On the Nonlinear Capacity with Memory of PS-QPSK and PDM-QPSK in WDM Non-Dispersion Managed Links

P. Serena, A. Bononi, G. Colavolpe

Università degli Studi di Parma, dept. Ingegneria dell'Informazione, v.le G. P. Usberti 181/A, 43124 Parma (Italy), ✉ paolo.serena@unipr.it

Abstract *We show a novel approach to numerically evaluate the information rate of long-memory nonlinear channels. We apply the technique to compare the spectral efficiency of PS-QPSK and PDM-QPSK in non-dispersion managed links.*

Introduction

The design of digital communication systems has been traditionally carried out with the aim of minimizing the uncoded bit error rate (BER). In the linear additive Gaussian noise channel and at large enough signal to noise ratio (SNR), this goal is achieved by maximizing the minimum Euclidean distance among the constellation symbols In this sense, polarization switched quadrature phase shift keying (PS-QPSK) has been shown to be more power-efficient than any other dual-polarization modulation format at equal bitrate[1]. However, PS-QPSK carries three bits/symbol, while for instance polarization division multiplexing quadrature phase shift keying (PDM-QPSK) carries four bits/symbol. Hence the increased power efficiency of PS-QPSK over PDM-QPSK comes at the expense of a $4/3$ larger symbol rate and thus larger bandwidth[2]. When forward error correction codes (FEC) are added, the spectrum further enlarges by the FEC overhead. Thus the bandwidth difference between PDM-QPSK and PS-QPSK can be used to equip the PDM-QPSK signal with a stronger FEC[3], and the question is which coded format at equal bandwidth is more power efficient.

To answer such a question we must resort to information theory. A fair comparison between two formats with coding can be made in terms of spectral efficiency (SE), defined as the net information rate (IR) per unit bandwidth (bit/s/Hz), as is routinely done in wireless communications[4], or, e.g., in deep space optical communications[5].

The nonlinear capacity of the fiber optic non-dispersion managed (NDM) channel has been recently studied under the memoryless assumption by simulation in[6] (by removing memory using digital back-propagation) and analytically in[7]. However, the NDM channel does have a huge memory. A quite general simulation technique to estimate the information rate in channels with memory was introduced in[8,9], and an early application of the algorithm to optical links appeared in[10].

In this work, we compare PDM-QPSK and PS-QPSK in terms of spectral efficiency in a nonlinear 20x100 km standard single mode fiber (SMF) NDM link using our original adaptation of the method in[8,9].

IR Simulation Technique

Figure 1 shows the IR simulation setup. We assume the transmission system is a finite state machine (FSM) with M states. State k is indicated in the figure by σ_k and corresponds to an input symbol sequence that uniquely identifies the channel output (or soft-information) $\vec{v}_N$, where N is the number of wavelength division multiplexed (WDM) channels. With two polarizations, such an output is a 4-D vector that we evaluate both in single channel propagation ($N = 1$) or with 7 interfering channels per side ($N = 15$). For $N > 1$, the IR for such a scenario can be found by using the technique in[8,9] that requires the availability of the optimal multiuser detector for N WDM channels. Since the complexity of this detector is too large, we proceed as in[9] assuming additive noise to the single channel propagation. We declare such a noise to be any deviation between the observed output $\vec{v}_{15}$ and the expected $\vec{v}_1$ (see Fig. 1). Moreover, we assume Gaussian statistics for the noise[7], with the 4×4 covariance matrix estimated during simulation. This way, we are accounting for the correlations introduced by intersymbol interference, while cross-distortions are assumed white, as if the receiver were unable to compensate for them.

The signal under investigation is modulated with a de Bruijn sequence (DBS), which is the shortest sequence containing all the states σ_k of the FSM. The IR estimation is finally carried out by Monte Carlo (MC) estimation until a relative error of 0.01 with Gaussian confidence 68% is observed. We test 10 different random seeds, each

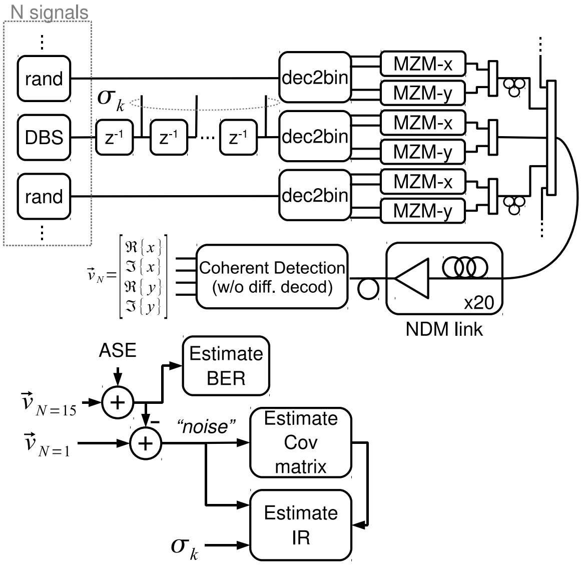

Fig. 1: Top: Numerical setup. DBS: de Bruijn sequence. dec2bin: decimal to binary conversion. MZM: Mach-Zehnder modulator. Bottom: simulative approach to estimate IR and BER.

corresponding to a different symbols pattern, including the seed of the DBS sequence, and then average the results.

Monte Carlo IR estimation for long-memory channels

The original method in[9] deals with a linear channel with finite impulse response equal to M taps. If B is the alphabet size of the modulation format, the total number of states of the FSM modeling the channel is B^M. When $M \gg 1$, the complexity of the FSM is too large, hence Arnold[9] introduced the concept of auxiliary channel based on a mismatched receiver. By using an auxiliary channel with a lower memory than M, Arnold[9] found a lower bound on the IR by individually testing such a channel with the reduced set of states and then by computing channel metrics using the true channel outputs.

Our adaptation of such auxiliary channel to huge-memory systems like NDM optical links instead proceeds with a block simulation using a DBS for the signal and averaging over the initial seed of the DBS. The idea is the following. A DBS of length B^M is a smart sequence that explores all possible states of the FSM in one block simulation. A shorter DBS as for the auxiliary channel cannot explore the entire state space of the true channel, but just a subset of it. Strictly speaking, using a shorter DBS than required by the FSM corresponds to partitioning the state space $\{\sigma_k\}$ in subsets and randomly choosing one state

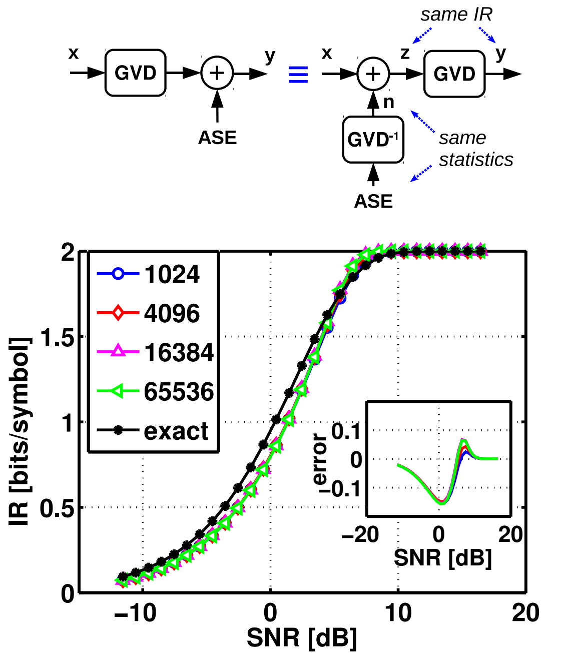

Fig. 2: IR of a linear channel impaired by 34000 ps/nm of GVD vs. de Bruijn sequence length ranging from $4^5 = 1024$ to $4^8 = 65536$ symbols. The estimated memory of the channel is 4^{860}. single-polarization QPSK signaling at 28 Gbaud. The top figure motivates why GVD should not impact the IR.

in each subset. The random choice depends on the seed of the DBS. This way, one can run an MC simulation of the IR by iterating over the DBS seeds. Like any MC simulation, the result is expected to stabilize for increasing number of seeds, indicating that the average behavior of the state space has been captured.

As a sanity check, we first tested our simulative technique by replicating the main results of[9], obtaining an excellent fit. Then we analyzed a single-polarization QPSK signal in a purely linear optical fiber impaired by group velocity dispersion (GVD). The transmitted signal was modulated at 28 Gbaud and cumulated a dispersion of 34000 ps/nm over the channel, corresponding to a memory of roughly 860 symbols. The GVD does not impact the IR of such a channel[11], as described by the equivalent block diagrams in Fig. 2. Thus the exact IR of such an experiment coincides with the memoryless case of zero GVD. We measured with our simulative approach the IR by iterating over 30 DBS seeds for 4 different DBS sequence lengths. Fig. 2 (bottom) shows both the simulated IR with GVD and the exact IR versus SNR. We note that our DBS-based procedure gives a reasonable error below 15% , confirming that the "curse of dimensionality" of long memory systems can be alleviated by MC simulations of the IR by iterating over the DBS seeds.

Spectral efficiency in NDM optical link

A 20×100 km NDM link was tested with a homogeneous WDM system with modulation format of either i) 37 Gbaud PS-QPSK , or ii) 28 Gbaud PDM-QPSK or iii) 37 Gbaud PDM-QPSK. Channel spacing was 50 GHz, for a total of $N = 15$ channels. We followed the simulative procedure detailed in Sec. 2 using a 4096-symbol DBS for the central channel, thus accounting for $M = 3$ in alphabet $B = 16$. Side channels had purely random symbols to better decorrelate their cross-effects. Before creating the WDM, we filtered each channel over a bandwidth of 50 GHz and randomly rotated the carrier state of polarization (SOP). The fibers had dispersion 17 ps/nm/km and nonlinear coefficient 1.3 1/W/km. Propagation was modeled by the Manakov equation. The GVD cumulated in the link was totally compensated by a purely linear fiber before capacity measurement. ASE noise was loaded at the end of the link with a unique noise source having noise figure 25 dB. The digital signal processing (DSP) used an anti-aliasing filter of bandwidth $0.6R$, being R the symbol rate. We also used data aided recovery of the average phase and polarization to preserve the soft information. This way, we did not use differential decoding.

Fig. 3 shows the central channel average Q-factor and SE vs. channel power. Note that the SE is defined using the channel spacing as bandwidth. At equal raw bitrate of 112 Gb/s, the 37 Gbaud PS-QPSK has a lower Q-factor than 28 Gbaud PDM-QPSK at very low powers because of a larger average distance among constellation symbols. At larger powers instead only the minimum distance matters, yielding a better Q-factor for PS-QPSK. However, comparing Q-factors and SE we observe opposite trends: while 37 Gbaud PDM-QPSK shows the worst Q-factor, it also shows the best SE. We thus conclude that, if an appropriate soft FEC performing close to capacity is available, it is preferable to fill the channel spacing by using a coded PDM-QPSK rather than resorting to a coded format such as PS-QPSK with smaller uncoded minimum distance but also less bits per symbol.

Conclusions

We estimated the information rate of a nonlinear NDM optical link by an effective simulative method that solves the problem of the large memory. We compared PS-QPSK and PDM-QPSK, and found almost equal SE at the same uncoded bitrate, while a superior SE for PDM-QPSK at same symbol rate. Hence, from an information theory point of view, over terrestrial lengths it is better to fill the channel spacing by using advanced codes and a format with high-enough bits/symbol rather than choosing a format with better uncoded Euclidean distance.

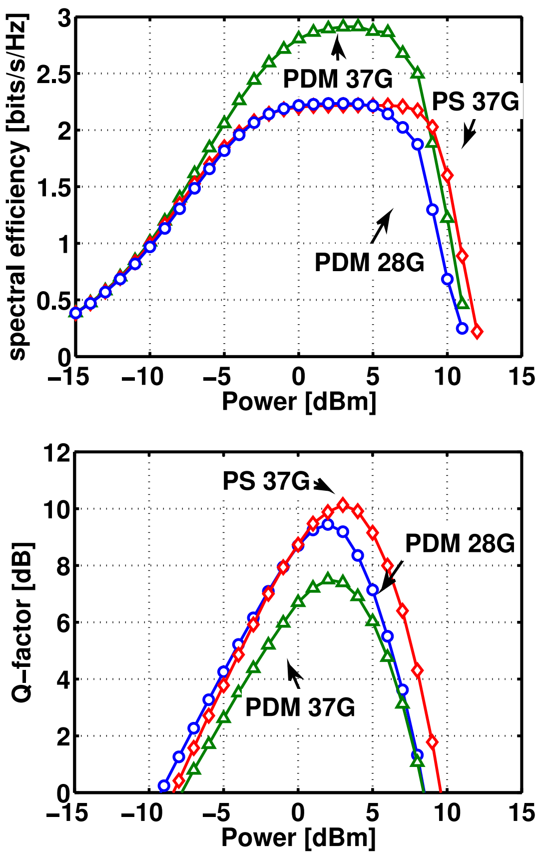

Fig. 3: SE and Q-factor vs power comparison of PDM-QPSK and PS-QPSK in a NDM link at equal raw bitrate (PS 37G and PDM 28G) and at equal baudrate.

References

1 M. Karlsson et al., Opt. Expr. **17**, 10814 (2009).
2 M. Sjödin et al., Proc OFC12, OTu2A.3 (2012).
3 B. Krongold et al., PTL, **24**, 203 (2012).
4 A. Barbieri et al., Trans. Com. **59**, 3223 (2011).
5 D. M. Boroson, MIT ISC-TR-2005-060 (2005).
6 R. Essiambre et al., JLT **28**, 662 (2010).
7 G. Bosco et al., Opt. Expr., **19**, 438 (2011).
8 H. D. Pfister et al., Proc GLOBECOM'01, 2992 (2001).
9 D. Arnold et al., Trans. Inf. Theory, **52**, 3498 (2006).
10 I. B. Djordjevic, Proc. of SPIE, **7621** (2010).
11 R. G. Gallager "Information Theory and Reliable Communication". Wiley, 1968.