Linear predictive receivers for fading channels

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The authors discuss maximum likelihood sequence detection (MLSD) based on prediction techniques for linearly modulated digital signals transmitted over fading channels. Efficient implementations of the sequence detector are investigated and a general formulation for computing the prediction coefficients is derived. Furthermore, the equivalence of different existing prediction-based receivers is shown.

Prediction based receiver: The derivation is carried out assuming a Rayleigh flat slowly fading channel and symbol-spaced sampling. Under these assumptions, the baseband scalar observation at epoch n is given by $r_n = a_n h_n + w_n$ where a_n denotes the information symbol (differentially encoded and belonging to a constellation with M symbols), h_n is the multiplicative fading whose correlation (perfectly known by the receiver) is assumed to obey the Clarke model [1], and w_n denotes a sample of additive white Gaussian noise with power spectral density N_0.

Let N be the length of the transmitted sequence and $\mathbf{r} \triangleq (r_1, r_2, ..., r_N)^T$, $\mathbf{h} \triangleq (h_1, h_2, ..., h_N)^T$, $\mathbf{w} \triangleq (w_1, w_2, ..., w_N)^T$ be the the entire received sequence, fading sequence and noise sequence, respectively. Letting $\mathbf{A} \triangleq \text{diag}\{a_1, a_2, ..., a_N\}$, we may write $\mathbf{r} = \mathbf{Ah} + \mathbf{w}$. The MLSD strategy is

$$\hat{\mathbf{a}} = \arg\max_{\mathbf{a}}\{f_{\mathbf{r}}(\mathbf{r}|\mathbf{a})\} \qquad (1)$$

where $\mathbf{a} \triangleq (a_1, a_2, ..., a_N)$ is the entire transmitted sequence and $f_r(\mathbf{r}|\mathbf{a})$ denotes the probability density function of $\mathbf{r}$ given $\mathbf{a}$. This function may be expressed as

$$f_{\mathbf{r}}(\mathbf{r}|\mathbf{a}) = (\pi^N \det \mathbf{R}_{\mathbf{r}}(\mathbf{a}))^{-1} \exp(-\mathbf{r}^H \mathbf{R}_{\mathbf{r}}(\mathbf{a})^{-1}\mathbf{r}) \qquad (2)$$

in which $\mathbf{R}_r(\mathbf{a}) \triangleq E\{\mathbf{rr}^H|\mathbf{a}\} = \mathbf{AR}_h\mathbf{A}^H + N_0\mathbf{I}$, having denoted the correlation matrix of the process $\mathbf{h}$ by $\mathbf{R}_h \triangleq E\{\mathbf{hh}^H\}$. Following [2], we let $\mathbf{r}_n = \triangleq (r_n, r_{n-1}, ..., r_1)$, $\hat{r}_n(\mathbf{a}) \triangleq E\{r_n|\mathbf{r}_{n-1}, \mathbf{a}\}$, $e_n(\mathbf{a}) \triangleq r_n - \hat{r}_n(\mathbf{a})$, $d_n(\mathbf{a}) \triangleq E\{|e_n(\mathbf{a})|^2|\mathbf{a}\}$, $y_n(\mathbf{a}) \triangleq e_n(\mathbf{a})/\sqrt{[d_n(\mathbf{a})]}$. The sequence $y_n(\mathbf{a})$ is the innovation process and allow us to express eqn. 2 as follows:

$$f_{\mathbf{r}}(\mathbf{r}|\mathbf{a}) = \prod_{n=1}^{N} f_r(r_n|\mathbf{r}_{n-1}, \mathbf{a}) = \pi^{-N}\left(\prod_{n=1}^{N}\frac{1}{d_n(\mathbf{a})}\right)\exp\left\{-\sum_{n=1}^{N} y_n(\mathbf{a})\right\} \qquad (3)$$

Assuming a constant prediction order, denoted by ν, and given the proper set $\mathbf{p}(\mathbf{a}) \triangleq (p_1(\mathbf{a}), p_2(\mathbf{a}), ..., p_\nu(\mathbf{a}))^T$ of prediction coefficients, which depends on the information sequence $\mathbf{a}$, the estimate of r_n is

$$\hat{r}_n(\mathbf{a}) = \sum_{m=1}^{\nu} r_{n-m} p_m(\mathbf{a}) \qquad (4)$$

To determine the vector $\mathbf{p}(\mathbf{a})$ and the mean square error $d_n(\mathbf{a})$, the following set of linear equations must be solved:

$$\mathbf{R}_{n,\nu+1}(\mathbf{a}) \cdot \mathbf{q} = \begin{bmatrix} \mathbf{0}_\nu \\ d_n \end{bmatrix} \qquad (5)$$

where $\mathbf{q} \triangleq (p_\nu, p_{\nu-1}, ..., p_1, 1)^T$, 0_ν, is a column vector of ν zeros and $R_{n,\nu+1}(\mathbf{a})$ is the following minor of matrix $R_r(a)$

$$\mathbf{R}_{n,\nu+1}(\mathbf{a}) \triangleq \begin{bmatrix} R_{n-\nu,n-\nu} & R_{n-\nu,n-\nu+1} & \cdots & R_{n-\nu,n} \\ R_{n-\nu+1,n-\nu} & R_{n-\nu+1,n-\nu+1} & \cdots & R_{n-\nu+1,n} \\ \vdots & \vdots & \ddots & \vdots \\ R_{n,n-\nu} & R_{n,n-\nu+1} & \cdots & R_{n,n} \end{bmatrix} \qquad (6)$$

Defining $\mathbf{a}_n = (a_n, a_{n-1}, ..., a_{n+1-\nu})$, minor $\mathbf{R}_{n,\nu+1}$ actually depends only on the subsequence $\{a_{n-1} \vdots a_n\}$. Hence, the linear system of eqn. 5 yields exactly $M^{\nu+1}$ couples $(\mathbf{p}, d_n)$, defining $M^{\nu+1}$ different branch metrics in eqn. 3. Taking the logarithm of eqn. 3, we obtain the following sequence sequence metric:

$$\Lambda_N(\mathbf{a}) \triangleq \sum_{n=1}^{N}\left\{|y_n(\mathbf{a}_{n-1} \vdots a_n)|^2 + \log[d_n(\mathbf{a}_{n-1} \vdots a_n)]\right\} \qquad (7)$$

which leads to a search which can be performed on a trellis diagram with M^ν states (the state is defined by $\mathbf{a}_{n-1}$). This metric approaches the optimal metric for increasing values of the prediction order ν.

Equivalent detectors: We may give a different expression to eqn. 5 by defining a new set of prediction coefficients $\mathbf{p}' \triangleq [p_1 a_{n-1}/a_n, p_2 a_{n-2}/a_n, ..., p_\nu a_{n-\nu}/a_n]^T$. The linear system of eqn. 5 is equivalent to the following:

$$\begin{cases} (\mathbf{R_h} + N_0\tilde{\mathbf{A}})\mathbf{p}' + \mathbf{c}_h = 0 \\ d_n = |a_n|^2[\mathbf{c}_h^T\mathbf{p}' + R_h(0)] + N_0 \end{cases} \qquad (8)$$

where $\tilde{\mathbf{A}} \triangleq \text{diag}\{1/|a_{n-1}|^2, 1/|a_{n-2}|^2, ..., 1/|a_{n-\nu}|^2\}$, $\mathbf{c}_h \triangleq [R_h(1), R_h(2), ..., R_h(\nu)]^T$ and $R_h(i) \triangleq E\{h_n h_{n+i}^*\}$. If the constellation symbols have constant (and unit) amplitude, eqn. 8 can be further simplified as

$$\begin{cases} (\mathbf{R_h} + N_0\mathbf{I})\mathbf{p}' + \mathbf{c}_h = 0 \\ d_n = \mathbf{c}_h^T\mathbf{p}' + R_h(0) + N_0 \end{cases} \qquad (9)$$

where $\mathbf{I}$ is the identity matrix of proper order. Although for a generic linear modulation the set $\mathbf{p}'$ and d_n depend on a constellation symbol modulus (see eqn. 8), they do not for phase shift keying (PSK). Hence, for PSK we can compute the vector $\mathbf{p}'$ and d_n separately from the sequence detection process making use of eqn. 9.

It can easily be shown that eqn. 8 or eqn. 9 is satisfied by the prediction coefficient of the fading process h_n given the samples of the sequence z_n defined as $z_n \triangleq r_n/a_n = h_n + w_n/a_n$. Therefore, the coefficients $\mathbf{p}'$ allow us to express $\hat{h}_n$ as

$$\hat{h}_n = \sum_{m=1}^{\nu} z_{n-m} p'_m \qquad (10)$$

which highlights the relationship between the two sets of prediction coefficients $\mathbf{p}$ and $\mathbf{p}'$. The vector $\mathbf{p}$ can be recovered as

$$\mathbf{p} = \left[p'_1\frac{a_n}{a_{n-1}}, p'_2\frac{a_n}{a_{n-2}}, \ldots, p'_\nu\frac{a_n}{a_{n-\nu}}\right] \qquad (11)$$

As a general result, the detection strategy based on the innovations approach leads to a prediction problem which can be described in a twofold perspective. The first approach, based on eqn. 5, is followed in [2] and consists of a prediction of the current observation r_n given the past observations $\{r_{n-1}, ..., r_{n-\nu}\}$. The second one is based on the prediction of h_n by means of $\{z_{n-1}, ..., z_{n-\nu}\}$ and is described by eqn. 8 and eqn. 9. This approach is adopted in [3, 4].

Using eqns. 7, 8 and 10 we obtain the following identity

$$\Lambda_N(\mathbf{a}) = \sum_{n=1}^{N}\left\{\frac{\left|r_n + \sum_{i=1}^{\nu} p_i r_{n-i}\right|^2}{d_n} + \log d_n\right\} \qquad (12)$$

$$= \sum_{n=1}^{N}\left\{\frac{\left|r_n + a_n\sum_{i=1}^{\nu} p'_i z_{n-i}\right|^2}{|a_n|^2\epsilon_h + N_0} + \log(|a_n|^2\epsilon_h + N_0)\right\} \qquad (13)$$

where $\epsilon_h \triangleq \mathbf{c}_h{}^T\mathbf{p}' + R_h(0)$ represents the prediction error of the fading process based on the sequence z_n. The metrics in eqn. 12 and eqn. 13 are equivalent: the former envisions the direct prediction of the observation, whereas the latter realises a prediction of the fading process which affects the observation.

For symbols with constant amplitude, the term d_n is independent of the transmitted sequence and the above metrics can be simplified:

$$\Lambda_N(\mathbf{a}) = \sum_{n=1}^{N}\left|r_n + \sum_{i=1}^{\nu} p_i r_{n-i}\right|^2 = \sum_{n=1}^{N}\left|r_n + a_n\sum_{i=1}^{\nu} p'_i z_{n-i}\right|^2 \qquad (14)$$

Numerical results: Further insights into the properties of the prediction-based receivers can be obtained by assessing the receiver performance for different orders of prediction. The fading variations are characterised by a normalised Doppler band $B_D = f_D T = 0.01$, where f_D is the maximum Doppler shift and T is the symbol interval. Computer simulations (not reported here) have been carried out assuming differentially encoded quaternary PSK (QPSK) ($M = 4$), $\nu = 4$ and $\nu = 10$. The performance for the two considered prediction orders is almost coincident, in agreement with [3], where ν = 3, or at most ν = 4, is claimed to be sufficient.

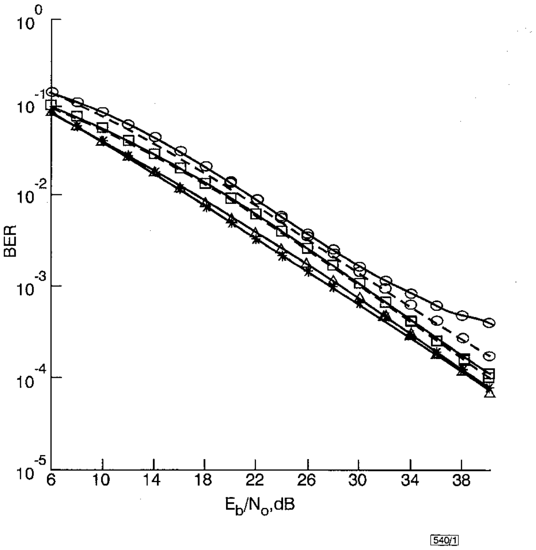

Fig. 1 *Bit error rate comparison between time-discrete observation model and time-continuous model for QPSK and ν = 4*

– – – – time-discrete model
——— time-continuous model
○——○ B_D = 0.100
□——□ B_D = 0.050
△——△ B_D = 0.010
✳——✳ B_D = 0.005
○– – –○ B_D = 0.100
□– – –□ B_D = 0.050
△– – –△ B_D = 0.010
✳– – –✳ B_D = 0.005

Fig. 1 shows, for QPSK and ν = 4, a comparison between the time-discrete observation model adopted in this Letter, which holds for slow fading, and a time continuous observation, which correctly models fast fading channels as well. In fact, in a flat fading channel, a fast time variation causes inter symbol interference (ISI) in the observation. This comparison shows that this ISI has almost no influence on the receiver performance for slowly fading channels (B_D = 0.005, 0.01, 0.05). For fast fading (B_D = 0.1), the difference in performance between the two models is still moderate, even for E_b/N_0 = 40dB.

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Noncoherent SPRT-based acquisition scheme for DSSS

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A noncoherent sequential PN code acquisition scheme is proposed. The out-of-phase and on-phase sequences are properly modelled to avoid significantly high error probabilities occurring with the conventional SPRT-based acquisition. In addition, data modulation and frequency offset can be effectively overcome using this technique.

Introduction: By modelling the acquisition problem as testing between two hypotheses, a sequential test technique under coherent demodulation environments has been proposed and analysed [1]. However, it is, in practice, almost impossible to achieve coherent demodulation, because the signal-to-noise ratio (SNR) before despreading is very low. Several sequential probability ratio tests (SPRTs) designed for PN code acquisition under noncoherent demodulation environments have also been discussed [2 – 4], but under the assumption that the out-of-phase sequence could be modelled as a zero sequence. However, such an assumption may not be very practical, for reasons explained in the following. The modified technique proposed here is then described and simulated.

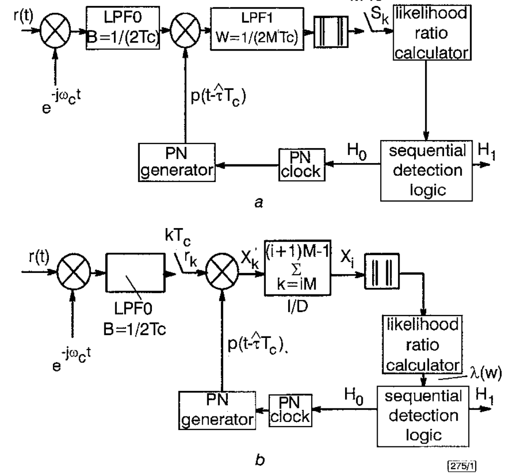

Fig. 1 *Conventional and proposed noncoherent sequential acquisition techniques*

a Conventional
b Proposed

Conventional sequential acquisition techniques: The conventional noncoherent acquisition technique based on SPRT is shown in Fig. 1*a*. The received signal is first down-converted by means of a noncoherent local carrier, $e^{j\omega_c t}$, and then passed through a lowpass filter LPF0. The resulting signal is then cross-correlated with the local code sequence. The cross-correlation signal is passed through a second lowpass filter LPF1. The output of LPF1 is then fed into an envelope detector. The envelope samples S_k are obtained by sampling the output of the envelope detector at a rate low enough (i.e. $1/M'T_c$) that the samples can be considered to be independent. Finally, the envelope samples enter the likelihood ratio calculator and the sequential detection logic.

However, in such a structure, the lowpass filter LPF1 may be very difficult to design. If its bandwidth is too wide, say $W \simeq 1/2T_c$, the cross-correlation of the incoming and the local sequences under the out-of-phase condition cannot be reduced at all. This leads to a high false alarm probability. The bandwidth W of this filter must be narrow enough, e.g. $W \ll 1/2T_c$, to reject residual cross-correlation under the out-of-phase conditions, because the likelihood ratio calculator and sequential detection logic are designed based on the assumption that the out-of-phase sequence can be modelled as a zero sequence here. Conversely, the SPRT algorithm is derived with the assumption that the samples entering the likelihood ratio calculator are sufficiently spaced, e.g. at $M'T_c$ where $M' \gg 1$, and can be considered independent, but with that