An Overstructured Graph for Reduced-State Forward-Backward Algorithms

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Abstract — An overstructured graph (OSG) for a finite state machine (FSM) and a novel message-passing algorithm (MPA) are proposed. This allows a unified graphical approach to the design of reduced-state (RS) forward-backward (FB) algorithms.

I. BRIEF REVIEW: FORWARD-BACKWARD ALGORITHM

The FB algorithm is an algorithm that efficiently produces soft outputs (a posteriori probabilities, APPs) on inputs/outputs of an FSM using the sum-product operation on a priori soft inputs of the same quantities [1, 2]. The algorithm can be described as a message-passing algorithm over a junction tree [3]. Fig. 1 (solid lines only) shows an example of such a junction tree where $a_k$, $x_k$, $v_k$ and $y_k$ represent the input, the output, the state, and the transition at time $k$ respectively. The FB algorithm can be obtained by activating nodes in the forward (left-to-right) and then backward (right-to-left) directions.

II. OVERSTRUCTURED GRAPH AND ITS NOVEL MPA

Let $a_k$, $x_k$, $s_k$ and $t_k$ be the input, the output, the state and the transition of the FSM at epoch $k$ respectively, where $t_k = (s_k, a_k, s_{k+1}, x_k)$ is valid if $s_{k+1} = S_k(s_k, a_k)$ and $x_k = O_k(s_k, a_k)$. The “next-state” $S_k(\cdot)$ and “output” $O_k(\cdot)$ functions are determined by the FSM structure. Let us define the expanded transition $y_k = (s_k, a_k^{L-k}, x_k^{L-k+2})$, where $u_k = (u_{k+1}, u_{k+2}, \ldots, u_L) \ (k \leq k_L \in \{0, x\})$ and $L_2 \geq 0$ is the expansion parameter. Note that the output sequence $x_k^{L-k}$ associated with a valid transition $y_k$ has to be consistent with $s_k$ and $a_k^{L-k}$ relative to $x_k$. To construct the OSG, each node $t_k$ in the junction tree is replaced by a node $y_k$ and extra edges between node $a_k$ ($x_k$) and node $y_k$ are added if $a_k(x_k) \in y_k$. For a simple FSM [2], $s_k = a_{k-1}^{L-k}$ and $y_k = (s_k, a_k^{L-k+2}, x_k^{L-k+2})$. In Fig. 1 (both solid and dashed lines), an example of the OSG for a simple FSM with $L = 2$ and $L_2 = 1$ is shown. The label for the directed message along each edge is the mutual information between the two connected nodes. More precisely, the edge connecting $y_{k-1}$ and $y_k$ is labeled by $v_k = a_k^{L-k+1}$, which represents the expanded state.

For an APP algorithm, the extrinsic soft output can be expressed as $SO[u] = \left(\sum_{t=0}^{L-1} x_t \cdot S_t[x_t] \cdot \frac{S_{L-t}[u]}{S_t[u]}\right)$, where the summation is operated over all valid sequences $x_t \in \Omega$ compatible with $u$, $N$ represents the length of the information sequence, and $S[x_t]$ is the input soft output of the quantity in brackets. Since the graph is loopless, the standard MPA [3] cannot provide the desired soft output. To provide the exact soft output, a novel MPA is proposed for the OSG. Let $A[v_k]$ and $B[v_k]$ be the forward and backward messages (i.e., state metric) along the edge labeled by $v_k$ and $SL_k[u_{L-k}]$ (SO)$k$ be the message from node $u_{L-k}$ ($y_k$) to node $y_k$.

Defining the weight exponents $w^k_1$, $w^k_2$, $w^k_1$, and $w^k_2$, the message-updating formulas are

\[ SL_k[u_{k-L}] = SL_k[u_{k-L}] \prod_{m=L}^{L} (SO_m[u_{k-m}])^{w^k_1(m, l)} \]

\[ A[v_{k+1}] = \sum_{y_k+v_{k+1}} A[v_k]M(a_k^{L-k} x_k^{L-k+2}, y_k, w^k_1) \]

\[ B[v_k] = \sum_{y_k+v_{k+1}} B[v_{k+1}]M(a_k^{L-k} x_k^{L-k+2}, y_k, w^k_1) \]

\[ SO_k[u_k] = \sum_{y_k+v_{k+1}} [A[v_{k+1}] B[v_{k+1}]]M(a_k^{L-k} x_k^{L-k+2}, y_k, w^k_2(x_k)) \]

\[ SO[u_k] = \prod_{m=L}^{L} (SO_m[u_k])^{w^k_2(m, l)} \]

where $M(a_k^{L-k} x_k^{L-k+2}, y_k, w^k_1) \cdot \prod_{m=L}^{L} (SL_m[u_{k-m}])^{w^k_2(m, l)} \cdot \prod_{m=L}^{L} (SO_m[u_{k-m}])^{w^k_1(m, l)} = \prod_{m=L}^{L} (SL_m[u_{k-m}])^{w^k_2(m, l)}$ in (1) and (5), $m \in \{L-1, L-2, \ldots, L\}$

if $u = a_k$ or $m \in \{L-2, L-3, \ldots, 0\}$ if $u = x_k$, and the range of $h$ in (4) is determined by the range of $m$ in (6). It is shown in [4] that the desired soft output (SO) can be obtained by selecting appropriate sets of weight exponents, activation schedules, and initializations.

Through the OSG and the above message-updating formulas, it is shown in [4] that various RS-FB algorithms (both novel and existing RS-FB algorithms) can be realized by applying the forward-backward activating schedule and appropriately choosing the sets of the weight exponents, the expansion parameter $L_2$, and state reduction techniques.

REFERENCES


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