Constant SNR-Error Step-Size Selection Rule for Numerical Simulation of Optical Transmissions

S. Musetti, P. Serena and A. Bononi

Università di Parma, dept. Ingegneria e Architettura, 43124, Parma (Italy), ✉ simone.musetti@studenti.unipr.it

Abstract *We propose a novel power-independent step size selection rule for the split-step Fourier method which targets a desired simulation error on the signal to noise ratio of coherently received signals.*

Introduction

The numerical simulation of the nonlinear Schrödinger equation (NLSE) is generally implemented by the split step Fourier method (SSFM). A trustworthy SSFM should simulate the NLSE with the desired accuracy whatever the optical link under investigation. Targeting a desired accuracy is crucial in nowadays time-consuming simulations of fully loaded wavelength division multiplexing (WDM) systems, where a SSFM fine tuning is mandatory to save computational effort.

Over the past years some effort has been put into ways to quantify the trade-off accuracy/runtime[1–4]. The logarithmic step size[1] and the nonlinear phase[2] criteria are two popular SSFM step selection rules that efficiently stretch the step along propagation to follow the loss profile. On the contrary, the walk off criterion[2] puts focus on the scaling properties of inter-channel dispersion regardless of the loss profile. Some ways to mediate between these two requirements have been proposed in[2,4], respectively based on adaptive step rule[2] and on global simulation error scaling properties[4].

In this paper we review the SSFM error in the general framework of the Gaussian noise (GN) model[5], which focuses on the signal to noise ratio (SNR). We propose that the SSFM should target a given relative error in SNR rather than in electric field for a reliable simulation. Contrary to methods[1–4] where the number of SSFM steps scales with the transmitted power, with our proposal we get a power-independent number of steps.

Moreover, we introduce a simple universal parameter at constant SNR relative error for trustworthy SSFM simulations.

SSFM Error on SNR

The SSFM error can be seen as a perturbation to the information signal similarly to amplified spontaneous emission (ASE) and nonlinear interference (NLI). This way we can express the SNR of the received estimated sample as:

$$\frac{1}{\text{SNR}} = \frac{1}{\text{SNR}_{\text{ASE}}} + \frac{1}{\text{SNR}_{\text{NLI}}} + \frac{1}{\text{SNR}_{\text{SSFM}}} \tag{1}$$

where the subscripts refer to the SNR considering ASE, NLI and SSFM error alone, respectively. SNR_{SSFM} is the only term depending on the numerical implementation of SSFM and should be maximized. Since SNR is universally expressed in decibels, a trustworthy simulation should target a constant SNR error in [dB] by varying system parameters. Such an idea is sketched in Fig. 1.

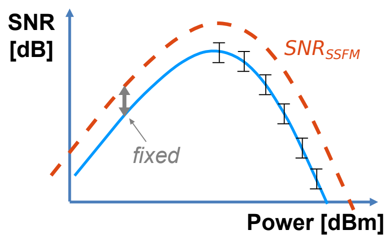

Fig. 1: In our proposal, SNR due to SSFM error only (dashed) should be a fixed multiple of the true SNR (solid) to give constant error bars in [dB] scale.

According to GN model theory[5], the variance of SSFM error is expected to scale as P^3, P being the signal power, like the scaling of the NLI. Therefore, for increasing power we can tolerate more absolute SSFM error at fixed relative accuracy on SNR. Unfortunately, this criterion is not followed by common step-selection rules[1–4] where the step size is decreased for increasing transmitted power.

The two degrees of freedom to set up SSFM simulations are i) the first step of the propagation and ii) the step updating rule with distance. The step updating rule should account only for the fiber loss profile, while the first step should account only for dispersion, since the dispersion induced error on SSFM is expected to be statistically identical from step to step.

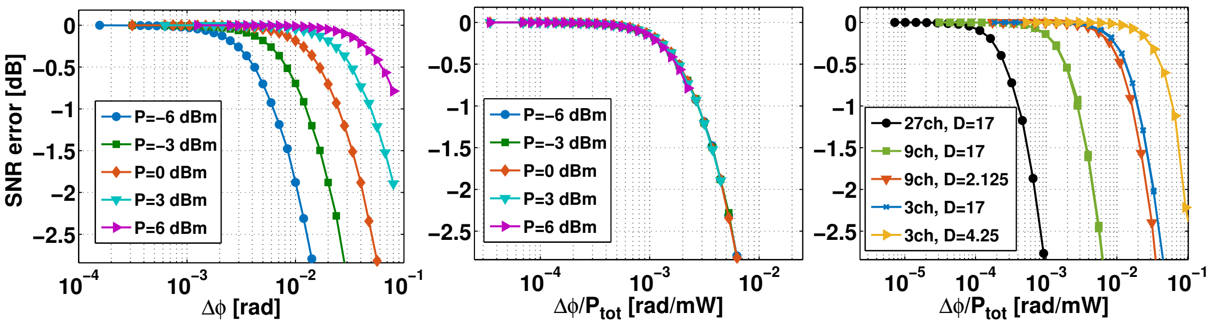

Fig. 2: SNR error (ΔSNR) in 20×100 km link at variable channel power vs (Left): cumulated nonlinear phase $\Delta\phi$ per SSFM step; (Center): $\Delta\phi$ normalized to total input WDM average power P_{tot}. 9 channel WDM signal. Dispersion $D = 17$ ps/(nm·km). (Right): ΔSNR vs $\Delta\phi/P_{\text{tot}}$ per step at variable dispersions and WDM bandwidths. In this last case $\Delta\phi/P_{\text{tot}}$ does not yield overlapping curves. D is the fiber dispersion [ps/(nm·km)].

Numerical Results

Results are expressed in terms of relative error on SNR, i.e., $\Delta\text{SNR} = \text{SNR}/\text{SNR}_{\text{true}}$, where SNR is given in (1). SNR_{true} is the most accurate result available and was obtained for decreasing first step lengths by factors $\sqrt{2}$ until observing convergence of SNR within a tolerance of 0.001 dB. All the remaining steps were scaled according to the method under analysis.

We analyzed a WDM comb at variable number of channels $N_{\text{ch}} \in [3,\ 9,\ 27,\ 54]$ with spacing 37.5 GHz, up to a WDM bandwidth of 2 THz. Each channel was modulated with polarization division multiplexing quadrature phase shift keying (PDM-QPSK) with root raised cosine pulses at symbol rate $R = 32$ GBaud and roll-off $r = 0.01$. Channel power was varied among $P \in [-6,\ -3,\ 0,\ 3,\ 6]$ dBm. The number of transmitted symbols per channel was set longer than the maximum walk-off between the edge channels of the WDM comb, with a minimum of 4096 symbols. To correctly capture at least first order four wave mixing (FWM) each symbol was discretized at a rate twice the WDM bandwidth. The optical link was dispersion uncompensated (span length 100 km; attenuation $\alpha = 0.2$ dB/km; nonlinear coefficient $\gamma = 1.3$ 1/(W·km)) with dispersion $D \in [17,\ 8.5,\ 4.25,\ 2.125]$ ps/(nm·km). Span loss was recovered after each span, while dispersion was fully compensated at the coherent receiver. ASE noise was not included in this work, since it is generally loaded at the receiver side, hence without impacting the SSFM results. SSFM solved the Manakov NLSE by using a symmetric SSFM and step-update rule according to i) the nonlinear phase criterion[2], or, ii) constant local error[4]. Finally, the signal was coherently detected by using a matched filter and a data aided 1-tap least squares butterfly equalizer.

Fig. 2(Left) shows ΔSNR vs. the nonlinear phase parameter per step $\Delta\phi = \gamma P_{\text{p}} L_{\text{eff}}(h)$, with L_{eff} the effective length in the step of length h, and P_{p} WDM peak power. In this graph the number of channels was 9. $\Delta\phi$ is the parameter to setup SSFM according to the nonlinear phase criterion. The smaller the value of $\Delta\phi$ the longer the simulation but the better is the accuracy in SNR. However, the curves are not coinciding at different channel powers P, thus indicating that the nonlinear phase criterion does not grant a constant relative SNR error. Normalizing $\Delta\phi$ to the total input WDM average power $P_{\text{tot}} = N_{\text{ch}} \cdot P$ indeed collapses all curves into one as shown in Fig. 2 (Center). This minor modification, besides yielding a more universal parameter for the SSFM, confirms that SSFM error scales with P^3 as the NLI. Most important, Fig. 2 (Center) shows that the accuracy, hence the computational effort, is independent of transmitted power, as desired.

However, besides the normalization of $\Delta\phi$, the nonlinear phase criterion is unaware of group velocity dispersion (GVD), as visible in Fig. 2 (Right) where we varied the dispersion and the number of channels. To solve such a problem we propose to scale the first step h_1 to keep a fixed maximum FWM phase matching coefficient in that step: $\Phi_{\text{FWM}} = h_1 \left|\beta_2\right| (2\pi B_{\text{WDM}})^2$, with β_2 the fiber dispersion and B_{WDM} the WDM signal bandwidth. The rationale is that the SSFM should track the worst case of GVD variation along distance, hence the highest FWM phase-matching coefficient. All the remaining steps should be scaled as in[4] to keep a constant local error per step.

Fig. 3 shows ΔSNR versus Φ_{FWM} in the first step at different number of channels (Left), or different values of chromatic dispersion (Right).

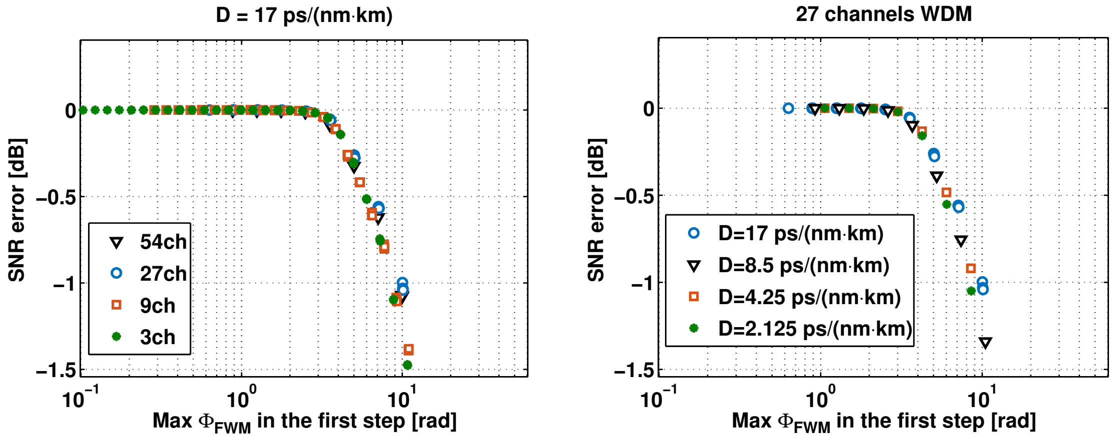

Fig. 3: SNR error (ΔSNR) vs max Φ_{FWM} in the first SSFM step of a 5×100 km link by varying (Left): the number of channels and (Right): the fiber dispersion. Constant local error step updating rule. Each marker refers to set of curves at different input powers in the range $[-6, 6]$ dBm. The collapse of the curves indicates that Φ_{FWM} is a good parameter to setup the first SSFM step.

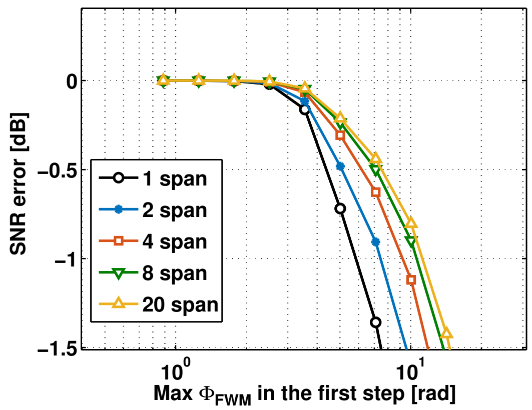

Fig. 4: SNR error (ΔSNR) vs max Φ_{FWM} in the first step at variable number of spans ($\times 100$ km). 27 channels WDM signal. Dispersion $D = 17$ ps/(nm·km). Input power per channel $P = -6$ dBm. Constant local error step updating rule.

Each marker in Fig. 3 is a whole set of curves regarding different input powers. Results here are evaluated after 5 spans since 54 channels simulations at larger distances were not feasible in practical time. However, this choice is a worst case for accuracy as we will show next. The collapsing of the curves supports the choice of Φ_{FWM} as the right parameter to set up simulations at variable bandwidth and fiber dispersion.

Fig. 4 shows the dependency of the SNR error on the number of spans. The main observation is that the relative SNR error is decreasing for increasing number of spans. The explanation is that SSFM error cumulates along distance almost incoherently. On the contrary, NLI shows some spatial coherence, i.e., correlation, along propagation[5], thus growing faster than SSFM error. The single span is thus a worst case for setting the accuracy of simulations and can be taken as a conservative choice to setup SSFM.

Conclusions

We analyzed the scaling properties of the numerical error in SSFM simulations focusing on coherent WDM systems up to bandwidths of 2 THz. We found out that the SSFM error scales with the cube of power as much as NLI, such that the relative error on SNR is transmitted power independent. We thus suggest to avoid scaling the step size with the transmitted power as in popular methods like the nonlinear phase criterion. We showed that a good way to pick the first step of the simulation granting a constant SNR error is given by fixing the maximum FWM phase shift on the first step. Moreover, we showed that SSFM error grows along distance at a smaller rate than NLI, such that sizing SSFM in the single span case is a conservative choice.

References

[1] G. Bosco et al., “Suppression of Spurious Tones Induced by the Split-Step Method in Fiber Systems Simulation,” IEEE Photon. Technol. Lett., vol. **12**, no. 5, p. 489 (2000).

[2] O. V. Sinkin et al., “Optimization of the split-step Fourier method in modeling optical-fiber communications systems,” J. Lightw. Technol., vol. **21**, no. 1, p. 61 (2003).

[3] J. Shao et al., “Comparison of Split-Step Fourier Schemes for Simulating Fiber Optic Communication Systems,” IEEE Photon. J., vol. **6**, no. 4, (2014).

[4] Q. Zhang et al., “Symmetrized split-step Fourier scheme to control global simulation accuracy in fiber-optic communication systems,” J. Lightw. Technol., vol. **26**, no. 2, p. 302 (2008).

[5] A. Carena et al., “Modeling of the impact of nonlinear propagation effects in uncompensated optical coherent transmission links,” J. Lightw. Technol., vol. **30**, no. 10, p. 1524, (2012).