Soft-Output Detection of CPM Signals Transmitted Over Channels Affected by Phase Noise

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***Abstract*— We consider continuous phase modulations (CPMs) and their transmission over a typical satellite channel affected by phase noise. By modeling the phase noise as a Wiener process and adopting a simplified representation of an M-ary CPM signal based on the principal pulses of its Laurent decomposition, we derive the MAP symbol detection strategy. Since it is not possible to derive the exact detection rule by means of a probabilistic reasoning, the framework of factor graphs (FGs) and the sum-product algorithm is used. By pursuing the principal approach to manage continuous random variable in a FG, i.e., the canonical distribution approach, two algorithms are derived which do not require the presence of known (pilot) symbols, thanks to the intrinsic differential encoder embedded in the CPM modulator.**

I. Introduction

Continuous phase modulations (CPMs) form a class of signaling formats that are efficient in power and bandwidth [1]. Moreover, the recursive nature of the CPM modulator makes them attractive in serially concatenated schemes to be decoded iteratively [2], [3].

Several decomposition approaches for CPM signals were presented in the literature. One of the most appealing, from a detection point of view, is the Laurent decomposition [4], [5] in which a CPM signal is expanded as a sum of linearly modulated components. This decomposition has been used in [6] to simplify the receiver front-end and the number of trellis states in maximum a posteriori (MAP) *sequence* detection based on the Viterbi algorithm [6]. More recently, by using factor graphs (FGs) and the sum-product algorithm (SPA) [7], this result has been also extended to MAP *symbol* detection schemes suitable to be used in coherent iterative detection/decoding [8].

Although several soft-input soft-output (SISO) detection algorithms suitable for iterative detection/decoding have been recently designed for linear modulations transmitted over channels affected by a time-varying phase (see for example [9]–[11] and references therein), less attention has been devoted to CPM signals. An exception is represented by [12] where, based on the approach in [10], joint detection and phase synchronization is performed by working on the trellis of the CPM signal or on an expanded trellis and using *multiple* phase estimators in a per-survivor fashion.

In this paper, we adopt a Bayesian approach, i.e., the channel phase is modeled as a stochastic process with known statistics. In particular, we model the phase noise as a Wiener process. By using the FG/SPA framework, we derive the exact MAP *symbol* detection strategy under the simplified representation of a CPM signal as sum of the principal pulses of its Laurent decomposition.[1] We analyze the properties of this detection strategy finding that it can be implemented by using a *single* forward-backward estimator of the phase probability density function, followed by a symbol-by-symbol completion to produce the a posteriori probabilities of the information symbols. Then, by using the canonical distribution approach [13] we develop a couple of practical schemes to implement the forward-backward estimator. The resulting algorithms obviously work in a joint demodulation/phase tracking fashion, do not require the insertion of pilot symbols, and may be used as SISO blocks for iterative detection/decoding in concatenated schemes.

[1] As shown in [8], a coherent receiver designed according to this simplified representation entails only a minor performance degradation.

The remainder of this paper is organized as follows. In Section II we provide the signal model and briefly review the Laurent decomposition. By means of the FG/SPA framework, the exact MAP symbol detection strategy is derived in Section III. The practical implementation of this exact strategy is discussed in Section IV, and a couple of algorithms proposed. The relevant performance, is assessed in Section V and finally, some conclusions are drawn in Section VI.

II. Signal Model and Laurent Decomposition

The complex envelope of a CPM signal has the form [1]

$$s(t,\boldsymbol{\alpha}) = \sqrt{\frac{2E_S}{T}}\ \exp\{j2\pi h \sum_{n=0}^{N-1} \alpha_n q(t-nT)\} \qquad (1)$$

in which E_S is the energy per information symbol, T is the symbol interval, $h = r/p$ is the modulation index (r and p are relatively prime integers), the information symbols $\{\alpha_n\}$ are assumed independent and take on values in the M-ary alphabet $\{\pm 1, \pm 3, \ldots, \pm(M-1)\}$, $\boldsymbol{\alpha} = \{\alpha_n\}$ denotes the information sequence, and finally N is the number of transmitted information symbols. The function $q(t)$ is the *phase-smoothing response* and its derivative is the *frequency pulse*, assumed of duration LT.

Based on Laurent representation, the complex envelope of a CPM signal may be exactly expressed as [5]

$$s(t,\boldsymbol{\alpha}) = \sum_{k=0}^{Q^{\log_2 M}(M-1)-1} \sum_n a_{k,n} p_k(t-nT) \qquad (2)$$

in which M is assumed to be a power of two to simplify the notation, $Q \triangleq 2^{L-1}$, and the expressions of pulses $\{p_k(t)\}$ and symbols $\{a_{k,n}\}$ as a function of the information symbol sequence $\{\alpha_n\}$ may be found in [5] (see this reference for the general case of M non-power of two). By truncating the summation in (2) considering only the first $K < Q^{\log_2 M}(M-1)$ terms, we obtain an approximation of $s(t,\boldsymbol{\alpha})$:

$$s(t,\boldsymbol{\alpha}) \simeq \sum_{k=0}^{K-1} \sum_n a_{k,n} p_k(t-nT)\,. \qquad (3)$$

Most of the signal power is concentrated in the first $M-1$ components, i.e., those associated with the pulses $\{p_k(t)\}$ with

$0 \leq k \leq M-2$, which are denoted as *principal components* [5]. As a consequence, a value of $K = M-1$ may be used in (3) to attain a very good tradeoff between approximation quality and number of signal components and in fact, in [6] and [8] it was shown that MAP *sequence* or *symbol* detection receivers only based on principal pulses practically attain the performance of the corresponding optimal detector. A nice feature of the principal components is that their symbols $\{a_{k,n}\}_{k=0}^{M-2}$ may be expressed as a function of the information symbol α_n and of symbol $a_{0,n-1}$. As an example, symbol $a_{0,n}$ can be recursively computed as [5][2]

$$a_{0,n} = a_{0,n-1} e^{j\pi h \alpha_n} . \quad (4)$$

Symbols $\{a_{0,n}\}$ take on p values [5]. They belong to the alphabet $\mathcal{A}_o = \{e^{j2\pi hm}, m = 0, 1, \ldots, p-1\}$ when n is odd, or to the alphabet $\mathcal{A}_e = \{e^{j\pi h} e^{j2\pi hm}, m = 0, 1, \ldots, p-1\}$ when n is even.[3]

III. MAP Symbol Detection

We now consider the transmission of a CPM signal over a typical satellite channel affected by phase noise plus the additive white Gaussian noise (AWGN). The complex envelope of the received signal can be modeled as

$$r(t) = s(t, \boldsymbol{\alpha}) e^{j\theta(t)} + w(t) \quad (5)$$

where $w(t)$ is a complex-valued white Gaussian noise process with independent components, each with two-sided power spectral density N_0, and $\theta(t)$ is the phase noise introduced by the channel. We model the phase noise $\theta(t)$ as a time-continuous Wiener process with incremental variance over a signaling interval equal to σ_Δ^2. The assumption on the phase noise model will be relaxed in the numerical results. We also assume that the channel phase $\theta(t)$ is slowly varying such that it can be considered constant over the duration of the pulses $\{p_k(t)\}$. In other words we assume that

$$\begin{aligned}\int_{-\infty}^{+\infty} & r(t) p_k(t-nT) e^{-j\theta(t)}\, dt \\ &= e^{-j\theta_n} \int_{-\infty}^{+\infty} r(t) p_k(t-nT) e^{j[\theta_n - \theta(t)]}\, dt \\ &\simeq e^{-j\theta_n} \int_{-\infty}^{+\infty} r(t) p_k(t-nT)\, dt = e^{-j\theta_n} x_{k,n} \quad (6)\end{aligned}$$

having defined $\theta_n = \theta(nT)$ and $x_{k,n} = r(t) \otimes p_k(-t)|_{t=nT}$, where $\otimes$ denotes "convolution". Hence, under this hypothesis, the output, sampled at the symbol rate, of a bank of filters matched to the pulses $\{p_k(t)\}$ is a set of sufficient statistics for this detection problem (see also [6]). Since most of the signal power is concentrated in the principal components, we use a simplified set represented by the output of a bank of filters matched to the corresponding pulses $\{p_k(t)\}_{k=0}^{M-2}$. From (6), only the samples of $\theta(t)$ at discrete-time nT are significant. These samples satisfy the discrete-time Wiener model:

$$\theta_{n+1} = \theta_n + \Delta_n \quad (7)$$

where $\{\Delta_n\}$ are real independent and identically distributed Gaussian random variables with zero mean and standard deviation σ_Δ,[4] and θ_0 is uniformly distributed in $[0, 2\pi)$.

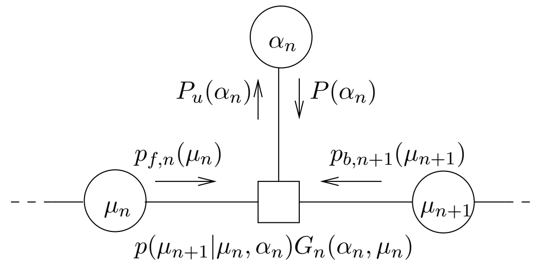

Fig. 1. Factor graph for the considered problem.

We now derive the MAP *symbol* detection strategy. To this purpose we first compute the joint distribution[5] $p(\boldsymbol{\alpha}, \mathbf{a}, \boldsymbol{\theta}|\mathbf{x})$ where $\mathbf{x} = \{\mathbf{x}_n\}$, with $\mathbf{x}_n = \{x_{k,n}\}_{k=0}^{M-2}$, $\mathbf{a} = \{\mathbf{a}_n\}$, with $\mathbf{a}_n = \{a_{k,n}\}_{k=0}^{M-2}$, and $\boldsymbol{\theta} = \{\theta_n\}$. Its expression is

$$p(\boldsymbol{\alpha}, \mathbf{a}, \boldsymbol{\theta}|\mathbf{x}) \mathrel{\tilde{\propto}} P(\boldsymbol{\alpha}) P(\mathbf{a}|\boldsymbol{\alpha}) p(\boldsymbol{\theta}) \prod_n G_n(\alpha_n, a_{0,n-1}, \theta_n) \quad (8)$$

where [6], [8][6]

$$G_n(\alpha_n, a_{0,n-1}, \theta_n) = \exp\left\{ \frac{1}{N_0} \mathrm{Re}\left[e^{-j\theta_n} \sum_{k=0}^{M-2} x_{k,n} a_{k,n}^* \right] \right\} \quad (9)$$

and symbol $\tilde{\propto}$ has been used to denote an approximate proportionality relationship. The approximation here is related to the fact that we are considering the principal components only. We can further factor the terms $P(\boldsymbol{\alpha})$, $P(\mathbf{a}|\boldsymbol{\alpha})$, and $p(\boldsymbol{\theta})$ in (8) as

$$P(\boldsymbol{\alpha}) = \prod_{n=0}^{N-1} P(\alpha_n) \quad (10)$$

$$P(\mathbf{a}|\boldsymbol{\alpha}) = P(a_{0,-1}) \prod_{n=0}^{N-1} I(a_{0,n}, a_{0,n-1}, \alpha_n) \quad (11)$$

$$p(\boldsymbol{\theta}) = p(\theta_0) \prod_{n=1}^{N-1} p(\theta_n|\theta_{n-1}) \quad (12)$$

where $I(a_{0,n}, a_{0,n-1}, \alpha_n)$ is an indicator function, equal to 1 if α_n and the pseudo-symbols $a_{0,n-1}$ and $a_{0,n}$ respect the constraint (4), and to zero otherwise, and $p(\theta_n|\theta_{n-1})$ is a Gaussian pdf in θ_n with mean θ_{n-1} and variance σ_Δ^2. In the following, we will denote a Gaussian pdf in the variable x, with mean η and variance ρ^2, as $g(\eta, \rho^2; x)$. Substituting (10), (11), and (12) into (8), clustering [7] the variables θ_n and $a_{0,n-1}$, i.e., defining $\mu_n = (a_{0,n-1}, \theta_n)$, we obtain the FG in Fig. 1, where we defined $p(\mu_{n+1}|\mu_n, \alpha_n) = p(\theta_{n+1}|\theta_n) I(a_{0,n}, a_{0,n-1}, \alpha_n)$. Since this FG does not contain cycles, the application to it of the SPA with a *non-iterative* forward-backward schedule produces the exact a posteriori probabilities $P(\alpha_n|\mathbf{x})$ necessary to implement the MAP *symbol* detection strategy. In the figure, we defined $P_u(\alpha_n)$ as the extrinsic a posteriori probability of α_n, i.e., $P_u(\alpha_n) = P(\alpha_n|\mathbf{x})/P(\alpha_n)$. With reference to the messages in the figure, the resulting forward–backward algorithm is characterized by the following recursions and

[2] Since in the next section the transmission over a channel affected by phase noise will be considered, we may assume that the initial symbol $a_{0,-1}$ is unknown to the receiver due to the initial channel phase uncertainty.

[3] When r is even, $\mathcal{A}_o$ and $\mathcal{A}_e$ coincide.

[4] Note that, since the channel phase is defined modulo 2π, the probability density function (pdf) $p(\theta_{n+1}|\theta_n)$ can be approximated as Gaussian only if $\sigma_\Delta \ll 2\pi$.

[5] We still use the symbol $p(.)$ to denote a continuous pdf with some discrete probability masses.

[6] Due to the above mentioned property of the principal components, G_n is a function of α_n, $a_{0,n-1}$, and θ_n only. We omitted the dependence on $\{x_{k,n}\}$ since these samples are known to the receiver.

completion:

$$p_{f,n+1}(a_{0,n},\theta_{n+1}) = \sum_{\alpha_n} P(\alpha_n)\int p_{f,n}(\check{a}_{0,n-1},\theta_n) \cdot G_n(\alpha_n,\check{a}_{0,n-1},\theta_n)g(\theta_n,\sigma_\Delta^2;\theta_{n+1})d\theta_n \quad (13)$$

$$p_{b,n}(a_{0,n-1},\theta_n) = \sum_{\alpha_n} P(\alpha_n)G_n(\alpha_n,a_{0,n-1},\theta_n) \cdot \int p_{b,n+1}(\check{\check{a}}_{0,n},\theta_{n+1})g(\theta_n,\sigma_\Delta^2;\theta_{n+1})d\theta_{n+1} \quad (14)$$

$$P_u(\alpha_n) = \sum_{a_{0,n}} \iint p_{f,n}(\check{a}_{0,n-1},\theta_n)p_{b,n+1}(a_{0,n},\theta_{n+1}) \cdot G_n(\alpha_n,\check{a}_{0,n-1},\theta_n)g(\theta_n,\sigma_\Delta^2;\theta_{n+1})d\theta_n d\theta_{n+1} \quad (15)$$

where in (13) and (15) $\check{a}_{0,n-1} = a_{0,n}e^{-j\pi h\alpha_n}$, whereas in (14) $\check{\check{a}}_{0,n} = a_{0,n-1}e^{j\pi h\alpha_n}$, and with the following initial conditions: $p_{f,0}(a_{0,-1},\theta_0) = 1/(p2\pi)$ and $p_{b,N}(a_{0,N-1},\theta_N) = 1/(p2\pi)$.

A proof of the following properties is omitted for a lack of space:

Property 1: for each $\ell = 0,\dots,p-1$,

$$p_{f,n}(a_{0,n-1}e^{j2\pi h\ell},\theta_n) = p_{f,n}(a_{0,n-1},\theta_n+2\pi h\ell) \quad (16)$$

$$p_{b,n}(a_{0,n-1}e^{j2\pi h\ell},\theta_n) = p_{b,n}(a_{0,n-1},\theta_n+2\pi h\ell) \quad (17)$$

Property 2: The extrinsic information in (15) is given by the sum of p terms (one for each value of $a_{0,n}$). All these terms assume the same value, i.e., they do not depend on $a_{0,n}$, for each given α_n.

From the first property it follows that it is not necessary to evaluate and store all pdfs $p_{f,n}(a_{0,n-1},\theta_n)$ and $p_{b,n}(a_{0,n-1},\theta_n)$ for different values of $a_{0,n-1}$. Defining

$$\bar{a}_n = \begin{cases} 1 & n \text{ odd} \\ e^{j\pi h} & n \text{ even} \end{cases} \quad (18)$$

it is sufficient to evaluate $\bar{p}_{f,n}(\theta_n) = p_{f,n}(a_{0,n-1} = \bar{a}_{n-1},\theta_n)$ and $\bar{p}_{b,n}(\theta_n) = p_{b,n}(a_{0,n-1} = \bar{a}_{n-1},\theta_n)$. From the second property, it follows that only one term in (15) needs to be evaluated. The MAP symbol detection strategy can therefore be simplified as follows:

$$\bar{p}_{f,n+1}(\theta_{n+1}) = \sum_{\alpha_n} P(\alpha_n)\int \bar{p}_{f,n}(\theta_n - 2\pi h\gamma_n) \cdot G_n(\alpha_n,\bar{a}_n e^{-j\pi h\alpha_n},\theta_n)g(\theta_n,\sigma_\Delta^2;\theta_{n+1})d\theta_n \quad (19)$$

$$\bar{p}_{b,n}(\theta_n) = \sum_{\alpha_n} P(\alpha_n)G_n(\alpha_n,\bar{a}_{n-1},\theta_n) \cdot \int \bar{p}_{b,n+1}(\theta_{n+1}+2\pi h\gamma_n)g(\theta_n,\sigma_\Delta^2;\theta_{n+1})d\theta_{n+1} \quad (20)$$

$$P_u(\alpha_n) \propto \iint \bar{p}_{f,n}(\theta_n - 2\pi h\gamma_n)\bar{p}_{b,n+1}(\theta_{n+1}) \cdot G_n(\alpha_n,\bar{a}_n e^{-j\pi h\alpha_n},\theta_n)g(\theta_n,\sigma_\Delta^2;\theta_{n+1})d\theta_n d\theta_{n+1} \quad (21)$$

where $\gamma_n = (\alpha_n+1)/2$, if n is odd, and $\gamma_n = (\alpha_n-1)/2$, if n is even, and with the following initial conditions: $\bar{p}_{f,0}(\theta_0) = 1/2\pi$ and $\bar{p}_{b,N}(\theta_N) = 1/2\pi$. Hence, we have a single forward-backward estimator of the phase probability density function and a final completion.

This exact MAP symbol detection strategy involves integration and computation of continuous pdfs, and it is not suited for direct implementation. A solution for this problem is suggested in [13] and consists of the use of *canonical distributions*, i.e., the pdfs $\bar{p}_{f,n}(\theta_n)$ and $\bar{p}_{b,n}(\theta_n)$ computed by the algorithm are constrained to be in a certain "canonical" family, characterized by some parameterization. Hence, the forward and backward recursions reduce to propagating and updating the parameters of the pdf rather than the pdf itself. In the next section, two low-complexity algorithms based on this approach will be described.

IV. Low-Complexity Algorithms

A. First Algorithm

A very straightforward solution to implement (19) and (20) is obtained by discretizing the channel phase [9], [11]. In this way, the pdfs $\bar{p}_{f,n}(\theta_n)$ and $\bar{p}_{b,n}(\theta_n)$ become probability mass functions (pmfs) and the integrals in (19), (20), and (21) become summations. When the number D of discretization levels is large enough, the resulting algorithm becomes optimal (in the sense that its performance approaches that of the exact algorithm).[7] Hence, it may be used to obtain a performance benchmark and will be denoted to as "discretized-phase algorithm" (*dp-algorithm*).

B. Second Algorithm

By observing that the Tikhonov distribution ensures a very interesting performance with a low complexity when used as a canonical distribution in detection algorithms for phase noise channels [11], pdfs $\bar{p}_{f,n}(\theta_n)$ and $\bar{p}_{b,n}(\theta_n)$ are constrained to have the following expressions

$$\bar{p}_{f,n}(\theta_n) = \sum_{m=0}^{p-1} q_{f,n}^{(m)} t\left(z_{f,n};\theta_n - \frac{2\pi}{p}m\right) \quad (22)$$

$$\bar{p}_{b,n}(\theta_n) = \sum_{m=0}^{p-1} q_{b,n}^{(m)} t\left(z_{b,n};\theta_n - \frac{2\pi}{p}m\right) \quad (23)$$

where, for each time index n, $\{q_{f,n}^{(m)}, m = 0,1,\dots,p-1\}$ ($\{q_{b,n}^{(m)}, m = 0,1,\dots,p-1\}$) and $z_{f,n}$ ($z_{b,n}$) are, respectively, p real coefficients and one complex coefficient, and $t(z;\theta)$ is a Tikhonov distribution with complex parameter z defined as

$$t(z;\theta) = \frac{e^{\mathrm{Re}\left[ze^{-j\theta}\right]}}{2\pi \mathrm{I}_0(|z|)} \quad (24)$$

$\mathrm{I}_0(x)$ being the zero-th order modified Bessel function of the first kind. Note that $\sum_{m=0}^{p-1} q_{f/b,n}^{(m)} = 1$ in order to obtain pdfs.

Three approximations are now introduced in order to derive a low complexity detection algorithm:[8]

i. the convolution of a Tikhonov and a Gaussian pdf is still a Tikhonov pdf, with a modified complex parameter [11], i.e.,

$$\int t(z;x)g(x,\rho^2;y)dx \simeq t\left(\frac{z}{1+\rho^2|z|};y\right) \quad (25)$$

ii. since, for large arguments, $\mathrm{I}_0(x) \simeq e^x$, we approximate

$$e^{\mathrm{Re}[ze^{-j\theta}]} \simeq 2\pi e^{|z|}t(z;\theta) \quad (26)$$

iii. let z be a complex number, $\{u_m, m = 0,1,\dots,p-1\}$ a set of complex numbers, and $\{q_m, m = 0,1,\dots,p-1\}$ a set of real numbers such that $\sum_m q_m = 1$, then the following approximation holds, especially when $|z|$ is sufficiently larger

[7] As a rule of thumb (confirmed by the results in [9]), the number of discretization levels must be at least $D = 8p$ in order to avoid any performance loss.

[8] A justification of these approximations is represented by the excellent performance of the resulting algorithm and by its very low complexity.

than each $|u_m|$ or when there is a $\overline{m}$ such that $q_{\overline{m}} \gg q_m, \forall m \neq \overline{m}$:

$$\sum_m q_m t\left(ze^{j\frac{2\pi}{p}m} + u_m;\theta\right) \simeq \sum_m q_m t\left(we^{j\frac{2\pi}{p}m};\theta\right) \quad (27)$$

where $w = z + \sum_\ell q_\ell u_\ell e^{-j\frac{2\pi}{p}\ell}$.

In order to illustrate the derivation of the proposed algorithm, we consider the case of a binary modulation, i.e., $M = 2$, and hence $K = 1$, although the generalization to the non-binary case is straightforward from a conceptual viewpoint. In this case

$$\frac{1}{N_0}\sum_{k=0}^{K-1} x_{k,n} a^*_{k,n} = \frac{1}{N_0} x_{0,n} a^*_{0,n} \quad (28)$$

and we define $y_n = \frac{x_{0,n}\bar{a}^*_n}{N_0}$. We now derive the reduced-complexity forward recursion. Substituting (9) in (19), assuming that $\bar{p}_{f,n-1}(\theta_{n-1})$ has the canonical expression (22), and using approximation (25), we obtain

$$\bar{p}_{f,n+1}(\theta_{n+1}) = \sum_{\alpha_n} P(\alpha_n) \sum_{m=0}^{p-1} q^{(m)}_{f,n} \int g(\theta_n, \sigma^2_\Delta; \theta_{n+1}) \cdot t\left(z_{f,n}; \theta_n - \frac{2\pi}{p}(r\gamma_n + m)\right) e^{\mathrm{Re}\left[y_n e^{-j\theta_n}\right]} d\theta_n \,. \quad (29)$$

By now changing the first summation index in $\ell = m + r\gamma_n$, using (25) and (26), discarding irrelevant multiplicative factors, and neglecting $|y_n|$ with respect to $|z_{f,n}|$, we have

$$\bar{p}_{f,n+1}(\theta_{n+1}) = \sum_\ell \left[\sum_{\alpha_n} P(\alpha_n) q^{(\ell - r\gamma_n)}_{f,n}\right] \cdot e^{z_{f,n} + y_n e^{-j\frac{2\pi}{p}\ell}} t\left(\frac{z_{f,n} e^{j\frac{2\pi}{p}\ell} + y_n}{1 + \sigma^2_\Delta |z_{f,n}|}; \theta_n\right) \quad (30)$$

This resulting $\bar{p}_{f,n+1}(\theta_{n+1})$ is not in the constrained form (22). However, by applying the approximation (27), we obtain the following updating equations for the parameters of the canonical distribution (22)

$$q^{(\ell)}_{f,n+1} \propto \left[\sum_{\alpha_n} P(\alpha_n) q^{(\ell - r\gamma_n)}_{f,n}\right] e^{z_{f,n} + y_n e^{-j\frac{2\pi}{p}\ell}} \quad (31)$$

$$z_{f,n+1} = \frac{z_{f,n} + y_n \sum_m q^{(m)}_{f,n+1} e^{-j\frac{2\pi}{p}m}}{1 + \sigma^2_\Delta |z_{f,n}|} \,. \quad (32)$$

It is worth noticing that, before the evaluation of the coefficient $z_{f,n+1}$, the p real coefficients $q^{(\ell)}_{f,n+1}$ evaluated through (31) have to be normalized so that their sum is 1. Since there is no a priori knowledge of the initial phase or of the initial symbol $a_{0,-1}$, the following initial values of the recursive coefficients result

$$q^{(\ell)}_{f,0} = 1/p$$
$$z_{f,0} = 0 \,.$$

In addition, since at the first step of the forward recursion the approximation (27) does not hold, we use the folliwing values for the recursive coefficients at time $n = 1$:

$$q^{(\ell)}_{f,1} = \delta_\ell$$
$$z_{f,1} = \frac{y_0}{1 + \sigma^2_\Delta |y_0|}$$

where δ_ℓ represents the Kronecker delta.

Similarly, it is also possible to find the backward recursive equations. Due to the lack of space, we report here only the final expressions

$$s^{(\ell)} = \sum_{\alpha_n} P(\alpha_n) q^{(\ell + r\gamma_n)}_{b,n+1} e^{z'_{b,n+1} + y_n e^{-j\frac{2\pi}{p}(\ell + r\gamma_n)}}$$

$$z_{b,n} = z'_{b,n+1} + y_n \sum_i s^{(i)} e^{z'_{b,n+1} + y_n e^{-j\frac{2\pi}{p}i}} e^{-j\frac{2\pi}{p}i} \quad (33)$$

$$q^{(\ell)}_{b,n} = \frac{s^{(\ell)}}{\sum_m s^{(m)}} \quad (34)$$

where $z'_{b,n+1} = \frac{z_{b,n+1}}{1 + \sigma^2_\Delta |z_{b,n+1}|}$ and coefficients $s^{(\ell)}$ have been introduced to simplify the notation (they do not need to be stored, since they are not involved in the completion stage). The initial values of the backward coefficients are (assuming that N is even)

$$q^{(\ell)}_{b,N} = 1/p$$
$$z_{b,N} = 0$$
$$q^{(\ell)}_{b,N-1} = \begin{cases} P(\alpha_{N-1} = -1) & \ell = 0 \\ P(\alpha_{N-1} = +1) & \ell = p - r \\ 0 & \text{else} \end{cases}$$
$$z_{b,N-1} = y_{N-1} \,.$$

Finally, substituting (22) and (23) into (21) and discarding irrelevant constants, the extrinsic information is evaluated as

$$P_u(\alpha_n) \propto \sum_\ell \sum_m q^{(\ell)}_{f,n} q^{(m)}_{b,n+1} \cdot \mathrm{I}_0\left(\left|z_{f,n} + z'_{b,n+1} e^{j\frac{2\pi}{p}(m - \ell - r\gamma_n)} + y_n e^{j\frac{2\pi}{p}(\ell + r\gamma_n)}\right|\right) \,. \quad (35)$$

In summary, this detection algorithm is based on three steps: a forward recursion in which, for each time epoch n, p real and one complex coefficients are evaluated based on (31) and (32), a backward recursion, based on (32) and (33), which proceeds similarly, and finally a completion (35), which consists of the sum of p^2 terms.[9] This algorithm entails a minor complexity increase with respect to the known-phase MAP *symbol* detector [8] and will be denoted to as "algorithm based on Tikhonov parameterization" (*Tikh-algorithm*).

V. Numerical Results

The performance of the algorithms described in the previous section is assessed by computer simulations in terms of bit error rate (BER) versus E_b/N_0, E_b being the received signal energy per information bit. The proposed algorithms are used as SISO blocks for iterative detection/decoding in serially concatenated CPM schemes

In Fig. 2 we consider the serial concatenation, through an interleaver, of a convolutional code (CC) and the minimum shift keying (MSK) modulation (i.e., a binary modulation with $h = 1/2$ and a rectangular frequency pulse of duration T). The outer code is a 4-state rate-$1/2$ CC with generators $(5, 7)$ (octal notation) and the interleaver has size 2048. A maximum of 10 iterations is allowed and the phase noise affecting the channel is modeled as a Wiener process with $\sigma_\Delta = 5$ degrees. In the figure the performance of the proposed algorithms is shown along with that of the algorithm proposed in [12]

[9] We would like to point out that the coefficients $q^{(m)}_{f,n}$ and $q^{(m)}_{b,n}$ can be evaluated and stored in the log-domain which is a convenient representation for practical hardware realizations.

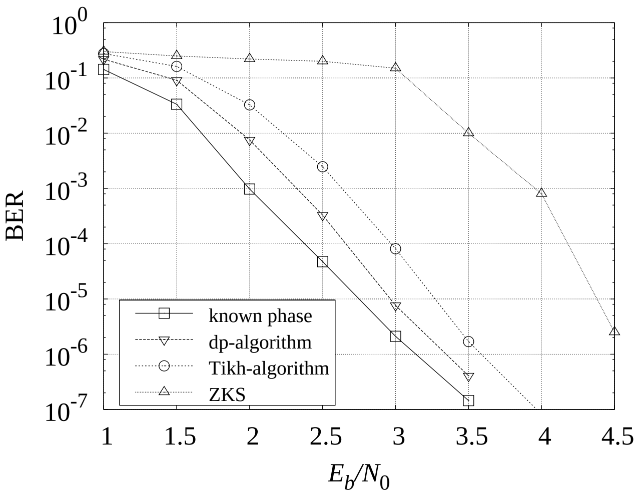

Fig. 2. Performance for the MSK system.

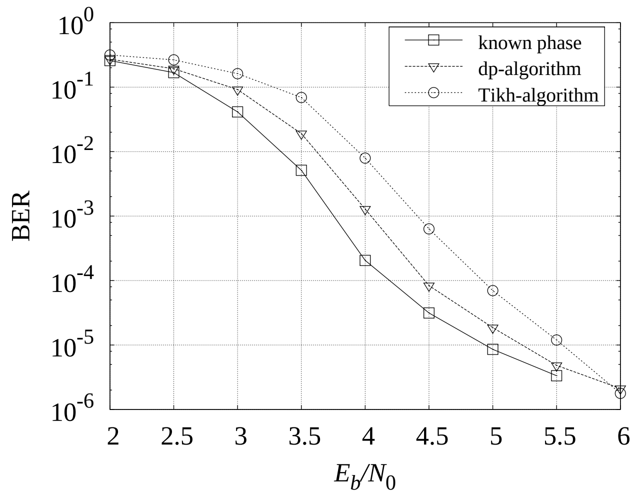

Fig. 3. Performance for the system employing a binary 2-RC modulation with $h = 1/4$.

and based on per-survivor processing (curve labeled ZKS). $D = 16$ discretization levels have been used for the *dp-algorithm* and no performance improvement has been observed for larger values of D. The ideal curve related to the perfect knowledge of the channel phase is also shown for comparison. It can be observed that, at a BER of 10^{-6}, the loss of the optimal *dp-algorithm* with respect to the known-phase case is approximately 0.25 dB, while the loss of the low complexity *Tikh-algorithm* is about 0.5 dB. More than 1 dB is gained with respect to the algorithm in [12], although this was, up to now, the most robust algorithm available in the literature. The increased robustness of our solutions is due to the Bayesian approach which requires the knowledge of parameter σ_Δ. However, this value can be easily estimated and is not critical, in the sense that even if not perfectly estimated, the resulting performance loss is negligible.

In Fig. 3 we consider a system employing the same CC and the same interleaver but a different CPM signal. In particular, we consider a binary CPM modulation with raised-cosine frequency pulse of duration $2T$ (2-RC) and with $h = 1/4$. A different phase noise model is assumed, namely the DVB-S2 compliant phase noise model assuming a baud rate of 10 MBaud [11]. Although the Wiener model does not apply to this case, the proposed algorithms work well with a properly optimized value of $\sigma_\Delta = 0.5$ degrees. $D = 32$ discretization levels have been used for the *dp-algorithm*. Despite the model mismatch, both the proposed algorithms exhibits very good performance (with a loss of the *Tikh-algorithm* of less than 0.5 dB), even if, as well known, for increasing L and decreasing h the resulting CPM modulation has a much higher sensitivity to phase noise.

VI. Conclusions

In this paper, the problem of MAP *symbol* detection for CPM signals transmitted over a channel affected by phase noise has been faced. The algorithm has been derived based on factor graphs and the sum-product algorithm and using the Laurent representation of a CPM signal as sum of linearly modulated components. In particular, only the principal components have been considered, since neglecting the other components only a minor degradation results. A simplified, although exact, version of the algorithm has been derived based on a forward-backward single estimation of the phase probability density function and a final completion. For the practical implementation of the forward-backward estimator, two algorithms have been proposed. The first one is based on the phase discretization and becomes optimal for a large enough number of discretization levels. To reduce the computational complexity, some approximations have been introduced in order to derive a new algorithm which exhibits a very good performance and a very low complexity.

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