The Impact of the Modulation Dependent Nonlinear Interference Missed by the Gaussian Noise Model

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Abstract *The impact of higher-order modulation-dependent nonlinear interference noise on system performance is studied in both dispersion-managed and unmanaged links, and compared to the predictions of the Gaussian Noise Model.*

Introduction

The Gaussian noise (GN) model was introduced as a simple tool to predict the variance of the nonlinear interference (NLI) in dispersion unmanaged (DU) optical links[1]. Despite its reasonable predictions of system reach, its accuracy is limited in short links because of the key assumption of a modulation-independent stationary Gaussian input field[2]. Such a problem in cross-channel interference (XCI) dominated links has been explained by some extra modulation dependent (MD) higher-order NLI terms neglected by the GN model[3,4].

Such a model was recently extended to include self channel interference (SCI) in[5].

In this paper, by extending the time-domain based approach of[6] to include MD higher-order XCI and SCI, we show that it is possible to i) quickly estimate the maximum reach with good accuracy even for dispersion managed (DM) links where the GN model fails, and ii) investigate performance of a wavelength division multiplexing (WDM) comb filling the whole 5 THz bandwidth of erbium doped fiber amplifiers where standard split-step Fourier (SSF) simulations are prohibitively long.

Models

The original idea of Mecozzi et al.[3] was to evaluate the variance of the received NLI by a regular perturbation (RP) solution of the nonlinear Schrödinger equation by assuming linearly modulated input digital signals. Such a variance is an extremely useful indicator of transmission quality, and can be converted to bit error rate (BER) assuming circular Gaussian noise statistics. The GN model, indeed, simplifies the problem by assuming a stationary Gaussian noise input signal in place of the true digital signal, an approximation motivated by practical observations[1]. The GN model thus disregards the impact of the true higher-order statistics of the constellation symbols, mainly expressed by their kurtosis[4,5,7].

GN and extended GN models have been derived in time domain[3,6] or frequency domain[1,4,5]. Unlike[5], in this work we took a time-domain approach to extend Mecozzi's XCI RP-model[3] to include both SCI and polarization division multiplexing (PDM), by following the same approach of[6] by working out the 4$^{\text{th}}$ and 6$^{\text{th}}$ order moments of MD NLI. The main advantage of the approach in[6] is that frequency integrals can be efficiently evaluated by fast Fourier transform (FFT), thus leaving a double integral in distance that can be efficiently evaluated as well by adaptive quadrature routines. The resulting model is thus based on just two minimal assumptions: i) a first order field perturbation is correct, ii) channel four wave mixing is negligible.

Results

We first concentrate on the normalized NLI coefficient a_{NL}, related to the NLI variance by $\sigma^2_{\text{NLI}} \triangleq a_{\text{NL}} P^3$, with P the channel power. Fig. 1 shows a_{NL} vs. number of channels of an homogeneous WDM comb composed of either PDM quadrature phase shift keying (PDM-QPSK) or PDM quadrature amplitude modulation (PDM-16QAM) of alphabet 16 signals. Optical link was 10×100 km, D=17 ps/nm/km, either DU or DM with residual dispersion per span of 30 ps/nm (DM30). Before transmission in the DM30 case we used a pre-compensating fiber of -390 ps/nm. Pulses were sinc-shaped with symbol rate 32 Gbaud, while channel spacing was 37.5 GHz. Symbols in Fig. 1 refer to lengthy 100-seed Monte Carlo (MC) SSF simulations as in[7], with a maximum of 16384 symbols per channel, limited to a maximum value of 51 channels for feasible execution times. Power was -4 dBm while SSF step was chosen to cumulate a peak nonlinear phase of $5 \cdot 10^{-4}$ rad per step. The good agreement between SSF simulations and the perturbative model is an indication that a first order perturbation is accurate. How-

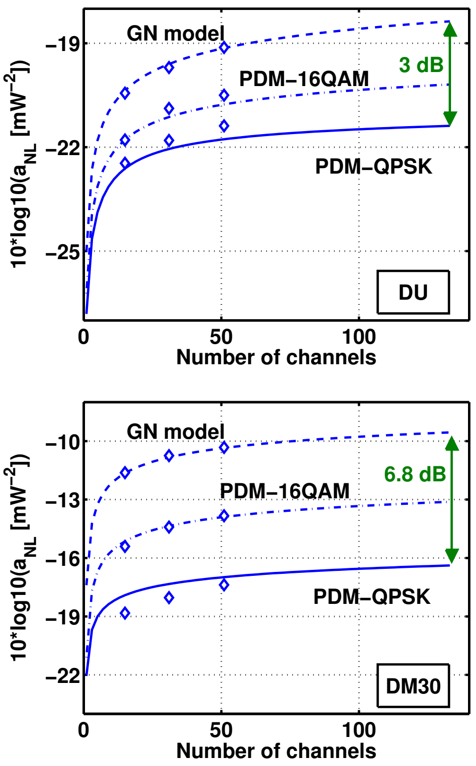

Fig. 1: Normalized NLI variance ($\sigma^2_{\mathrm{NLI}} \triangleq a_{\mathrm{NL}} P^3$, P: power) vs. number of channels. 10×100 km 32 Gbaud transmission, spacing 37.5 GHz. Lines: theory. Symbols: Split-step simulations. DU: dispersion-unmanaged. DM30: dispersion managed with 30 ps/nm/span.

ever, the RP theoretical MD NLI can be evaluated up to 133 channels (bandwidth 5 THz) where a SSF simulation is unfeasible. Main observations from the figure are the following: 1) GN model error increases for increasing number of channels: for instance, in the DU case for PDM-QPSK the gap at 5 channels is ~2 dB, while with 133 channels it is ~3 dB; 2) while the GN model curve well scales with the logarithm of the number of channels, the scaling of the MD NLI is slower, such that the SCI share of the overall penalty is greater than GN model predictions; 3) 16QAM is closer to the GN model due to a "more Gaussian" constellation; 4) as expected, GN model error is significant in the DM30 case.

In a second set of simulations we estimated the power yielding a BER of 10^{-3} vs. the number of spans. At a given number of span there are two powers achieving the target BER, one in the amplified spontaneous emission (ASE) noise dominated regime, one in the NLI dominated regime. We estimated these powers through MC simulations by counting at least 400 errors and averaging over 10 different random realizations of input states of polarization and symbol patterns. We used different values of noise figure to control the ASE asymptote (see Fig. 2), thus limiting the reach to feasible values, without affecting the conclusions. ASE was loaded at the receiver since for such systems the nonlinear signal-noise interaction is negligible[8].

The semi-analytical model including MD-NLI (solid lines in Fig. 2) or without (i.e., GN model, dashed lines in Fig. 2) was used to estimate the NLI variance, then converted to power using the circular-noise based relations of[9, eq. (8)]. Here pulses were non-return to zero with rise time 10%. Symbol rate was 10, 28, or 80 Gbaud, with varying channel spacing so that bandwidth efficiency was 0.56 (e.g., 50 GHz @ 28 Gbaud). In all cases we used 15 channels. The digital signal processing (DSP) at the receiver included trained least squared equalization with 15 taps and blind phase estimation with 27 taps. Symbols were differentially encoded.

Fig. 2 shows the results for both a DU link (top row) and a DM30 link (bottom row). In the DU link we observe that by including MD-NLI terms in the RP model the match with SSF simulations is excellent, with a minor error at 10 Gbaud where Gaussian approximation for the received statistics starts to break down. GN model error is almost 1 dB on the NLI asymptote. This value is smaller than the error on variance observed in Fig. 1 because the power P_{M} representing the NLI asymptote, where ASE noise is negligible, is[9] $P_{\mathrm{M}} = 1/\sqrt{S_0 a_{\mathrm{NL}}}$, S_0 being the signal to noise ratio yielding BER=10^{-3}. The presence of the square root halves variance errors in a dB scale. Regarding the reach, it was theoretically proved that an error of $\Delta\alpha$ [dB] in a_{NL} translates into an error of $\Delta N_0 = -\frac{\Delta\alpha}{3+\epsilon}$ [dB] in reach, $0 < \epsilon < 1$ being the noise accumulation factor[9]. Thus the dB error on reach is at least 1/3 smaller than the dB error on a_{NL}. For these reasons, reach estimations are well tolerant to NLI modeling errors, so that the GN model, despite its non negligible inaccuracy in predicting variance, works quite well in predicting the reach[1].

The bottom row of Fig. 2 refers to the DM30 case. Here the Gaussian assumption may be questionable, even at the receiver side. Not surprisingly, the GN model error is most of the times unacceptable, e.g., 50% in reach prediction at 28Gbaud PDM-QPSK. The complete RP model including MD-NLI shows instead good accuracy,

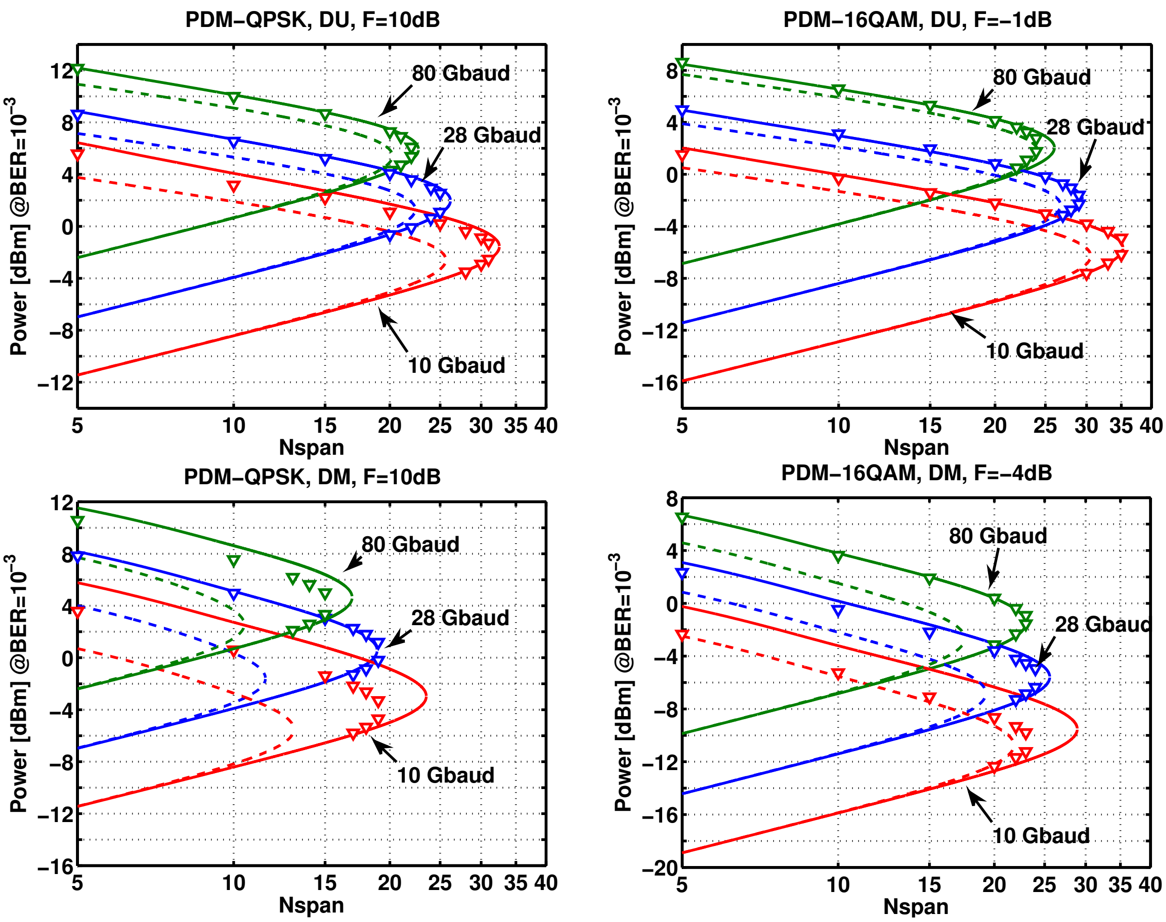

Fig. 2: Power yielding BER=10^{-3} vs. number of spans (100 km each). Solid lines: theory including modulation dependent higher-order NLI terms. Dashed lines: theory (GN Model). Symbols: Split-step Simulations. Noise figure F was set to limit the maximum number of spans to speed up simulations.

except at smaller symbol rates where the circularity of the received NLI statistics fails. For QAM, MD-NLI and GN curves are closer, because of the faster convergence to a Gaussian-like propagating field. However, it is worth mentioning that the GN model always provides a lower bound.

Conclusions

We extended the time-domain based model[3,4] to estimate the modulation-dependent NLI variance, by including single channel effects and dual polarization. We showed that such a model can be used to reasonably predict performance not only in dispersion free links, but also in DM links. The main advantage of our MD-NLI time-model is that it can be efficiently numerically evaluated to estimate performance with large number of channels where SSF simulations are unfeasible. We also discussed the reasons of the GN model accuracy in reach prediction despite its non negligible inaccuracy in NLI variance prediction.

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