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Analysis of Cross-Phase Modulation Induced Intensity Noise in High-Speed Dispersion Compensated Transmission Systems

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Abstract — **We introduce an improved theoretical model for the cross-phase modulation (XPM) induced intensity noise, and we evaluate the XPM-related impairments on 10 Gbit/s and 40 Gbit/s transmission systems with different dispersion compensation schemes.**

I. Introduction

Like any phase modulation, XPM generates intensity noise at the end of a transmission fiber because of the phase-to-intensity (PM/IM) conversion induced by group-velocity dispersion (GVD) [1]. In previous papers [2, 3], we introduced a new theoretical model for the XPM-induced intensity noise, based on the assumption of undistorted interfering channels, and we showed that it gives a very good approximation of the signal intensity noise in wavelength-division multiplexing (WDM) systems at 10 Gbit/s. In this paper, we improve such model, by assuming that the intensity of interfering channels is distorted by GVD only. The improved model well predicts the intensity noise also in systems at 40 Gbit/s. Using this model, we evaluate the XPM-related impairments in WDM transmission systems for different dispersion compensation schemes and we discuss the impact of several design parameters, such as bit rate, dispersion coefficient, dispersion slope, and residual dispersion.

II. Theoretical model

Consider one span of single-mode fiber of length L, with two co-propagating channels, s and p, having the same polarization. *Probe* channel s is continuous-wave (CW), while *pump* channel p is intensity modulated, being $P_p(0,\omega)$ the Fourier transform of its power at the beginning of the fiber. Let v_s and v_p be the group velocities of the two channels, and let $d_{sp} \triangleq 1/v_s - 1/v_p \cong D\,\Delta\lambda_{sp}$ be the walk-off parameter [4], with D the fiber dispersion at the probe wavelength and $\Delta\lambda_{sp}$ the channel spacing. The pump power at coordinate z along the fiber, in the assumption that the interfering channel intensity distortion is due to GVD only, has Fourier transform (with respect to a time frame moving with the probe group velocity) given by: $P_p(z,\omega) \triangleq P_p(0,\omega)e^{(-\alpha+j\omega d_{sp})z}\cos\left[\omega^2\frac{\lambda^2}{4\pi c}Dz\right]$. The imaginary argument of the exponential term accounts for the time shift due to channel walk-off, while the cosine term accounts for the intensity-to-intensity conversion induced by GVD in the assumption of small perturbations [1]. The probe phase induced at z through XPM by propagation of such pump over an infinitesimal segment dz is $d\theta_{sp}(z,\omega) = -2\gamma P_p(z,\omega)dz$. Such phase modulation enters the remaining $L-z$ km of fiber: if such fiber were purely linear, it would produce at its output a relative probe power distortion [1]

$$\frac{dP_{sp}(z,\omega)}{<P_s>} = -2\sin\left[\omega^2\frac{\lambda^2}{4\pi c}D(L-z)\right]d\theta_{sp}(z,\omega) \quad (1)$$

where $<P_s>$ is the time averaged output probe power, λ the probe wavelength and c the light velocity. If the infinitesimal intensity contributions add up, the total relative output power distortion on the probe is obtained by integrating (1) in dz over the fiber length:

$$\frac{\Delta P_{sp}(\omega)}{<P_s>} = P_p(0,\omega)H_{sp}(\omega), \quad (2)$$

where we defined the XPM/IM filter as:

$$\begin{aligned}
H_{sp}(\omega) &\triangleq 4\gamma\int_0^L \left\{e^{(-\alpha+j\omega d_{sp})z}\cos\left[\omega^2\frac{\lambda^2}{4\pi c}Dz\right]\cdot\right.\\
&\qquad\left.\cdot\sin\left[\omega^2\frac{\lambda^2}{4\pi c}D(L-z)\right]\right\}dz\\
&= 4\gamma\left\{\frac{1}{4j}e^{j\omega^2\frac{\lambda^2}{4\pi c}[D_r-D_a]}\frac{1-e^{\left(-\alpha+j\omega d_{sp}-2jD\omega^2\frac{\lambda^2}{4\pi c}\right)L}}{\alpha-j\omega d_{sp}+2jD\omega^2\frac{\lambda^2}{4\pi c}}\right.\\
&\quad-\frac{1}{4j}e^{-j\omega^2\frac{\lambda^2}{4\pi c}[D_r-D_a]}\frac{1-e^{\left(-\alpha+j\omega d_{sp}+2jD\omega^2\frac{\lambda^2}{4\pi c}\right)L}}{\alpha-j\omega d_{sp}-2jD\omega^2\frac{\lambda^2}{4\pi c}}\\
&\quad\left.+\frac{1}{2}\sin\left[\frac{\lambda^2}{4\pi c}\omega^2(D_r+D_a)\right]\frac{1-e^{(-\alpha+j\omega d_{sp})L}}{\alpha-j\omega d_{sp}}\right\} \quad (3)
\end{aligned}$$

where D_r is the residual dispersion accumulated from the beginning of the fiber to the end of the system, here $D_r = DL$, and D_a is the dispersion accumulated from the beginning of the system to the beginning of the fiber, here $D_a = 0$. Consider now the general case of a chain of M end-amplified fiber links, with $\alpha_i,\gamma_i,D_i,l_i,d_{sp}^{(i)}$ the attenuation, nonlinear and dispersion coefficients, the length and the walk-off parameter of the i-th link, $i = 1,...,M$, respectively. Suppose that the i-th amplifier has gain $G_p^{(i)}$ for the pump, so that the pump power at coordinate z of the k-th link, in our assumptions, is $P_p(L_k+z,\omega) = C_p^{(k)}P_p(0,\omega)e^{(-\alpha_k+j\omega d_{sp}^{(k)})z}\cos\left[\omega^2\frac{\lambda^2}{4\pi c}(D_a^{(k)}+D_kz)\right]$, where $L_k \triangleq \sum_{i=1}^{k-1}l_i$, $C_p^{(k)} \triangleq \prod_{i=1}^{k-1}e^{(-\alpha_i+j\omega d_{sp}^{(i)})l_i}G_p^{(i)}$, $C_p^{(1)} \triangleq 1$, and the accumulated dispersion from the beginning of the system to the beginning of the fiber is now $D_a^{(k)} \triangleq \sum_{i=1}^{k-1}D_il_i$. Reasoning as before, the XPM contribution $d\theta_{sp}^{(k)}(z,\omega) = -2\gamma_kP_p(L_k+z,\omega)dz$ generated at coordinate z of the k-th fiber enters a "purely linear equivalent fiber" so that its contribution to the relative output power distortion on the probe is $\frac{dP_{sp}^{(k)}(z,\omega)}{<P_s>} = -2\sin\left[\omega^2\frac{\lambda^2}{4\pi c}(D_r^{(k)}-D_kz)\right]d\theta_{sp}^{(k)}(z,\omega)$. The residual accumulated dispersion from the beginning of the k-th link to the end of the system is now $D_r^{(k)} \triangleq \sum_{i=k}^{M}D_il_i$. Integrating as before over the k-th fiber length and adding

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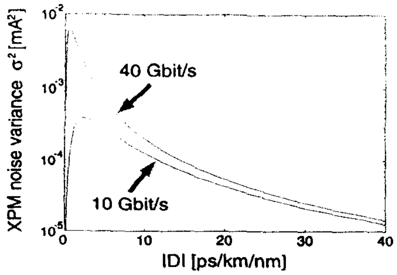

Fig. 1: XPM variance vs. dispersion coefficient.

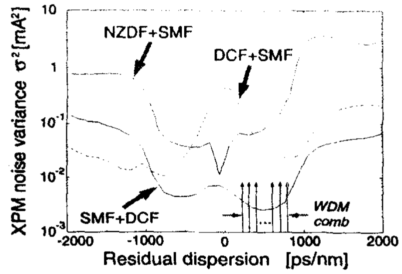

Fig. 2: XPM variance vs. residual dispersion.

the contributions of all fiber segments, the overall XPM/IM filter for the M links becomes: $H_{sp}(\omega) = \sum_{k=1}^{M} C_p^{(k)} H_{sp}^{(k)}(\omega)$, where $H_{sp}^{(k)}(\omega)$ is given by (3) with the appropriate parameters of the k-th fiber. The relative probe power distortion at the system end is given again by eq. (2). When several pump channels are present, the total relative probe power distortion can be written as the sum of the contributions due to each pump. The comparison with computer simulations carried out using the split-step Fourier method has shown that this improved model well predicts the intensity noise also at the bit rate of 40 Gbit/s.

III. Results and Discussion

If we treat the intensity noise caused by the random pump bits like additive intensity noise, its variance can be used in the evaluation of the Q factor [5], and is therefore a fundamental parameter of merit for estimating the system performance. Using the theoretical model, the overall XPM-induced intensity variance on signal s at the receiver is: $\sigma^2 = <P_s>^2 \sum_p \int_{-\infty}^{+\infty} S_p(0,\omega)\, |H_{sp}(\omega)|^2\, |H_R(\omega)|^2 d\omega$, where $S_p(0,\omega)$ is the power spectrum of the input power of channel p, $H_R(\omega)$ is the transfer function of the receiver electrical filter, and the summation is extended to the N-1 interferers, N being the number of multiplexed channels. In Fig. 1 we show the dependence of σ^2 ($N = 2$, $\Delta\lambda_{sp}$=0.8 nm) on the dispersion coefficient of a transmission fiber 100 km long (γ=2.2 W^{-1}km^{-1}, α=0.22 dB/km) at a bit rate R of 10 Gbit/s and 40 Gbit/s. The peak input power is 8 dBm, and the dispersion is perfectly compensated by an ideal linear fiber. The receiver electrical filter is ideal with bandwidth 0.65R. We see that the effect of XPM is stronger at 40 Gbit/s, and that, in general, it increases at low values of $|D|$, where the increase of the GVD-induced phase-to-intensity conversion dominates [1], and decreases at high dispersions, because of the walk-off filtering effect [4]. We note that, in the case of 10 Gbit/s, the maximum falls at $|D|$=2 ps/km/nm. We also note that σ^2 is independent of the sign of the dispersion.

Next, we evaluate a 5-span system with three diffent compensation schemes: in the first, the transmission fiber is standard single-mode (SMF), post-compensated at every span by a dispersion compensating fiber (DCF); in the second, the DCF is used for span pre-compensation; in the third, the transmission fiber is non-zero dispersion (NZDF), and a SMF is used for post-compensation. The length of the transmission fiber is 100 km, corresponding to a power loss of 22 dB. The fiber parameters are given in the table. The compensating fiber is placed between the two stages of a dual-stage amplifier [6]. The peak power at the input of the transmission fiber is 8 dBm, the one at the input of the compensating fiber is -2 dBm. In Fig. 2 we show the dependence of σ^2 on the total residual dispersion, for the three different schemes, at the bit rate of 10 Gbit/s. We see that, near the region of perfect compensation (zero residual dispersion), the system minimizing the XPM-induced intensity noise is SMF+DCF, while the inverse map DCF+SMF, which, in this region, optimizes the effect of SPM [7], performs worse in terms of XPM. We also see that all schemes show "valleys" of local minima of σ^2. In order to globally minimize the XPM impairments, the residual dispersion of each WDM channel should fall within the best "valley". For instance, because of the fiber dispersion slope, in the SMF+DCF scheme the residual dispersion of all channels of a 32 channels WDM comb, 0.8 nm spaced, falls in a range of about 1100 ps/nm. Employing instead a DCF with negative slope (-0.12 ps/km/nm^2), the range of residual dispersion is reduced to 550 ps/nm, and a better fit of the "valley" can be achieved (see Fig. 2).

Fiber	D [$ps/km/nm$]	D' (slope) [$ps/km/nm^2$]	γ [$1/W/km$]	α [dB/km]
SMF	17	0.07	2.2	0.22
NZDF	-2	0.07	2.3	0.22
DCF	-80	0.08	9.4	0.6

The peak of the variance at the zero residual dispersion for the SMF+DCF and the DCF+SMF schemes is due to the large nonlinearities generated in the second fiber of every span, which are not compensated at all. Such nonlinearities are large, in the first case because of the high nonlinear coefficient of the DCF, in the second one because of the high power at the input of the transmission fiber. Such peak can be lowered in the first scheme by reducing the power at the input of the DCF as far as possible, while the second scheme cannot be easily improved, and it is found to be dramatically limited by XPM.

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