An Alternative Analysis of Nonlinear Phase Noise Impact on DPSK Systems

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Abstract *We propose an alternative analysis of the impact of nonlinear phase noise in DPSK systems with realistic receivers, showing that ASE noise is Gaussian after practical optical filtering, which allows using known exact BER formulae.*

Introduction

The most limiting impairment for phase-modulated long-haul optical systems is nonlinear phase noise, i.e., the amplitude to phase-noise conversion due to the interaction between the transmitted signal and the amplified spontaneous emission (ASE) noise of the in-line amplifiers in presence of Kerr nonlinearities. Because of such interaction, also known as parametric gain (PG) [1], the Gaussian white ASE noise gets colored during propagation and its statistics are changed. Several models have been proposed for the performance evaluation of differential phase shift keying (DPSK) systems in presence of nonlinear phase noise [2–5]. For instance, Ho [3] computed the probability density function (PDF) of nonlinear phase noise at zero group velocity dispersion (GVD). In the same case, Mecozzi [2] computed the *n*-th order moments of the optical field before the receiver. In [4] the authors showed that the received phase difference noise can be assumed Gaussian when GVD is included. However, all these models do not realistically take into account the impact of practical optical and electrical filters on phase noise statistics at the receiver. In this work, we tackle the bit-error rate (BER) of DPSK modulated signals when realistic receivers are used, showing that optical filtering at the receiver recovers the Gaussian statistics of the optical ASE noise. To this aim, we prefer not to focus on the statistical behaviour of the received differential phase, but concentrate on the statistics of the real and imaginary optical ASE components. We prove that after optical filtering such components well follow a jointly Gaussian PDF both without and with the inclusion of fibre GVD. In the first case, the exact BER is computed and validated down to $BER = 10^{-10}$ making use of a multicanonical Monte Carlo (MMC) method [6]. In the second case, we already proposed a small-signal model for ASE noise statistics [1], that here we compare to MMC numerical simulations of 10 Gb/s dispersion-managed DPSK systems.

Theory

We now focus on the exact BER evaluation based on the sampled current $I_k = \Re\{E(t)E^*(t-T)\} \otimes h_R(t)\mid_{t=t_k}$, where $E(t)$ is the received signal (plus noise) after optical filtering, T is the bit time delay due to the Mach-Zehnder interferometer, $h_R(t)$ is the post-detection electrical filter and $\otimes$ indicates linear convolution. In the absence of GVD, even in nonlinear regime the ASE noise remains white over the bandwidth of the in-line amplifiers. If such bandwidth is much larger than that of the optical filter (B_o), as it usually happens, at the optical filter output the ASE noise turns out to be a complex Wiener process, which is Gaussian whatever the distribution of the input noise. Consequently, the statistics of the process $E(t)$ are completely identified by its covariance matrix and average value, and they are sufficient for BER evaluation. Hence, there is no need of computing the exact PDF of phase noise [3, 4]. The *n*-th order moments of the optical field are explicitly given in [2] for an infinite-span link before optical filtering and they are referred to a proper optical bandwidth B_M (at most that of the in-line amplifiers) which represents the PG bandwidth of the link. From linear system theory such moments can be related to those of $E(t)$. By exploiting the covariance matrix's eigenvalues and the average value of $E(t)$ the exact moment generating function (MGF) of I_k can be evaluated as in [1]. By means of the numerical inverse Laplace transform of such MGF the exact BER is also derived. For instance, in the case of a rectangular optical filter and an integrate and dump electrical filter, the MGF is:

$$MGF(s) = \frac{\exp\left(\frac{s\eta_R}{1-\beta_1 s} + \frac{s\eta_I}{1-\beta_2 s}\right)}{\left[\left(1-\beta_1^2 s^2\right)\left(1-\beta_2^2 s^2\right)\right]^{\frac{2M+1}{2}}} \qquad (1)$$

where: $M = B_o T$; η_R and η_I are the energy per bit of the real and imaginary parts of $E(t)$, including noise-induced depletion [2]; the terms $\beta_{1,2} = 2\lambda_{1,2}\sigma^2$ account for PG effects, being $\lambda_{1,2}$ the eigenvalues of the ASE covariance matrix and σ^2 the ASE one-sided power spectral density per component in absence of PG. In (1) the ASE on the orthogonal polarization has been neglected.

In presence of both GVD and Kerr nonlinearities, we will see that ASE can still be assumed Gaussian after optical filtering. Plausibility of such a result comes from the following observations: 1) with only the linear effect of GVD, ASE noise is exactly Gaussian; 2) with only self-phase modulation (SPM), as above demonstrated, ASE is Gaussian after the optical filter provided that $B_M \gg B_o$; 3) in the middle case, when GVD and SPM interact, the noise is filtered over a PG bandwidth depending on the system parameters. If such bandwidth is still larger than B_o, at the optical filter output a Wiener process is still obtained. However, it is worth noting that fibre dispersion introduces a memory effect which, during propagation, limits the noise PG-inflation and accelerates the convergence towards a Gaussian PDF. We prove these statements in the next section.

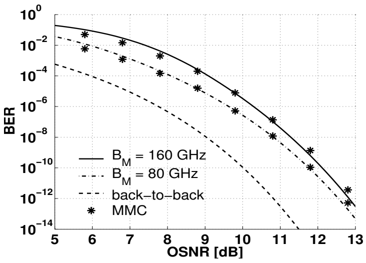

Fig. 1: *BER vs OSNR in absence of GVD.* $\Phi_{NL} = 0.2\pi$.

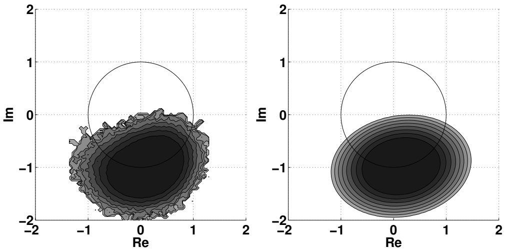

Fig. 2: *Joint PDF of ASE real and imaginary parts after optical filter. (Left) MC simulations; (right) theory.*

From these arguments, a small-signal assumption on ASE is reliable to derive the noise covariance matrix. In the hypothesis of a transmitted continuous wave, a closed-form of such a matrix can be found in [1] or, alternatively, it can be numerically inferred from the models in [5,6]. The BER is then evaluated with a Karhunen Loéve method for quadratic receivers [1].

Results and discussion

In Fig. 1 we compare our theoretical BER to MMC simulations in the case of a constant envelope DPSK signal propagating in absence of GVD on a $N \times 100$ km link, $N = 20$, with fibre attenuation $\alpha = 0.2$ dB/km and nonlinear coefficient $\gamma = 1.7$ 1/W/km. The average cumulated nonlinear phase $\Phi_{NL} = \gamma L_{eff} P_{av} N$ was 0.2π rad, being L_{eff} the fibre effective length and P_{av} the transmitted average power. At the receiver a 6-th order Butterworth optical filter with $B_0 = 15$ GHz and a 5-th order Bessel electrical filter with bandwidth of 6.5 GHz were used. Thanks to the efficient MMC numerical technique [6], $5 \cdot 10^4$ samples were enough to evaluate BER down to 10^{-12} with 15 iterations. We investigated the two cases $B_M = 80$ GHz and $B_M = 160$ GHz, which give at least 2 dB penalty for PG. Theory shows an excellent agreement with MMC for both bandwidths, confirming the validity of the Gaussian model even at low OSNR and with strong PG.

In Fig. 2 the contour plots of the joint PDF of real and imaginary ASE components are measured after a 6-th order Butterworth optical filter with $B_0 = 30$ GHz. A constant envelope signal with $\Phi_{NL} = 0.2\pi$ rad was propagated on a 20×100 km link with a transmission fibre dispersion $D_{Tx} = 4$ ps/nm/km. Each span had an in-line dispersion of -35 ps/nm. We applied our small-signal model [1] in the right figure, while the left one was obtained with $2 \cdot 10^{-6}$ samples of a pure Monte Carlo (MC) simulation which allowed to reliably resolve

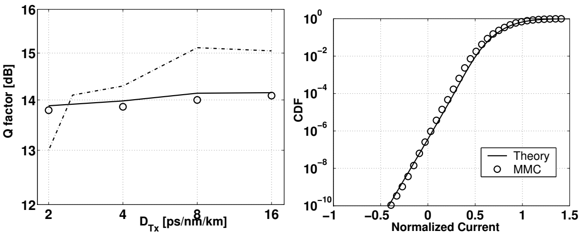

Fig. 3: *(Left) Q factor vs* D_{Tx}*; (right) average CDF for* $D_{Tx} = 2$ *ps/nm/km*

the PDF tails down to 10^{-5} (last level in the contours). Simulations well resemble the elliptical shape of the theoretical Gaussian PDF contours [1].

In Fig. 3 we show the Q factor $Q = \sqrt{2}\mathrm{erfc}^{-1}(2\mathrm{BER})$ of a single-channel DPSK signal propagating at 10 Gb/s on a 20×100 km link with varying D_{Tx}. The in-line dispersion was fixed to 88%, while the pre- and post-compensation dispersions were optimized for each D_{Tx}. The same receiver and Φ_{NL} of Fig.1 were used with a transmitted 32-bit pseudo-random bit sequence. The results given by our small-signal model (solid line) are compared to MMC simulations (circles). The Q factor of the back-to-back system was 15.1 dB. A good match between theory and MMC simulations is shown for each fibre dispersion. The reason of the good match is understood from the right figure, where the cumulative distribution function (CDF), averaged over the received 1s, is drawn vs. the normalized current for $D_{TX} = 2$ ps/nm/km. Theory (solid line) and MMC simulation (circles) are seen to well agree down to 10^{-10}. To clarify the role of PG on this DPSK system, in Fig. 3 (left) we also included (dashed line) the performance of a 37-channel system propagating on the same link in absence of PG and with a channel spacing of 50 GHz, where cross-channel nonlinearity dominates. We see that PG dominates over cross-channel nonlinearity for D_{Tx} larger than 2.5 ps/nm/km.

Conclusions

In this work we proposed an alternative analysis of the nonlinear phase noise impact on DPSK systems. The basic idea is focusing on the statistics of the real and imaginary ASE components before photo-detection, which are shown to recover their Gaussian shape after usual optical filtering. With only SPM, an exact BER formula can be derived by inverting the current MGF. When GVD is non-zero, a small-signal model for PG can be applied to BER evaluation. We validated our analysis by MMC simulations down to BER of 10^{-10}.

References

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