MAC Protocols and Transport Capacity in Ad Hoc Wireless Networks: Aloha versus PR-CSMA

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***Abstract*—Ad hoc wireless networks represent a new communication paradigm and could be an important means of providing ubiquitous communication in the future. Based on a recently developed communication-theoretic framework, in which the interaction between the medium access control (MAC) layer and the physical layer is taken into account, we investigate the performance of *circuit switched* ad hoc wireless networks. Upon the introduction of the concept of *effective transport capacity*, which represents the "actual" rate-distance product carried by the network and the maximum of which is *the* transport capacity of the network, an intuitive and simple approach for the evaluation of this quantity is proposed. In particular, two MAC protocols are considered: Aloha and per-route carrier sense multiple access (PR-CSMA). Numerical results indicate that for low values of the network traffic load the effective transport capacity achievable with Aloha is almost equal to that obtained in the ideal case without inter-node interference (INI). We also show the existence of a threshold value of the traffic load below which Aloha outperforms PR-CSMA, and above which the opposite is true.**

I. Introduction

Ad hoc wireless networks represent a new and exciting communication paradigm which could have multiple applications in future wireless communication systems. Fundamental performance limits of such a communication paradigm need to be studied. The concept of *transport capacity* has been introduced to quantify (taking into account the distance over which the information is transferred) the achievable transmission of information in the network.

In [1], the authors compute, through an information-theoretic approach, the *transport capacity* of stationary wireless networks. From the results in [1], it is immediate to conclude that under a *physical model* of noninterference (error-free transmission between two neighboring nodes is guaranteed if the signal-to-interference-and-noise ratio (SINR) is above a specific threshold), an upper bound on the transport capacity for a stationary wireless network with free-space path loss is $O(R_b\sqrt{AN})$, where R_b is the channel data-rate of a node, A is the network area, and N is the number of nodes in the area.

While the proposed information-theoretic approach is interesting and provides ultimate achievable limits, the influence of *physical layer* characteristics and of the *medium access control* (MAC) protocol on the achievable performance is not clear. A recently developed communication-theoretic framework for multi-hop ad hoc wireless networks [2] clearly shows how physical layer and MAC layer are interrelated. Upon the introduction of the concept of *effective* transport capacity in ad hoc wireless networks, representing the rate-distance product "actually" carried by the network, we propose a simple and intuitive approach for its evaluation in the case of *circuit switched* ad hoc wireless networks. In the case of stationary nodes and no inter-node interference (INI), the results predicted by our theoretical framework are in good agreement with the results obtained in [1] when one considers an arbitrary network under physical model. For the case with INI, two MAC protocols are considered: Aloha and per-route carrier sense multiple access (PR-CSMA), the latter representing an extension of classical CSMA [3] to circuit-switched ad hoc wireless networks. The principle of operation of both MAC protocols is presented, and their performance is evaluated and compared to that of the ideal case. It is is shown that for low traffic load Aloha MAC protocol guarantees an effective transport capacity identical to that of the ideal case, whereas the effective transport capacity sustainable with PR-CSMA is lower. However, *the* transport capacity (i.e., the maximum possible value of the effective transport capacity for given number of nodes and network area) with PR-CSMA is larger than that with Aloha.

The remainder of the paper is structured as follows. In Section II, a model for circuit switched ad hoc wireless networks is presented. In Section III, some useful results of the communication-theoretic framework proposed in [2] are recalled. In Section IV, the concept of *single-route* effective transport capacity is introduced, while in Section V the aggregate (effective) transport capacity is considered. In Section VI, a comparative analysis of the performance of the considered MAC protocols is presented, and Section VIIconcludes the paper.

II. Circuit Switching in Ad Hoc Wireless Networks: Basic Approach and Preliminaries

In a realistic communication scenario, multi-hop communication in ad hoc wireless networks is affected by inter-node

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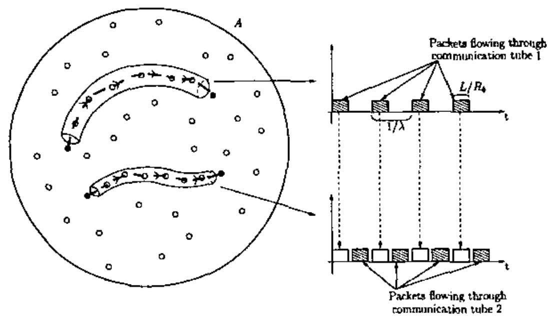

Fig. 1. *Communication tubes* with data packets flowing inside them.

interference (INI). Various approaches can be used to combat this effect, such as the use of *spread spectrum* techniques [4]. In this paper, we do not pursue this possibility further, but limit the analysis to two *random access* schemes: Aloha [5] and PR-CSMA, which will be described in more detail in the following. At this point, we describe the overall network communication scenario. In the proposed circuit-switched multi-hop ad hoc wireless network communication scenario, we do not consider any *retransmission* mechanism, such as *automatic repeat request* (ARQ)[2]. We note that retransmission does not represent an energy-conserving strategy, since a fundamental constraint in ad hoc wireless networks is limited energy available at each node. Since no retranmission is considered, each collision between packets has to be analyzed in terms of interference. To this end, we use a novel *bit-level* interference analysis for random access schemes proposed in [2], [6].

In this paper, we consider a novel network communication scenario based on *circuit* switching with packetized transmission. The basic principle of operation is described in the following. A multi-hop communication route between a source node and a destination node is initially created, through a route discovery mechanism based on *broadcast percolation* [7]. The nodes in the created route are *reserved* for this communication only, so that this route can be visualized as a *communication tube*, which can bend (if the nodes are mobile) without breaking—provided that efficient *local route maintenance* is performed [8]. The creation of a private path between source and destination resembles *circuit switching* [3]. At this point, the source simply "throws" its data packets into the tube, so that they are sent to the destination node. A graphical example, with two communication tubes inside which packets are flowing, is shown in Fig. 1. Observe, from Fig. 1, that in each tube there are "gaps" between consecutive packets. In the rest of the paper, we will assume that the packet transmission is Poisson distributed with parameter λ—this implies that the average inter-arrival rate between two consecutive packets is $1/\lambda$. In other words, instead of considering "pure" circuit switching (where transmission in each tube is continuous), we propose a hybrid scheme, whereby data transmission is packetized. This is a simple way to reduce interference, allowing a random access scheme without retransmission to have acceptable performance. In fact, as shown in the timing diagrams in Fig. 1, if L/R_b (the packet duration) is sufficiently smaller than $1/\lambda$ (the inter-arrival time), the packets transmitted in the two tubes may not overlap, reducing significantly the inter-route interference. To be more precise, this idea applies to the proposed Aloha MAC protocol scheme. The PR-CSMA scheme, on the other hand, completely eliminates the interference by activating almost always only one route (i.e., communication tube) at a time. This is due to the fact that, in the scheme with PR-CSMA, once a route is activated by a specific source node, all the other nodes "sensing" the presence of an on-going transmission will refrain from sending their packets. Note that in any case, the condition $\lambda L \leq R_b$ has to be satisfied in order for the circuit-switched ad hoc wireless network to properly work.

[2]Note that in the following we consider uncoded transmission, but the proposed analysis can be straightforwardly extended to the case of coded transmission. No checksum operation is considered after the possible decoding block, since no packet dropping is assumed.

III. An Analytical Framework

On the basis of the communication-theoretic framework developed in [2], we consider a node distribution characterized by the presence of N nodes placed at the vertices of a square grid inside a circular area A. Denoting by $\rho_S = N/A$ the node spatial density, it is easy to show that the minimum inter-node distance, denoted by r_L, can be written as $r_L \approx 1/\sqrt{\rho_S}$. In the rest of the paper we will assume that a multi-hop communication route is formed by a sequence of minimum length hops—this is the most effective strategy for minimizing the end-to-end BER [2], [6] as well as the transmission power. Indicating by p_L the BER at the end of a single link, assuming that i) there is regeneration (i.e., detection and possibly error correction) at each intermediate node, and that ii) the uncorrected errors made in successive links accumulate, it is possible to show [2] that the BER at the end of the n-th link of a multi-hop route, indicated by $P_b^{(n)}$, can be expressed as

$$P_b^{(n)} \approx 1 - (1 - p_L)^n. \quad (1)$$

An expression for the average BER can be obtained by evaluating (1) for an average number of hops. Assuming that the number of hops is uniformly distributed between 1 and $n_h^{\max} = 2\sqrt{N/\pi}$, the average number of hops becomes $\overline{n}_h = \mathrm{E}\{n_h\} = \lfloor\sqrt{N/\pi}\rfloor$. Hence,

$$\overline{P}_b = P_b^{(\overline{n}_h)} \approx 1 - (1 - p_L)^{\lfloor\sqrt{\frac{N}{\pi}}\rfloor}. \quad (2)$$

Expression (2) shows the dependence of the BER, at the end of an average multi-hop route in an ad hoc wireless network, on the number of nodes N and the link BER p_L. In particular, the link BER p_L depends, among other parameters, on the SNR at the ending node of the link, indicated by SNR_L. We assume that the transmitted signal is simply affected by free-space loss. Hence, according to Friis free space formula [9], the received signal power at the end of a minimum length hop, indicated by $P_r^{(r_L)}$, can be expressed as follows:

$$P_r^{(r_L)} = \frac{\alpha P_t}{r_L^2} \approx \alpha \rho_S P_t = \frac{G_t G_r \lambda_c^2}{(4\pi)^2 f_l} \rho_S P_t \quad (3)$$

where P_t is the transmitted power from each node, G_t and G_r are the transmitter and receiver antenna gains, $\lambda_c = c/f_c$ is the wavelength corresponding to the carrier frequency f_c (c is the speed of light), and $f_l \geq 1$ is a loss factor.

Two distinct cases can be distinguished, based on the absence or presence of INI—the former represents an ideal case, while the latter represents a more realistic case.

- *Ideal case (no INI).* The link SNR can be written as

$$\text{SNR}_L^{noINI} = \frac{P_r^{(r_L)}}{P_{thermal}} \quad (4)$$

  where $P_{thermal}$ is the thermal noise. Denoting the transmission bandwidth by B and recalling the concept of *noise figure* F of a receiver [9], one can write that $P_{thermal} = FkT_0B$, where $k = 1.38 \times 10^{-23}$ J/K is the Boltzmann's constant and T_0 is the room temperature ($T_0 \approx 300$ K).
- *Realistic case (INI).* Since interfering signals come from other nodes, we make the preliminary simplifying assumption that the interfering signals can be treated as additive white noise independent from the thermal noise. Indicating by P_{INT} the interference power (an expression for which will be provided later, depending on the specific MAC protocol and based on a bit-level analysis [2]), the SNR at the end of a minimum link length can be written as

$$\text{SNR}_L^{INI} = \frac{P_r^{(r_L)}}{P_{thermal} + P_{INT}}. \quad (5)$$

We refer to *full connectivity*, in an average sense, when at the end of an average multi-hop communication route the BER is lower than a maximum tolerable value $P_b^{\max}$. Since the link BER p_L is a decreasing function of the link SNR, from (2) it is possible to conclude that, in order for $\overline{P}_b$ to be lower than $P_b^{\max}$, SNR_L has to be larger than a minimum value, indicated by $\text{SNR}_L^{\min}$, which depends on $P_b^{\max}$ and N. In the following, without loss of generality, numerical results will be presented in the case of uncoded binary phase shift keying (BPSK) transmission [10] over an additive white Gaussian noise with free space loss. In this case, the link BER can be written as

$$p_L = Q\left(\sqrt{2\text{SNR}_L}\right) = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{2\text{SNR}_L}}^{\infty} e^{-x^2/2} dx \quad (6)$$

and the minimum link SNR required to guarantee an end-to-end BER $P_b^{\max}$ is the following:

$$\text{SNR}_L^{\min} = \frac{1}{2} \left\{ Q^{-1} \left[1 - (1 - P_b^{\max})^{\sqrt{\pi/N}} \right] \right\}^2 \quad (7)$$

where $Q^{-1}(\cdot)$ represents the inverse function of $Q(\cdot)$.

The *maximum sustainable number* of hops $n_{sh}^{\max}$ corresponding to a final BER $P_b^{\max}$, i.e., the maximum number of hops such that the final BER is lower than $P_b^{\max}$, can be written as [11], [12]

$$n_{sh}^{\max} = \left\lfloor \frac{\ln(1 - P_b^{\max})}{\ln(1 - p_L)} \right\rfloor. \quad (8)$$

Since, on average, a communication route is formed by a sequence of $\lfloor\sqrt{N/\pi}\rfloor$ hops, the *average sustainable number of hops* can be defined as

$$\overline{n}_{sh} \triangleq \min\left\{ n_{sh}^{\max}, \left\lfloor \sqrt{N/\pi} \right\rfloor \right\} = \begin{cases} \left\lfloor \sqrt{N/\pi} \right\rfloor & , \text{ if } \text{SNR}_L \geq \text{SNR}_L^{\min} \\ \left\lfloor \frac{\ln(1-P_b^{\max})}{\ln(1-p_L)} \right\rfloor & , \text{ if } \text{SNR}_L < \text{SNR}_L^{\min} \end{cases}. \quad (9)$$

In other words: (i) if the link SNR is larger than the minimum value $\text{SNR}_L^{\min}$, then the average number of hops is $\lfloor\sqrt{N/\pi}\rfloor$ (note that the maximum sustainable number of hops is larger); (ii) if the link SNR is lower than $\text{SNR}_L^{\min}$, then the number of hops which can be sustained is lower than the average number. This implies that in the latter case full connectivity (in an average sense) is lost.

IV. Single-Route Transport Capacity

Based on the notion of average sustainable number of hops, the average sustainable communication path length $\overline{r}_{PATH}$ that a bit has to travel from a source node to its destination node is

$$\overline{r}_{PATH} = \overline{n}_{sh} r_L \approx \overline{n}_{sh} \sqrt{\frac{A}{N}}. \quad (10)$$

At this point, we introduce the concept of *effective* transport capacity, representing the *actual* rate-distance product which is being sustained by the network. If only a single route at a time were active in the wireless network, the effective transport capacity of the network would be given by the *single-route* effective transport capacity, i.e., by the rate-distance product carried by this single route. In particular, the single-route effective transport capacity can be written as

$$C_{T,e}^{(sr)} = \lambda L \overline{r}_{PATH} \quad (11)$$

where λL represents the average data-rate at which the source node is transmitting, i.e., the traffic load per source node[3]. A fundamental underlying assumption in (11) is that *only* the source node contributes actual information (the λL bits generated, on average, every second and considered in $C_{T,e}^{(sr)}$ come from the source node). In this sense, the intermediate nodes act as *relay nodes*, but they do not contribute to the effective transport capacity in terms of supplementary information bits. If only one route is active in the network, it is possible to assume that there is no INI[4]. The single-route effective transport capacity can then be further written

[3] In the remainder of the paper, the traffic load per source node, i.e., λL, will simply be indicated as traffic load.

[4] Considering the case of a single route active at a time, a communication scenario without INI underlies the assumption that successive links of the same communication route do not interfere with each other.

as follows:

$$C_{T,e}^{(sr)} = \lambda L \,\overline{n}_{sh}^{noINI} \sqrt{\frac{A}{N}}. \quad (12)$$

Since $\overline{n}_{sh}^{noINI}$ does not depend on λL, it is immediate to conclude that the single-route transport capacity, defined as the maximum of the effective single-route transport capacity, can be written as follows:

$$C_T^{(sr)} = \max_{\lambda L, R_b : \lambda L \leq R_b} C_{T,e}^{(sr)} = \max_{R_b} R_b \,\overline{n}_{sh}^{noINI} \sqrt{\frac{A}{N}}. \quad (13)$$

V. Aggregate Transport Capacity

We propose a simple approach for the evaluation of the *aggregate* transport capacity[5] of an ad hoc wireless network. Since the average number of hops per communication route is $\lfloor\sqrt{N/\pi}\rfloor$, there can be at most $N/\lfloor\sqrt{N/\pi}\rfloor \approx \lfloor\sqrt{N\pi}\rfloor$ *disjoint* active communication paths—it is possible to show that a network communication scenario with disjoint communication routes maximizes the effective transport capacity [2]. It might happen that the maximum number of sustainable hops is lower than $\lfloor\sqrt{N/\pi}\rfloor$. In this case, one might argue that the number of disjoint routes could be larger than $\lfloor\sqrt{N\pi}\rfloor$. However, in order to obtain a connectivity-based description of the effective transport capacity in the network, in the following we assume that there can be $N_{ar} \leq \lfloor\sqrt{N\pi}\rfloor$ disjoint routes in the network, so that: (i) if there is full connectivity, then each route is formed by $\lfloor\sqrt{N/\pi}\rfloor$ successive hops (all the nodes in the network are used); (ii) if there is not full connectivity, then each route is formed by an average sustainable number of hops lower than $\lfloor\sqrt{N/\pi}\rfloor$. Based on these considerations, the effective transport capacity, indicated by $C_{T,e}$, can be defined as follows[6]:

$$C_{T,e} \triangleq \lambda L \,\overline{r}_{PATH} N_{ar}. \quad (14)$$

Depending on the particular communication scenario, the average sustainable number of hops, number of active routes, and, consequently, the transport capacity, can be determined. In the following, we consider the ideal case (without INI) and the two cases (with INI) corresponding to Aloha and PR-CSMA MAC protocols.

A. Ideal Case (no INI)

The effective transport capacity is simply obtained by multiplying the single-route effective transport capacity (which assumes no INI) with the number of disjoint routes, given by $\lfloor\sqrt{N\pi}\rfloor$. In particular, the effective transport capacity can be written as a function of the number of nodes (for fixed data-rate) or as a function of the data-rate (for fixed number of nodes), respectively, as follows:

$$C_{T,e}^{noINI} \approx \lambda L \,\overline{n}_{sh}^{noINI} \sqrt{\pi A}$$
$$= \lambda L \min\left\{\sqrt{N/\pi}, \left\lfloor \frac{\ln(1-P_b^{\max})}{\ln(1-p_L)} \right\rfloor\right\} \sqrt{\pi A}. \quad (15)$$

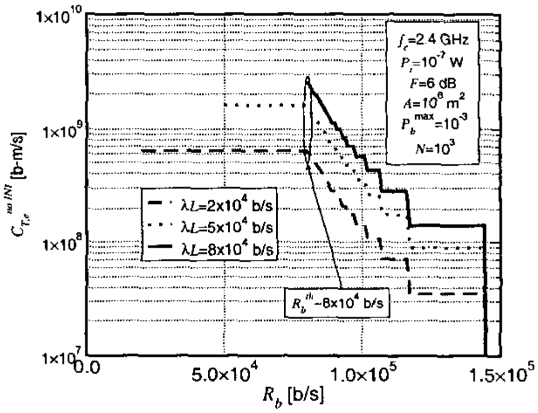

Fig. 2. *Aggregate* transport capacity versus data-rate in the *ideal* (no INI) case.

As in the single-route case, in this case as well, since $\overline{n}_{sh}^{noINI}$ does not depend on λ, the transport capacity is simply obtained by substituting λL by R_b, and then by maximizing with respect to R_b.

The behavior of the effective aggregate transport capacity as a function of the data-rate is shown in Fig. 2, for various values of the traffic load λL—note that for each value of the product λL the valid data-rate range is given by $R_b \geq \lambda L$. The major network parameters are set as indicated in Fig. 2. It is immediate to notice that the effective transport capacity increases, for increasing λL, up to the maximizing value corresponding to $(\lambda L)^{noINI,\max} = R_b^{noINI,\max} = 80$ kb/s. For larger values of the traffic load, the effective transport capacity is lower than the maximum value. It is interesting to observe that for $\lambda L \leq (\lambda L)^{noINI,\max}$ the effective transport capacity is constant (and maximum, with respect to R_b) in the data-rate range $(\lambda L, R_b^{noINI,\max})$. The behavior of the aggregate transport capacity as a function of the number of nodes is shown in Fig. 3. It is immediate to notice the existence of a threshold value N^{th} above which the effective transport capacity is proportional to $\sqrt{N}$ and reaches the upper bound proposed in [1] and below which the effective transport capacity does not reach (due to the loss of connectivity) this information-theoretic upper bound.

Thus, our result further refines the results proposed in [1], by taking into account a prescribed maximum end-to-end BER over an average multi-hop path. In [1], the authors consider a *hard* distinction between the case where no errors are made and the case where no transmission is possible. We propose a different approach. In fact, in the ideal case under examination

[5] In the rest of the paper, the term transport capacity will refer to aggregate transport capacity

[6] In the numerical evaluation of the derived expression for the transport capacity, the floor operation is not considered, in order to ease the comparison with results appeared in the literature (e.g., the expression for the transport capacity derived in [1]).

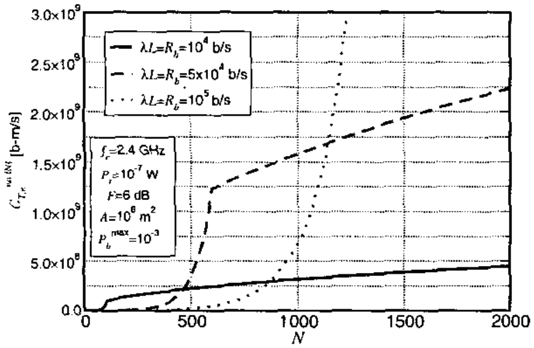

Fig. 3. Effective *aggregate* transport capacity versus number of nodes in the *ideal* (no INI) case, for $\lambda L = R_b$.

we still assume that there is no INI (this is equivalent to considering a SNIR above threshold for any communication link, for example using perfectly orthogonal spreading codes or perfectly functioning directional antennas), but we take into account the *cumulative* error effect due to the multiple hops. In this sense, we consider the degradation, in terms of detection performance, determined by channel impairments.

B. Aloha MAC Protocol

The basic principle of Aloha MAC protocol, originally introduced in [5] for single-hop wireless networks, is the provision that each node, without sensing the channel, transmits whenever it has information to transmit. It is easy to see that this protocol can be potentially used in multi-hop ad hoc wireless networks.

In [2], [12], a novel bit-level interference analysis is proposed, and the BER performance with Aloha MAC protocol is analyzed. In the following, we use part of the results in [2], [12] to evaluate the effective transport capacity with Aloha MAC protocol. In particular, it is possible to show that the interference power appearing in the SNR expression (5) can be written as follows:

$$P_{INT}^{Aloha} \approx \alpha P_t \rho_S \left(1 - e^{-\lambda D_P}\right) \Delta_A(N) \qquad (16)$$

where $D_P = L/R_b$ (L is the number of bits per packet) is the packet duration, and

$$\Delta_A(N) \triangleq \left[\sum_{i=1}^{i_{\max}} \left(\frac{6}{i^2} + 8\sum_{j=1}^{i-1}\frac{1}{i^2+j^2}\right) - 1\right] \qquad (17)$$

where $i_{\max} \approx \lfloor\sqrt{N}/2\rfloor$ is the maximum tier number in a square grid network. It is easy to show that in the case of Aloha MAC protocol, the link SNR is a monotonically increasing function of the transmitted power and node spatial density, and it is always lower than a maximum value $\mathrm{SNR}_L^{Aloha,\max}$, which can be written as:

$$\mathrm{SNR}_L^{Aloha,\max} = \lim_{P_t,\rho_S \to \infty} \mathrm{SNR}_L^{Aloha} = \frac{1}{\left(1 - e^{-\lambda D_P}\right)\Delta_A(N)}. \qquad (18)$$

In particular, for large product λD_P (low data-rate R_b and/or large λL product), $\mathrm{SNR}_L^{Aloha,\max}$ decreases, and it can become lower than the value $\mathrm{SNR}_L^{\min}$ required for full connectivity. The condition previously formulated as $\mathrm{SNR}_L^{Aloha,\max} \geq \mathrm{SNR}_L^{\min}$ can be re-written, for large P_t or ρ_S, as follows:

$$R_b \geq \frac{\lambda L}{\ln\left[\frac{\Delta_A(N)\mathrm{SNR}_L^{\min}}{\Delta_A(N)\mathrm{SNR}_L^{\min}-1}\right]}. \qquad (19)$$

If (19) is not satisfied, then, regardless of the transmitted power and node spatial density, connectivity is lost. For a moderate to large number of nodes N, the quantity $\Delta_A(N)$ is large, and then the minimum data-rate required for full connectivity can be large as well [2]. Finally, the effective transport capacity[7] can be written as

$$C_{T,e}^{Aloha} \approx \lambda L \,\overline{n}_{sh}^{Aloha} \sqrt{\pi A}. \qquad (20)$$

At this point, there is a major difference for the computation of the transport capacity with respect to the ideal (no INI) case. In fact, while in the ideal case $\overline{n}_{sh}^{noINI}$ does not depend on λL, from (16) it is immediate to conclude that $\overline{n}_{sh}^{Aloha}$ depends on the product λL. Hence, the transport capacity, given by

$$C_T^{Aloha} = \max_{\lambda L, R_b : \lambda L \leq R_b} C_{T,e}^{Aloha} \qquad (21)$$

needs to be numerically evaluated.

In Fig. 4, the effective transport capacity is shown, as a function of the data-rate R_b, for various values of the traffic load λL. In particular, for traffic load $\lambda L \leq 130$ b/s the maximum value (with respect to the data-rate) of the effective transport capacity is increasing for increasing traffic load, whereas for traffic load larger than 130 b/s, the maximum value of the effective transport capacity decreases. Unlike the ideal case, in this case there are not well defined values of the traffic load and data-rate maximizing the effective transport capacity. Numerical results show that increasing the transmitted power increases the transport capacity (as the overall maximum of the effective transport capacity). However, the effective transport capacity does not increase without limit by increasing the transmitted power. In Fig. 5, the effective transport capacity is shown, as a function of the traffic load λL, for various values of the transmitted power P_t. In particular, as one can see from Fig. 5, increasing the transmitted power beyond 10^{-4} W does not lead to any increase of the transport capacity. The highest possible value of the transport capacity is achieved for

[7] We emphasize that expression (20) is actually an upper bound on the effective transport capacity with Aloha, in the sense that, based on the packet generation rate, not all possible disjoint routes could be activated (there could be less than $\lfloor\sqrt{N\pi}\rfloor$ source nodes ready to transmit).

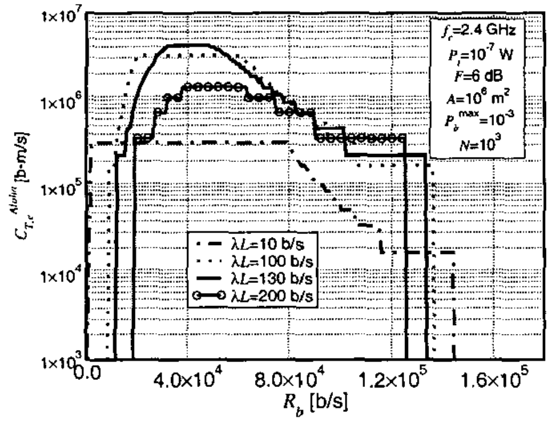

Fig. 4. Effective transport capacity versus data-rate in the case with Aloha MAC protocol. Various values of the traffic load λL are considered.

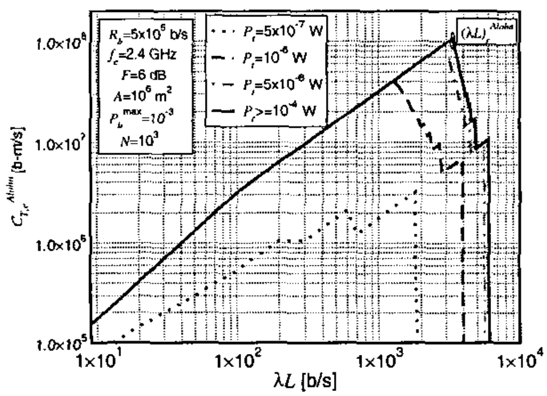

Fig. 5. Effective transport capacity versus traffic load λL for various values of the transmitted power P_t.

a critical value of the traffic load $(\lambda L)_c^{Aloha}$, which, based on (19), can be written as follows:

$$(\lambda L)_c^{Aloha} = R_b \ln \left[\frac{\Delta_A(N)\text{SNR}_L^{\min}}{\Delta_A(N)\text{SNR}_L^{\min} - 1} \right]. \qquad (22)$$

For the particular values of the network parameters considered in Fig. 5, it follows that $(\lambda L)_c^{Aloha} \approx 3.39$ kb/s—note that this maximum sustainable traffic load is much lower than the data-rate $R_b = 500$ kb/s used in the network.

C. PR-CSMA MAC Protocol

The evaluation of the effective transport capacity of an ad hoc wireless network when using the PR-CSMA MAC protocol can be carried out as in the previous subsection. However, a fundamental observation has to be made. In fact, in the case of no INI or in the case of Aloha MAC protocol, we assumed that $\lfloor \sqrt{N\pi} \rfloor$ communication routes can be active at the same time. This assumption is not valid in the case of PR-CSMA MAC protocol. This means that as soon as one route becomes active, the nodes which are not involved in this communication route refrain from transmitting when they sense the on-going transmission[8]. The underlying assumption is that communication routes are created *before* they are actually activated–this could be based, for example, on the use of a specific control channel, separate from the data channel, for the *topological* creation of a communication route. Therefore, once a source node has created a communication route, it actually has to start the transmission. At this point, we assume that a source node senses the channel before transmitting: if no transmission is going on, then it starts transmitting, i.e., it activates the route. In [2], [12], it is shown, through a bit-level interference analysis, that the interfering power in this case can be written as

$$P_{INT}^{PR-CSMA} \approx \alpha P_t \rho_S \Delta_C(N, \lambda, \rho_S) \qquad (23)$$

where

$$\Delta_C(N,\lambda,\rho_S) \triangleq \left\{ \sum_{i=1}^{i_{\max}} \left[\frac{4}{i^2}\left(1 - e^{-\lambda 2 i \tau_m}\right) + \frac{2}{i^2}\left(1 - e^{-\lambda 2\sqrt{2} i \tau_m}\right) + 8 \sum_{s=1}^{i-1} \frac{1}{i^2+s^2} \cdot \left(1 - e^{-\lambda 2\sqrt{i^2+s^2}\tau_m}\right) \right] - \left(1 - e^{-\lambda 2 \tau_m}\right) \right\} \qquad (24)$$

where $\tau_m = r_L/c$ (c is the speed of light) is the propagation time between two nodes at the minimum distance r_L. As in the case with Aloha MAC protocol, in this case as well there exists a maximum attainable SNR ($\text{SNR}_L^{PR-CSMA,\max}$), but it is possible to show [12] that it is usually much larger than $\text{SNR}_L^{Aloha,\max}$—this is intuitively obvious as PR-CSMA MAC protocol significantly reduces the inter-node interference.

For reasonable network dimension and traffic loads [2], there is usually only $N_a^{PR-CSMA} = 1$ active route in the network, i.e., the communication scenario coincides with a single-route network communication scenario. Hence, the effective transport capacity with PR-CSMA MAC protocol can be written as

$$C_{T,e}^{PR-CSMA} \approx \lambda L \, \bar{n}_{sh}^{PR-CSMA} \sqrt{\frac{A}{N}} \qquad (25)$$

where the number of sustainable hops $\bar{n}_{sh}^{PR-CSMA}$ has formally the same expression as (9). The transport capacity can then be written as follows:

$$C_T^{PR-CSMA} = \max_{\lambda L, R_b : \lambda L \leq R_b} C_{T,e}^{PR-CSMA}. \qquad (26)$$

In Fig. 6, the effective transport capacity in the case with PR-CSMA MAC protocol is shown as a function of the data-rate, for various values of the traffic load λL. It is immediate to notice that the obtained curves are a scaled

[8] This could be implemented by assuming that all relay nodes in an active communication route do not stop transmitting, once the source node has taken possession of the medium, until the transmission has ended.

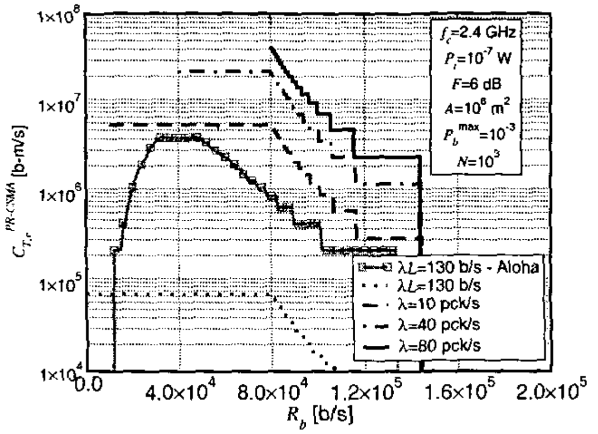

Fig. 6. Effective transport capacity versus data-rate in the case with PR-CSMA MAC protocol. Various values of the traffic load λL are considered.

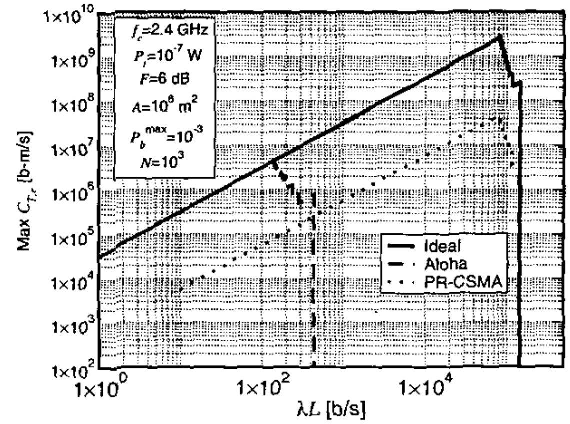

Fig. 7. Maximum effective transport capacity versus traffic load λL: comparison between Aloha, PR-CSMA, and ideal case.

version of those obtained in Fig. 2 for the ideal case—in fact, in this case as well the transport capacity, i.e., the maximum of the effective transport capacity, is obtained for $(\lambda L)^{PR-CSMA,\max} = (R_b)^{PR-CSMA,\max} = 80$ kb/s. For comparison, in Fig. 6 the behavior of the effective transport capacity with Aloha MAC protocol for the maximizing traffic load ($(\lambda L)^{Aloha,\max}$ =130 b/s) is also shown. Comparing this curve with the corresponding curve in the PR-CSMA case, it is immediate to conclude that for sufficiently high data-rate, the maximum value of the effective transport capacity in the Aloha case far exceeds the maximum value in the PR-CSMA case—note, however, that for low data-rate, where the inter-node interference becomes significant [2], the effective transport capacity with PR-CSMA MAC protocol is much larger than that with Aloha MAC protocol. We further compare the two considered MAC protocols in the next section.

VI. Comparison of MAC Protocols: Discussion

After analyzing the performance, in terms of (effective) transport capacity, of each considered MAC protocol, a direct comparison provides further insights regarding the behavior of a circuit switched ad hoc wireless network. In particular, the performance of the MAC protocols is compared based on three useful quantities: i) the maximum effective transport capacity[9] obtainable (suitably setting the transmission data-rate) for each value of the traffic load λL; ii) the minimum data-rate $R_b^{\min}$ necessary to maximize the effective transport capacity at each traffic load; iii) the data-rate range (starting from $R_b^{\min}$) over which the effective transport capacity is maximized.

In Fig. 7, the maximum effective transport capacity as a function of the traffic load is shown. Considering the ideal case without INI, it is immediate to see that the maximum achievable effective transport capacity is a linear function of the traffic load λL up to the maximizing value $(\lambda L)^{noINI,\max} =$ 80 kb/s—this was expected from Fig. 2. For traffic loads larger than $(\lambda L)^{noINI,\max}$, since $R_b \geq \lambda L$, the thermal noise power is so large that the effective transport capacity rapidly drops to zero. Considering Aloha MAC protocol, the maximum effective transport capacity coincides with that obtained in the ideal case for traffic loads lower than the value $(\lambda L)^{Aloha,\max}$ in (22). For larger traffic loads, the transport capacity rapidly goes to zero. From Fig. 7, it is immediate to recognize that the transport capacity with Aloha (i.e., the global maximum of the curve relative to Aloha) is almost three orders of magnitude lower than that in the ideal case. When considering PR-CSMA MAC protocol, for low traffic values (lower than $(\lambda L)^{Aloha,\max}$), the achievable effective transport capacity is almost two orders of magnitude lower than in the ideal and Aloha cases. However, for increasing traffic loads, while the effective transport capacity with Aloha MAC protocol drops to zero, the effective transport capacity with PR-CSMA keeps on increasing. In fact, as expected, the maximum achievable transport capacity with PR-CSMA is a scaled version (the scaling factor corresponds to the number of active routes $\lfloor\sqrt{N\pi}\rfloor$) of the effective transport capacity in the ideal case.

In Fig. 8, the minimum data-rate necessary to maximize the effective transport capacity for a given traffic load is shown as a function of the traffic load. While the minimum data-rate is, as expected, the same in the ideal and PR-CSMA cases, the minimum required data-rate in the Aloha case is significantly larger. This is due to the fact that the data-rate needs to be increased to reduce the INI caused by multiple access with Aloha MAC protocol.

Finally, in Fig. 9, the data-rate range, over which the effective transport capacity is maximized, is shown as a function of the traffic load λL. As for the minimum data-rate required to maximize the effective transport capacity, in this case as well the curves for the ideal and PR-CSMA network communication scenarios are identical, whereas the curve for the Aloha case is significantly lower. In other words, for a given traffic load λL, the data-rate range, over which the effective transport capacity is maximized, reduces

[9]Note that this quantity should not be confused with the transport capacity, which represents the global maximum, also with respect to λL.

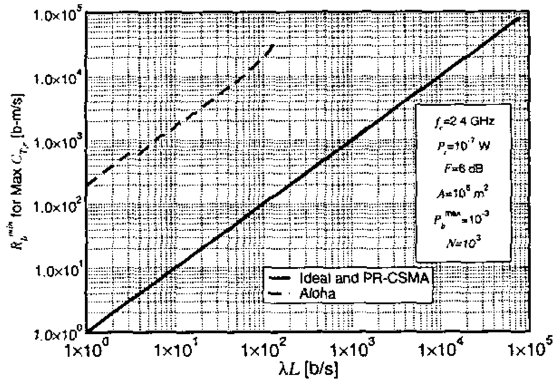

Fig. 8. Minimum data-rate $R_b^{\min}$ to achieve maximum effective transport capacity versus traffic load λL: comparison between Aloha, PR-CSMA, and ideal case.

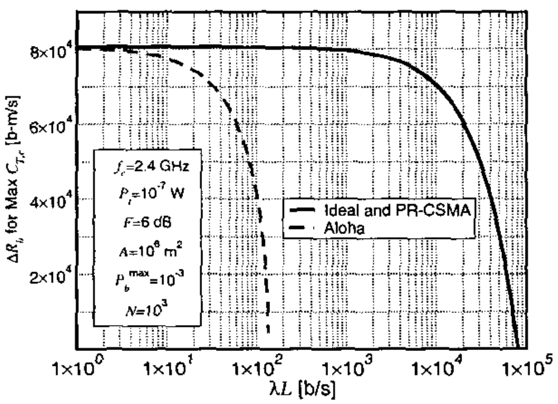

Fig. 9. Data-rate range (starting from $R_b^{\min}$) for maximum effective transport capacity versus traffic load λL: comparison between Aloha, PR-CSMA, and ideal case.

when considering Aloha MAC protocol, implying that in this case the transmission data-rate has to be carefully selected depending on the traffic load in the network.

VII. Concluding Remarks

In this paper, a novel communication-theoretic approach to the analysis of circuit-switched ad hoc wireless networks has been proposed. By introducing the concept of effective transport capacity (whose maximum represents the transport capacity of the network), the performance of ad hoc wireless networks was analyzed. The obtained results provide a new perspective on MAC design in ad hoc wireless networks as well as on results that previously appeared in the literature. More precisely, there exists a threshold N^{th}, in terms of the number of nodes, above which the transport capacity increases proportionally to $\sqrt{N}$ and below which it rapidly drops to zero. The performance obtained in the ideal case was compared to that obtained in a realistic network communication scenario with INI, considering two random access protocols, namely Aloha and PR-CSMA MAC protocols. The main findings can be summarized as follows.

- For very low values of the traffic load λL the effective transport capacity achievable with Aloha MAC protocol is identical with that obtained in the ideal case, and significantly outperforms that with PR-CSMA MAC protocol. However, for increasing traffic loads, the effective transport capacity with PR-CSMA supersedes that of Aloha (which rapidly goes to zero).
- In order to maximize the effective transport capacity, it is shown that the data-rate has to belong to a specific range. While this data-rate range is identical in the ideal and PR-CSMA cases, in the case of Aloha MAC protocol the lower extremum of the data-rate range is larger (in order to reduce the INI) and the range is narrower. Hence, when using Aloha MAC protocol one has to be careful with the choice of the data-rate on the basis of the particular value of the traffic load.

While the approach considered in this paper is limited to stationary wireless networks, it is also of interest to study and quantify the impact of mobility on the performance of ad hoc wireless networks [2], [13], and to consider other routing strategies, such as packet switching [14].

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