Optimal Common Transmit Power in Ad Hoc Wireless Networks

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Abstract—Power conservation is one of the most important issues for ad hoc wireless networks where nodes are likely to rely on limited battery power. Transmitting at unnecessarily high power not only reduces the lifetime of the nodes and the network, but also introduces excessive interference. It is in the network designer’s best interest to have each node transmit at the lowest possible power while preserving network connectivity. In this paper, we investigate the optimal common transmit power, defined as the minimum transmit power used by all nodes necessary to guarantee network connectivity. In particular, we show that for a given route BER and node spatial density, there exists a global optimal data rate at which the transmit power can be globally minimized. Moreover, we also show that there exists a critical node spatial density at which the optimal transmit power is the minimum possible for a given data rate and a given route BER.

I. INTRODUCTION

In an ad hoc wireless network, where nodes are likely to operate on limited battery life, power conservation is an important issue. Conserving power prolongs the lifetime of a node and also the lifetime of the network as a whole. In addition, transmitting at low power reduces the amount of excessive interference. The fundamental question which naturally arises is: what is the optimal transmit power to use? This is the fundamental question that we try to answer in this paper. Obviously, a suitable criterion of optimality must be introduced.

The connectivity level of an ad hoc wireless network depends on the transmit power of the nodes. If the transmit power is too small, the network might be disconnected (i.e., there are multiple disconnected clusters of nodes instead of a single connected network). However, as discussed earlier, transmitting at excessively high power is inefficient because of the mutual interference in the shared radio channel and the limited battery life. Thus, it is intuitively clear that the optimal transmit power is the minimum power sufficient to guarantee network connectivity [1–3].

To achieve the ultimate power saving, the transmit power of a node should be adjusted on a link-by-link basis [2–4]. However, due to the absence of a central controller, performing power control on a link-by-link basis in a large-scale peer-to-peer ad hoc wireless network is a complicated task. A simpler solution, which is more viable for implementation, is to have all the nodes use a common transmit power. In addition, the performance difference, in terms of traffic carrying capacity, between adjusting the power locally and employing a common transmit power is small, especially when the number of nodes is large [1].

In this paper, we evaluate the optimal transmit power for an ad hoc wireless networking scenario where all nodes use a common transmit power. Similar works which consider common transmit power exist [5–7]. Nonetheless, these works follow a graph-theoretic approach which only takes into account the distances between nodes. More specifically, the authors consider that two neighboring nodes can communicate if they are within communication range of each other, and two nodes that are not neighbors can communicate if there is a multi-hop path connecting them. However, we point out that although there may be a path connecting two nodes, a communication between them may not be possible as the QoS in terms of tolerable BER at the end of a multi-hop route may not be satisfied. We discuss this in more detail in Section III. In this paper, as opposed to the conventional graph-theoretic approach, the optimal transmit power sufficient to maintain network connectivity is found according to a physical layer-oriented quality of service (QoS) constraint given by the maximum tolerable bit error rate (BER) at the end of a multi-hop route with an average number of hops.

In [1], the authors focus on common power control as well, i.e., they consider a scenario where all nodes use the same power, but an analytical closed-form expression for the optimal transmit power is not given. Moreover, in this paper, we investigate the interrelation between optimal transmit power, data rate, and node spatial density. In particular, our results show that, for a given node spatial density and a desired BER threshold, there exists a global optimal data rate for which the optimal transmit power is the minimum possible. Similarly, we also show that, for a given data rate and a desired BER threshold, there exists a global optimal node spatial density in correspondence to which the optimal transmit power is the minimum possible.

The rest of this paper is organized as follows. In Section II,
we describe the model and the assumptions which will be used in the derivation of the optimal transmit power. In Section III, we define network connectivity. The minimum transmit power sufficient to maintain network connectivity is analyzed in Section IV. Numerical results, along with their implications, are presented in Section V. Finally, we conclude the paper in Section VI.

II. MODEL AND ASSUMPTIONS

In this section, we describe the basic ad hoc wireless network communication model and the basic assumptions behind the derivation in the remainder of this paper.

A. Network Topology

Throughout the paper, we consider a scenario where \( N \) nodes are distributed over a surface with finite area \( A \). The node spatial density is defined as the number of nodes per unit area and is denoted as \( \rho_s = N/A \). To avoid edge effects, we assume the network surface to be the surface of a torus with length \( 2R \) on each edge, as shown in Fig. 1. However, the analytical technique presented in this paper can be applied to other types of surfaces as well.

In this paper, we consider ad hoc wireless networks with square grid topology. However, the analysis can also be extended to the case with random topology, by following the approach proposed in [8]. In a network with square grid topology, each node has four nearest neighbors at a fixed distance. Although we only consider ad hoc wireless networks with stationary nodes, the extension to a scenario where nodes are mobile can be done following the approach proposed in [9]. Examples of networks with static nodes are sensor networks [10] and wireless mesh networks [11].

B. Routing

In this paper, we assume a simple routing strategy such that a packet is relayed hop-by-hop, through a sequence of nearest neighboring nodes, until it reaches the destination. In addition, we assume that a source node discovers a route prior to data transmission [12]. Discovery of a multi-hop route from a source to a destination is a crucial phase in a wireless networking scenario with flat architecture. The focus of this paper, however, is on the characterization of the steady-state behavior of on-going peer-to-peer multi-hop communications. Therefore, we will assume that a route between source and destination exists.

Due to the regularity of a grid topology, the distance to the nearest neighbor, denoted by \( r_{\text{link}} \), is fixed, and a route is constituted by a sequence of hops with equal length. It can be shown [13, 14] that \( r_{\text{link}} = 1/\sqrt{\rho_s} \).

C. MAC Protocol

In this paper, we consider a simple reservation-based MAC protocol introduced in [13] and defined as reserve-and-go (RESGO). The main idea of this protocol is that, after reserving a multi-hop route to its destination, a node starts transmitting regardless of the activity of the other nodes not belonging to that reserved route.

D. BER at the End of a Multi-Hop Route

Conceptually, a multi-hop route can be viewed as a sequence of links. Assuming that the transmit power on each link attenuates with \( r^\gamma_{\text{link}} \), where \( \gamma \) is the pathloss exponent [16], and that the total additive interference noise can be modeled as white, the signal-to-noise ratio (SNR) at the receiving node of the link can generally be written as

\[
\text{SNR}_{\text{link}} = \frac{\alpha P_t}{r_{\text{link}}^\gamma B (P_{\text{therm}} + P_{\text{int}})}
\]

where \( \alpha = \frac{G_tG_r\sigma^2}{(4\pi f_c)^2} \); \( G_t \) and \( G_r \) are transmitter and receiver antenna gains, \( f_c \) is the carrier frequency, \( P_t \) is the transmit power, \( P_{\text{therm}} \) is the additive white Gaussian thermal noise power, \( P_{\text{int}} \) is the interference power, \( R_b \) is the data rate (dimension: \([\text{bits/s}]\)), and \( B \) is the bandwidth (dimension: \([\text{Hz}]\)). The ratio \( R_b/B \) corresponds to the spectral efficiency of the used modulation format [17].

The thermal noise power \( P_{\text{therm}} \) can be written as \( FkT_0B \), where \( F \) is the noise figure, \( k = 1.38 \times 10^{-23} \text{ J/K} \) is the Boltzmann’s constant, and \( T_0 = 300 \text{ K} \) is the room temperature [16]. In the case of binary phase shift keying (BPSK), which will be considered in the remainder of this paper, it follows that \( B = R_b \).

The interference power \( P_{\text{int}} \) experienced by the receiver in a link depends on (i) the number of nodes concurrently transmitting in the network and (ii) the positions of these interfering nodes relative to the receiver. The number of nodes concurrently active in the network is governed by the MAC protocol and the traffic load at each node. The total interference power \( P_{\text{int}} \) is the sum of the powers from the interfering nodes. In reality, the interference power experienced by the receiving node in each link of a route will vary from link to link. For analytical purposes, we compute the SNR on each

1This MAC protocol was incorrectly referred to in [13] as Aloha MAC protocol, for its resemblance, in terms of route activation independent from the activity of other nodes in the network, with the classical Aloha MAC protocol [15]. However, there are significant differences which make the proposed protocol different from the classical Aloha MAC protocol: (i) multi-hop route reservation and (ii) no use of retransmission techniques.
link using the average interference power $E[P_{\text{int}}]$, instead of the actual interference power. Due to spatial invariance (torus assumption), without any loss of generality, we could compute the amount of interference experienced by a receiving node as if it were at the center of the network. Fig. 2 illustrates a scenario where the receiving node is at the center of the network and the other nodes are grouped in concentric square tiers. Since each node transmits with a common power $P_b$, if all nodes simultaneously transmit, it can be shown that total interference power experienced by the receiver at the center of the network is [13]

$$ I_{\text{grid}} = \alpha P_b I_{\text{grid total}} $$

where

$$ I_{\text{grid total}} = \frac{1}{\gamma_{\text{link}}} \sum_{i=1}^{i_{\text{max}}} \left[ \frac{4}{\gamma} + \frac{4}{(\sqrt{2})^{\gamma}} + \frac{8}{\sum_{j=1}^{i-1} (\sqrt{2} + j)^{\gamma}} \right] $$

and $i_{\text{max}}$ is the maximum tier order. For sufficiently large values of $N$, it can be shown that $i_{\text{max}} \approx \sqrt{N}/2$ [13].

In reality, not all nodes will be transmitting simultaneously. Considering RESGO MAC protocol, and assuming that each node generates packets with fixed length $L$ (dimension: b/pck) according to a Poisson process with parameter $\lambda$ (dimension: [pck/s]), the probability of interference is equal to the probability that a node transmits during a vulnerable interval of duration $L/R_b$ [13]. This probability can be written as

$$ p_{\text{tran}} = 1 - e^{-\frac{L}{R_b}}. $$

The average interference power can then be given as follows

$$ E[P_{\text{int}}] = p_{\text{tran}} E[I_{\text{grid total}}] = (1 - e^{-\frac{L}{R_b}}) I_{\text{grid total}}. $$

Considering BPSK signaling and assuming that the interfering noise is Gaussian, it can be shown that the BER on each link can be written as

$$ \text{BER}_{\text{link}} = Q \left( \frac{2\alpha P_b}{\gamma_{\text{link}} R_b B} \left( P_{\text{therm}} + E[P_{\text{int}}] \right) \right), $$

where $Q(x) \triangleq \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$. Finally, assuming (in a conservative way), that errors in a link are not corrected and not recovered in the subsequent links, the BER at the destination node of an $n$-hop route can be written as

$$ \text{BER}_{\text{Rn}} = 1 - (1 - \text{BER}_{\text{link}})^n. \quad (7) $$

III. CONNECTIVITY

As discussed earlier, the optimal common transmit power is the minimum power sufficient to preserve network connectivity. In this section, we formalize the definition of network connectivity. Conceptually, an ad hoc wireless network is often viewed as a graph, where vertices represent the nodes and edges represent the links connecting neighboring nodes. From a graph-theoretical perspective, a network is strongly connected if there is a path (possibly multi-hop) connecting any node to any other node in the network. However, using this notion of connectivity for an ad hoc wireless network, where a communication channel is error-prone, can be misleading. Since the wireless links are susceptible to errors, the QoS in terms of route BER deteriorates as the number of hops in a route increases. Consequently, the performance may be unacceptable although there is a sequence of links to the destination. To take the physical layer characteristics into account, in this paper, we consider network connectivity from a communication-theoretic viewpoint. In particular, a network is said to be connected if any source node can communicate with a BER lower than a prescribed value $\text{BER}_{\text{th}}$ to a destination node placed at the end of a multi-hop route with an average number of hops [18]. To be conservative, in this paper, we consider an ideal worst-case scenario where an information bit is relayed on each link of a route toward a destination without retransmissions. Had the retransmissions been considered, the BER observed at the end of a route would be better than in the case without retransmissions.

In addition, note that this notion of connectivity corresponds to requiring that, on average, a communication between a source and a destination can be guaranteed with a desired quality. However, it does not guarantee that a source can communicate with every node in the network with this QoS. A more stringent connectivity requirement, such that a source can communicate with every node in the network with the desired QoS, can also be enforced. The approach proposed in this paper can be straightforwardly extended by considering the BER at the end of a multi-hop route with the maximum possible number of hops. We now analyze the average number of hops in the case of grid topology.

With the torus assumption, one can assume that a source node is at the center of the network (see Fig. 2). The average number of hops can be obtained by counting the number of hops on a route from the source to each destination node and finding the average value. Assuming that each destination is equally likely and using the fact that $i_{\text{max}} \approx \sqrt{N}/2$ for sufficiently large $N$, the average number of hops on a route can be written as [19]

$$ \overline{\gamma}_{\text{grid}} \approx \frac{\sqrt{N}}{2} + \frac{1}{\sqrt{N}} + \frac{3}{2} \approx \frac{\sqrt{N}}{2}. \quad (8) $$

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### Table I

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of nodes in the network (N)</td>
<td>1089 nodes</td>
</tr>
<tr>
<td>Area of the network (A)</td>
<td>10^4 m²</td>
</tr>
<tr>
<td>Packet length (L)</td>
<td>10^3 bits</td>
</tr>
<tr>
<td>Packet arrival rate at each node (λ)</td>
<td>1 pack/s</td>
</tr>
<tr>
<td>Pathloss exponent (γ)</td>
<td>3</td>
</tr>
<tr>
<td>Carrier frequency (fc)</td>
<td>2.4 GHz</td>
</tr>
<tr>
<td>Room temperature (Tb)</td>
<td>300 K</td>
</tr>
<tr>
<td>Noise figure (F)</td>
<td>6 dB</td>
</tr>
</tbody>
</table>

### IV. Optimal Common Transmit Power

In this section, we derive the optimal common transmit power for ad hoc wireless networks with grid topology. In a network with grid topology, the common transmit power used by each node should be large enough so that the BER at the end of a multi-hop route with an average number of hops \( n_{grid} \), given by equation (8), is lower than the maximum tolerable value, denoted as \( \text{BER}_{th} \). This implies that the following inequality must be satisfied:

\[
1 - \left[ Q\left(\frac{2αP_t}{\tau_{link}(P_{thrm} + E[\frac{P_{int}^{4}}{\sqrt{2}}])}\right)^{n_{grid}}\right] \leq \text{BER}_{th} \tag{9}
\]

After some straightforward analysis (details omitted here due to space limitation), one obtains the following expression for the optimal transmit power:

\[
P_t^* = \frac{0.5}{\alpha} \left(\frac{\Psi - \left(1 - e^{-\frac{1}{\sqrt{n_{grid}}}}\right)}{\tau_{link}}\right)^{-1} \tag{10}
\]

where

\[
\Psi \triangleq \left[ Q\left(\frac{1}{1 - \left(1 - \text{BER}_{th}\right)^{1/n_{grid}}}\right)\right]^2 \tag{11}
\]

and \( n_{grid} \) is given by (3). The expression given in (10) corresponds to the optimal transmit power for a given data rate \( R_t \), node spatial density \( \tau_{link} \approx 1/\sqrt{n_{grid}} \), number of nodes in the network \( \sqrt{n_{grid}} \), antenna gains and carrier frequency (\( α \) depends on them).

### V. Results and Discussion

Numerical results, along with their implications, are presented and discussed in this section. The values of the major network parameters are given in Table I, unless specified otherwise.

#### A. Global Optimal Transmit Power and Data Rate

In Fig. 3, the optimal common transmit power is shown as a function of the data rate. The optimal power-data rate curves are shown for various values of the threshold BER. It can be observed that the optimal power-data rate curve is concave. Consequently, there exists a global optimal power-data rate pair for a given value of \( \text{BER}_{th} \). Recall that every point in each power-data rate curve shown in Fig. 3 guarantees the corresponding end-to-end BER. Hence, the global optimal operating point, from a power consumption perspective, is the point on that curve with the lowest possible transmit power.

The reason why there exists a global optimal power-data rate pair can be explained as follows. At data rates lower than that of the global optimal point, the duration of a packet transmission is large, since the data rate is low. As a result, the vulnerable time during which a receiving node can experience interference is large, and high transmit power is thus required to sustain the desired BER level. At data rates larger than that of the global optimal point, a packet is transmitted at a high data rate, resulting in a shorter vulnerable interval, and therefore lower interference. However, the thermal noise power increases and the minimum transmit power required to sustain network connectivity has to increase correspondingly.

This suggests that the data rate also plays an important role in the design of ad hoc wireless networks—that is, for a given node spatial density, if the data rate is carefully chosen, the transmit power can be minimized, prolonging network's lifetime.

We are currently working on the extension of this approach to random topologies. Our preliminary results (omitted here due to space limitation) show that a similar behavior can be observed in a scenario with 2-dimensional Poisson topology. However, in the case of random topology, a much higher transmit power is required to sustain network connectivity. This is due to the fact that the hop length is random. In other words, a long hop is likely to be present in a multi-hop route, and this significantly increases the end-to-end BER.

Another interesting issue is to see what happens when different links on a multi-hop route may have different pathloss exponents. Further research is needed to shed light on these more realistic scenarios.

#### B. Critical Node Spatial Density

Fig. 4 illustrates the optimal power as a function of node spatial density in a network with square grid topology where RESGO MAC protocol is used. In particular, two values of \( \text{BER}_{th} \) are considered. In this case, the data rate is fixed to \( R_t = 1 \text{ Mbits} \) and the traffic load per node is \( λ = 15 \text{ pack/s} \).
where network connectivity cannot be achieved, regardless of the transmit power, beyond a critical node spatial density. This phenomenon coincides with the network connectivity behavior predicted by the theory of percolation [20].

REFERENCES


VI. CONCLUSIONS

In this paper, we have investigated the optimal common transmit power for ad hoc wireless networks. In particular, the optimal common transmit power has been defined as the minimum transmit power sufficient to preserve network connectivity. An analytical closed-form expression for the optimal common transmit power is derived. This is particularly useful for network planning as it allows one to determine the minimum power to use while keeping the network connected.

We have observed that for a given data rate and a given maximum tolerable BER at the end of a multi-hop route, there exists an optimal transmit power. In addition, we have also observed the existence of a global optimal data rate for which the optimal common transmit power is the minimum. This suggests that a careful choice of the data rate can significantly help in power saving, prolonging the battery life.

Finally, we have shown that there exists a global optimal node spatial density at which the transmit power can be globally minimized. Moreover, we have observed a behavior