A NEW CONTROL FOR DOUBLE-STAGE OPTICAL PMD COMPENSATORS AND ITS GEOMETRICAL INTERPRETATION

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Abstract *A control algorithm for double-stage optical PMD compensators is devised, based on the spherical geometry interpretation of the concatenation rule. Outage probability of 10^{-5} is achieved for mean line DGD of 40% the bit time.*

Introduction

Optical Polarization Mode Dispersion (PMD) compensation is usually accomplished by cascading one or more compensation *stages*, each made of a polarization controller followed by a differential delay element – such as a Polarization Maintaining Fiber – with fixed or variable Differential Group Delay (DGD) /1/. Various double-stage compensators are designed and analyzed in the literature with the aim of minimizing the PMD vector up to second order, as, e.g., in /2/.

The target of our compensation algorithm is the equalization of the fiber Jones matrix $U(\omega)$ at the frequencies $\pm\omega_0=\pm\pi/T$, where T is the bit time, which implies that $U(\omega)$ does not deviate significantly from its central frequency value $U(0)$, on the signal bandwidth $[-\omega_0;+\omega_0]$. Note that equalization at some given frequency is typically performed by tapped-delay-line electrical equalizers.

To achieve our goal, we use tools from *spherical trigonometry*: explicit equations set the control parameters of the compensator, showing that its first stage must have a fixed DGD equal to one bit time T. We obtain our numerical results by explicitly setting the compensator controls to the values imposed by the proposed algorithm. Although this requires knowledge of some parameters of the line Jones matrix, still we believe that convergence to the same control values may be achieved in similar double-stage compensators with large fixed-delay in the first stage and driven in feedback, e.g., so as to maximize the eye opening /2/.

Compensator theory

We describe the unitary line fiber Jones matrix with the formalism used in /3/, as

$$U(\omega)=U_l(\omega)U_0=\exp\{-j(\Delta\phi(\omega)/2)\,[\mathbf{b}(\omega)\cdot\boldsymbol{\sigma}]\}\,U_0 \qquad (1)$$

where $U_0=U(0)$ is factored out, so that PMD is included in the *left-extracted* exponential matrix $U_l(\omega)$. In (1), $\boldsymbol{\sigma}$ is the *spin vector*, whose elements are the three Pauli matrices; the positive angle $\Delta\phi$ is the *retardation* and the unit-magnitude Stokes vector **b** is the *eigenmode*. At any frequency, the fiber rotates the input state of polarization (ISOP) on the Poincaré sphere by an angle $\Delta\phi$ around **b**.

In /3,4/ we introduced the *rotation model* for $U(\omega)$, which accounts for all orders of PMD. It assumes that $\Delta\phi(\omega)=\Delta\phi_\omega\omega$ is linear with frequency and that $\mathbf{b}(\omega)$ describes a piece of circular trajectory on the Poincaré sphere as ω varies, at constant angular speed. Such model can be fitted to Jones matrices obtained from the standard Random Waveplate Model (RWM) over bandwidths of the order of the inverse mean DGD $1/\langle\Delta\tau\rangle$ /3,4/. Its parameters have been statistically characterized /4/ and related to the usual PMD-vector description /3/, which is less suitable for analyzing Jones matrices of actual fibers. While U_0 is irrelevant for PMD, the left-extracted matrix $U_l(\omega)$ amounts to the identity matrix I at $\omega=0$. Our objective is to equalize $U(\omega)$ so that $U_l(\pm\omega_0)=I$ *also* at two opposite optical frequencies $\pm\omega_0=\pm\pi/T$.

To this aim, we apply a two-step algorithm:

i) let $\mathbf{e}_{c1}$ be the unit-magnitude Stokes eigenmode of the first stage and $\Delta\tau_{c1}$ its DGD. We impose the following conditions on the first stage:

$$\mathbf{e}_{c1} \text{ counter-aligned to } [\mathbf{b}(+\omega_0)+\mathbf{b}(-\omega_0)] \qquad (2)$$

$$\Delta\tau_{c1}\omega_0=\pi \qquad (3)$$

Note that, from (3) and our choice of ω_0, the DGD of the first stage is fixed and equals one bit-time T. To visualize the effects of choices (2-3), we resort to the *concatenation rule for the eigenmodes and retardations* of a series of Jones matrices, which is expressed in terms of spherical trigonometry /3/. Fig.1(left) shows how to find the eigenmode $\mathbf{b}_T(+\omega_0)$ and retardation $\Delta\phi_T(+\omega_0)$ of the *line+first stage* Jones matrix $U_T(+\omega_0)$, on the Poincaré sphere, starting from the eigenmode $\mathbf{b}(+\omega_0)$ of the line and $\mathbf{e}_{c1}$ of the first stage. We construct a spherical triangle from the vertices $\mathbf{b}(+\omega_0)$ and $-\mathbf{e}_{c1}$ and their adjacent angles $\Delta\phi(+\omega_0)/2$ (internal) and $\Delta\tau_{c1}\omega_0/2=\pi/2$ (external): the eigenmode $\mathbf{b}_T(+\omega_0)$ and the angle $\Delta\phi_T(+\omega_0)/2$ result from the third vertex.

If the line matrix is such that $\Delta\phi(-\omega_0)=-\Delta\phi(+\omega_0)$, which holds for the rotation model, then the construction for $\mathbf{b}_T(-\omega_0)$ is the mirror image of the one just described, as shown by the dashed line spherical triangle, so that $\mathbf{b}_T(-\omega_0)=\mathbf{b}_T(+\omega_0)$.

Although fibers emulated with the RWM can deviate from the rotation model, the principle of operation of the compensator is still valid. Fig. 1(right) shows the trace of the line eigenmode $\mathbf{b}(\omega)$ on a bandwidth $[-\omega_0;+\omega_0]$ computed before and after the application of the first stage to a RWM emulated fiber: the typical *bending* of the eigenmode trace reduces the system matrix to a nearly first-order PMD matrix.

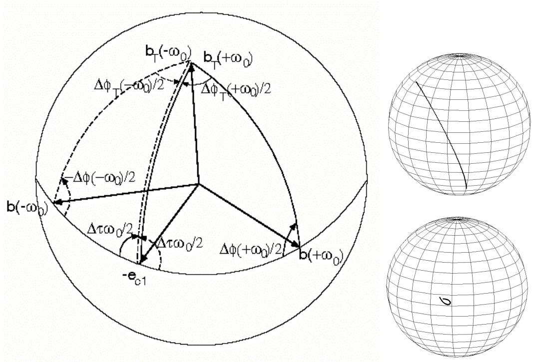

Fig.1 Left) Geometrical construction showing the principle of operation of the first stage. Right) application to a RWM emulated fiber.

ii) let $\mathbf{e}_{c2}$ be the eigenmode of the second stage and $\Delta\tau_{c2}$ its (variable) DGD. We now impose that $\mathbf{e}_{c2}$ is opposite to $\mathbf{b}_T(\pm\omega_0)$ and that $\Delta\tau_{c2}=\Delta\phi_T(\omega_0)/\omega_0$, so as to perform first-order compensation of the Jones matrix $U_T(\pm\omega_0)$. Explicitly, we set:

$$\Delta\tau_{c2}= (2/\omega_0)\mathrm{ArcCos}(-\mathrm{Sin}(\Delta\phi(\omega_0)/2)\ \mathbf{e}_{c1}\cdot\mathbf{b}(+\omega_0)) \qquad (4)$$

$$\mathbf{e}_{c2}= -[\mathrm{Cos}(\Delta\phi(\omega_0)/2)\ \mathbf{e}_{c1} + \mathrm{Sin}(\Delta\phi(\omega_0)/2)\ \mathbf{e}_{c1}\times\mathbf{b}(+\omega_0)] / \mathrm{Sin}(\Delta\tau_{c2}\omega_0/2) \qquad (5)$$

After such two steps, we have equalized the line Jones matrix at $\pm\omega_0$. It can be shown that conditions (2-3) are both necessary and sufficient for such equalization.

Simulation results

We evaluated the Outage Probability (OP) versus the average line DGD $<\Delta\tau>$, following the semi-analytical technique described in /4/, where OP is defined here as the probability that the sensitivity penalty (SP), at BER=10^{-10}, exceeds 3dB. To summarize the method described in /4/, 10Gb/s NRZ transmission was performed over 363 *representative* sample fibers, extracted from a pool of 500,000 DRW emulated fibers. For each fiber sample, propagation was repeated using 62 different ISOPs, uniformly tiling the Poincaré sphere, for a total of more than 20,000 SP calculations. Propagation and SP evaluation was then repeated, using the same ISOPs, on 648 fibers synthesized with the Rotation Model, for the purpose of covering those cases of very large high-order PMD not occurring in the half-million set of DRW fibers.

Using SP data from the 648 Rotation Model samples and the 363 DRW samples, the leftmost curve in Fig.2 reports the OP for the uncompensated case, showing that, for an OP=10^{-5} (5 minutes outage per year), the maximum tolerable mean DGD is 15ps. Such curve, already obtained in /4/, is reported here for comparison with the OP obtained using our double-stage compensator driven as described above, corresponding to the rightmost curve, which shows that, for the same target OP, the maximum tolerable $<\Delta\tau>$ reaches 40ps. For completeness, we also report in Fig.2 the OP curve obtained using a single stage compensator with a fixed DGD set at 60ps.

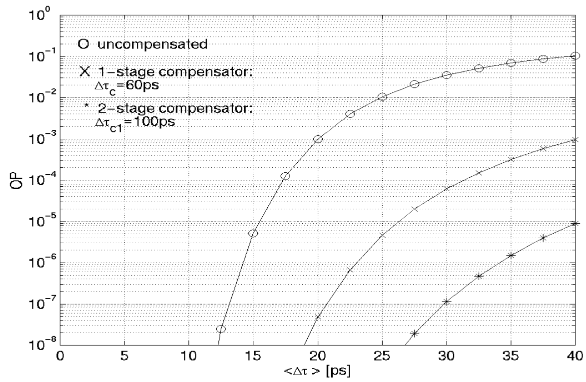

Fig.2 Outage Probability vs. mean DGD of the transmission fiber.

Conclusions

We described a control algorithm for a double-stage optical PMD compensator, whose target is the equalization of the fiber Jones matrix $U(\omega)$ at the frequencies $\pm\omega_0=\pm\pi/T$. The enabling concept for the compensation strategy is the eigenmodes *concatenation rule* /3/, which is expressed in terms of spherical trigonometry. We described how the tasks of the two stages are conceptually different: the first stage compensates for the depolarization of the line eigenmodes, i.e., eliminates higher-order PMD, while the second stage compensates for the residual PMD, which is nearly first-order. Though we refer to a well-known compensator structure, another element of novelty is the large DGD ($\Delta\tau_{c1}$=T) that is needed in the first stage to achieve our goal.

References

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