Transient Gain Dynamics
in Saturated Counter-pumped Raman Amplifiers

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Outline
✓ Motivation
✓ New State-Variable Model
✓ Results
✓ Conclusions
Motivation

- Saturation-induced gain transients (Cross-gain modulation):
  Chen and Wong, OAA 2001, paper OMC2

- Physical reason: pump-signal walkoff
  signal front grows and depletes pump
  ⇒ signal body finds less pump than front

- Need to Quantify:
  Power surges, time constants of transients

☞ Need a system model!
Propagation Equations

\[
\begin{align*}
\left\{ \begin{array}{l}
\left( \frac{\partial}{\partial z} + \frac{1}{v} \frac{\partial}{\partial t} \right) S_j(t,z) &= \left[ -\alpha_j + g_j P(t,z) \right] S_j(t,z) \quad j = 1, \ldots, N \\
\left( \frac{\partial}{\partial z} - \frac{1}{v} \frac{\partial}{\partial t} \right) P(t,z) &= \left[ \alpha_p + \sum_{j=1}^{N} \hat{g}_j S_j(t,z) \right] P(t,z)
\end{array} \right.
\end{align*}
\]

where: \( S_j \) signal [W]; \( v \) speed [m/s]; \( g_j \) Raman gain [1/W/m]; \( \alpha \) attenuation [1/m]; \( P \) pump [W]; \( \hat{g}_j \triangleq g_j \lambda_j / \lambda_p \)

**Neglect:**

- ASRS noise
- Rayleigh Backscattering
- direct signal-signal crosstalk
Implicit Solution

Solution of propagation equations in the signals and pump

\[ t_s = t - z/\nu, \quad t_p = t + z/\nu \]

is

\[
\begin{align*}
    S_j(t_s, z) &= S_j^{in}(t_s) \exp\{-\alpha_j z + g_j \int_0^z P(t_s + d z', z')dz'\} \\
    P(t_p, z) &= P_0 e^{-\alpha_p(L-z)} e^{-\Gamma(t_p,z)}
\end{align*}
\]

where: \( S_j^{in}(t_s) \) input signal [W]; \( d \triangleq 2/\nu \) [s/m] walkoff parameter;
\( P_0 \) launched pump [W]; \( L \) amplifier length [m]; and

\[
\Gamma(t_p,z) \triangleq \sum_{j=1}^N \hat{g}_j \int_{z}^{L} S_j(t_p - d z', z')dz'
\]

is the pump depletion in the pump time frame.
At moderate pump depletion $e^{-\Gamma} \approx 1 - \Gamma$. Signal power at output is thus

$$S_{j}^{\text{out}}(t_s) = S_{j}^{\text{in}}(t_s) \exp \left\{ -\alpha_j L + Q_j (1 - x(t_s)) \left( 1 - e^{-\alpha_p L} \right) \right\}$$

where $Q_j \triangleq \frac{g_j}{\alpha_p} P_0$ and

$$x(t_s) \triangleq \frac{1}{L_{e f f}^{(p)}} \int_0^L e^{-\alpha_p (L - z')} \Gamma(t_s + dz', z') \, d z'$$

is the \textit{relative change in injected pump power sensed by the signals at retarded time $t_s$}.

\textit{Once $x(t_s)$ is known, all WDM output signals are known!}
The State Equation

The state variable $x(t_s)$ satisfies the following implicit integral equation

$$x(t_s) = \sum_{j=1}^{N} S_{j}^{out}(t_s, x(t_s)) \otimes h_j(t_s)$$

where $\otimes$ denotes convolution, and $h_j$ is the impulse response of a linear filter:

$$h_j(t) = \frac{\hat{g}_j}{d L_{eff}^{(p)} G_j(L)} \left[ \int_0^{L-t} e^{-\alpha p(L-z')} G_j \left( z' + \frac{t}{d} \right) dz' \right] \cdot p(t)$$

where $G_j(z)$ is the unsaturated gain-versus-$z$ profile, and $p(t) =$
A **closed-form** of such filter exists. For a DCF with \( \alpha_j = 0.46 \) dB/km, \( \alpha_p = 0.6 \) dB/km, \( g_j = 2 \) [W\(^{-1}\)km\(^{-1}\)] get: (- - exponential approx.)
Amplifier Block Diagram

RIN filter: Cfr Fludger et al, JLT Aug ’01.

Can be implemented in any Block-Diagram Simulator!
Results: Single Channel

Two Pulses of 1 and 0.1 mW, 400 µs each. Ampli: L=14 km, DCF.

(Left) state variable, and (Right) output signal power, for increasing levels of pump power $P_0 = 0.64, 0.70, 0.77, 0.86, 0.97$ W. Solid lines: exact solution (+DRB); Dahed lines: model.
Results: WDM

Three staggered pulses (1 mW, 800 $\mu$s each) on 3 channels, $\Delta \lambda = 0.4$ nm. L=14 km, DCF.

(Left) state variable, and (Right) output signal powers, for pump power $P_0 = 0.86$ W. Solid lines: exact solution (+SS xtalk, +DRB); Dahed lines: model.
Power surges

The power Sag across a pulse is:

\[
\left. \frac{S_{out}(0)}{S_{out}(x^{ss})} \right|_{dB} \approx (10 \log_{10} e) \left[ \hat{g}_s \alpha_p \left( \sum_{i=1}^{N} S_{out}^{out}(x^{ss}) \right) \right]
\]
Time constants

Using exponential approximation $h_j(t) \approx h_{j0} e^{-\frac{\alpha_p}{d} t}$, state equation becomes an ordinary differential equation (ODE), similar to that for EDFAs:

$$\dot{x}(t) = -\frac{x(t)}{\tau} + \sum_{j=1}^{N} h_{j0} S_{j}^{\text{out}}(t, x(t))$$

with $\tau \triangleq d/\alpha_p$ playing the role of the fluorescence time.

Using standard linearization technique (Y. Sun et al., JLT, July 97) we find:

$$\tau_{\text{eff}} \approx \frac{d/\alpha_p}{1 + \left[\frac{\alpha}{\alpha_p} \left(\sum_{i=1}^{N} S_{i}^{\text{out}}(x^{ss})\right)\right]}$$
Conclusions

✔ XGM present also in Raman amplifiers

✔ New state-variable model of counter-pumped saturated Raman amplifiers

✔ Closed-form expressions for power surges and time constants of transients