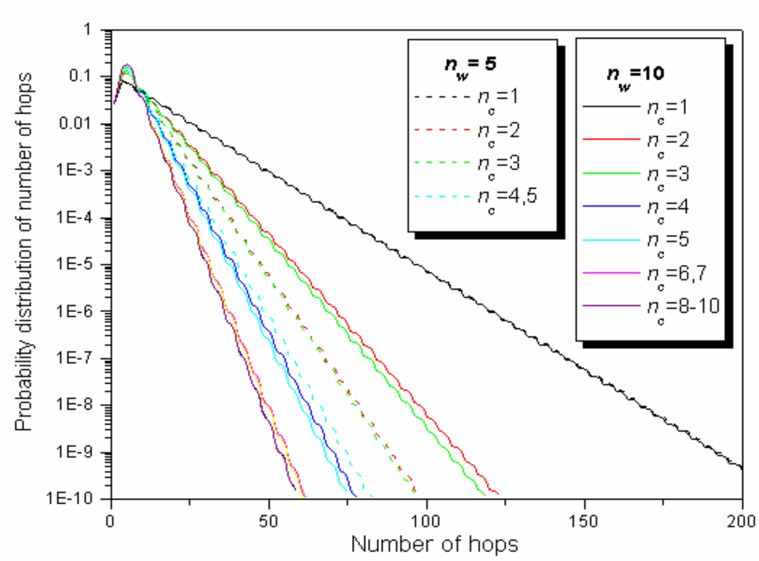


Fig. 3. Probability mass function of number of hops versus number of hops.

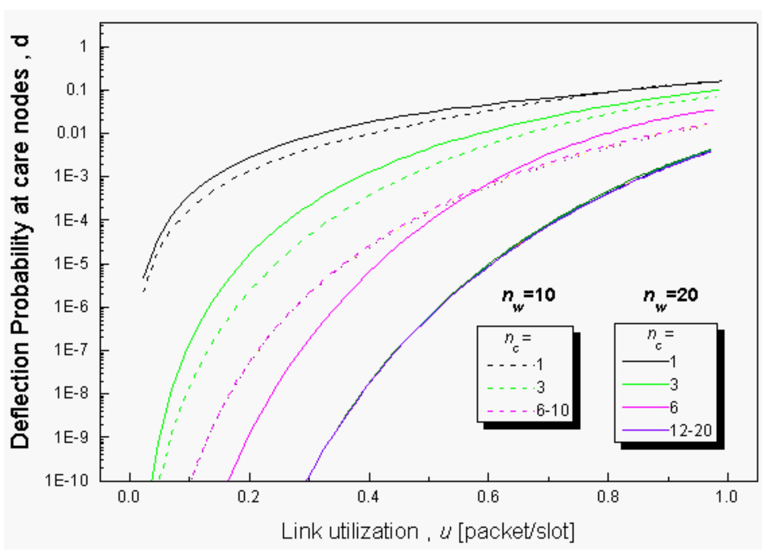
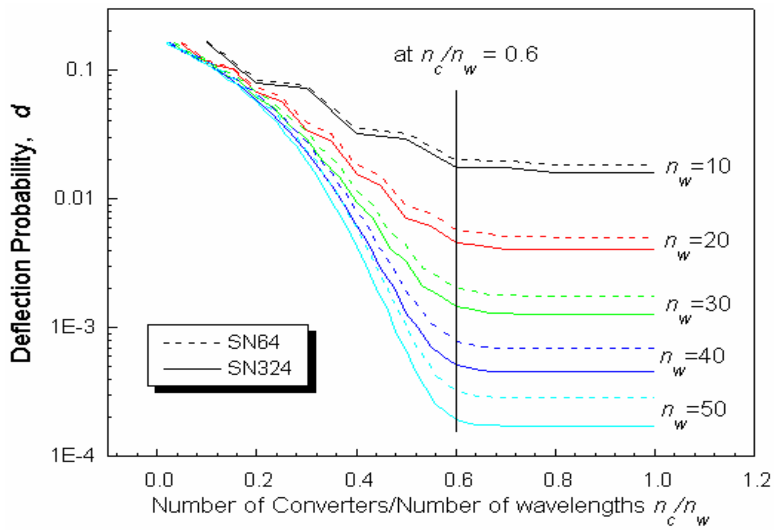


Fig. 4. Deflection probability at care nodes d versus link utilization u [packets/slot].

deflection probability is independent of the network size under this circumstance. It is noted that the deflection probability for smaller networks is greater than that of large networks because the ratio of traffic per node is greater and this causes packets to be deflected more frequently.



(a)

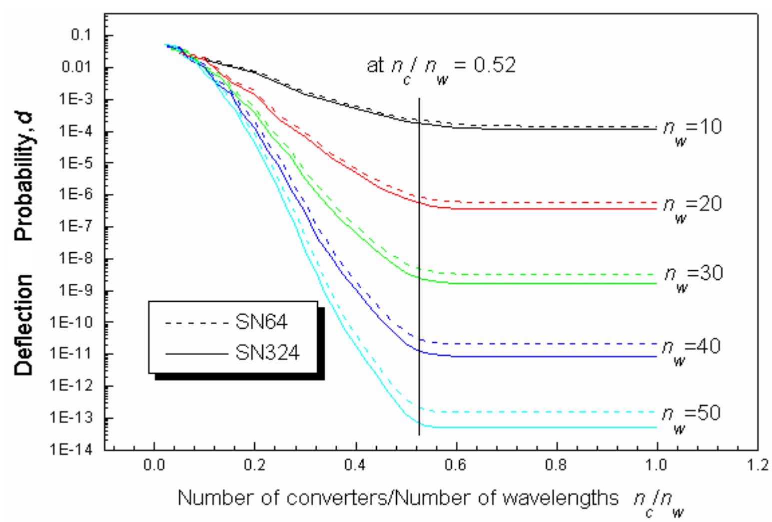


Fig. 5. (a) Deflection probability d versus number of converters/number of wavelengths (n_c/n_w) when $u = 1.0$. (b) Deflection probability d versus Number of number of converters/number of wavelengths (n_c/n_w) when $u = 0.5$.

Figure 5(b) shows the deflection probability with a value of link utilization that is used in practical network design, which is $u = 0.5$. Compared with Fig. 5(a), Fig. 5(b) presents a smoother curve for $n_w = 10$ and 20. The optimum ratio of n_c/n_w when $u = 0.5$ also has a smaller value of 0.52 compared with the optimum ratio of n_c/n_w of Fig. 5(a) when $u = 1.0$.

5. Conclusions

We have performed a detailed analysis of multihop wavelength-convertible ShuffleNet with limited number of wavelength converters employing deflection routing. From our study, we conclude that the required number of wavelength converters is only 60% or less than the number of optical carriers (wavelengths) in each node in order to reduce the propagation delay in a fully loaded optical network. Any additional wavelength converters will not be useful in reducing the overall network deflection probability to obtain the maximum throughput per wavelength. Thus, from this analysis, the network cost can be optimized in terms of number of wavelengths and wavelength converters used.

Acknowledgment

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