

# On Non-Cooperative Block-Faded Orthogonal Multiple Access Schemes with Correlated Sources

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**Abstract**—In this paper, we study the performance of non-cooperative multiple access systems with noisy separated channels, where *correlated* sources communicate to an access point (AP) through block-faded links. In the considered scenario, perfect channel state information (CSI) is assumed at the receiver while no CSI is available at the transmitters. We first consider uncoded transmissions from the sources to the AP, which exploits the source correlation to carry out joint channel detection (JCD). In this scenario, we propose an analytical approach to evaluate the achievable performance in terms of average bit error rate (BER). We then investigate the impact of coding, considering the same fixed coding scheme at each source. Then, we consider an extrinsic information transfer (EXIT) chart-based framework to optimize the design of concatenated and low-density parity-check (LDPC) codes for JCD schemes.

**Index Terms**—Source correlation, block fading, joint source channel coding (JSCC), joint channel detection (JCD), low-density parity check (LDPC) and concatenated codes, code optimization.

## I. INTRODUCTION

**I**N this paper, we focus on distributed radio communication systems where source nodes transmit directly to a common remote destination. This model applies to many scenarios, such as, for example, cellular networks, wireless local area networks with one access point (AP), wireless sensor networks (WSNs), etc. In many application scenarios the information which resides in different nodes is intrinsically correlated. A significant illustrative example where this situation typically arises is given by WSNs [1]. The design of efficient transmission schemes of correlated signals, observed at different nodes, to one or more collectors is one of the main design challenges in these networks. In the case of a single collector node (as in the remainder of our work), the corresponding system model is often referred to as reach-back channel [2], [3]. In some cases, there is only one source of interest, while the other sources act as helpers by sending correlated information (which is called side-information) to help the reproduction of the first source. This problem is referred to as *m-helper* problem. A power-distortion analysis of the *1-helper* problem over a Gaussian multiple access channel is proposed in [4], where it is shown that, under some circumstances, the uncoded transmission may

lead to a much larger power-distortion region than the separate source and channel coding approach. In the following, all sources will be of interest.

In the case of orthogonal additive white Gaussian noise (AWGN) channels, the separation between source and channel coding is known to be optimal [2], [5]. This means that the ultimate performance limits can be achieved by first compressing each source up to the Slepian-Wolf (SW) limit, by means of distributed source coding (DSC), and then utilizing two independent capacity-achieving channel codes (one per source) [6]. An alternative solution to exploit source correlation is based on the use of joint source channel coding (JSCC) schemes, where the correlated sources are not source encoded but only channel encoded [7].

In both DSC and JSCC cases, no cooperation among the sources is required. The absence of direct cooperation between source nodes is attractive in scenarios where the communication links between them may be noisy. If one compares JSCC and DSC schemes by fixing the information rate at the sources, the channel codes used in JSCC schemes must be less powerful (i.e., they can have higher rates) than those used in DSC schemes. This weakness can be compensated by exploiting the correlation between the sources at the decoder, which jointly recovers the information signals sent by the source nodes, so that the achievable performance can approach the ultimate theoretical limits. For this reason, this approach is also referred to as joint channel detection<sup>1</sup> (JCD) [8]–[11]. In JCD schemes, the sources are encoded independently of each other (i.e., for a given source neither the observation at the other sources nor the correlation model are available at the encoder) and transmitted through the channel. Correlation between the sources is, instead, assumed to be known at the (common) receiver. A rate-distortion analysis for the transmission of correlated sources over a fast fading multiple access channel with partial channel state information (CSI) available at both the transmitters and the receiver, is addressed in [12].

In this paper, we study the performance of non-cooperative multiple access systems with noisy separated channels, where correlated sources communicate to an AP in the presence of block-faded links. Perfect CSI is assumed at the receiver while no CSI is available at the transmitters. We first consider a JCD scenario with uncoded transmissions from the sources to the AP, i.e., a scenario where neither source nor channel coding is performed at the sources and the source correlation

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<sup>1</sup>We remark that the acronym JCD is typically used for joint channel *decoding*. However, since we will also consider this approach in uncoded scenarios, we believe that the acronym joint channel *detection* is more general.

is exploited only at the AP. In this setting, we propose a novel analytical approach to evaluate the achievable performance in terms of average bit error rate (BER) at the AP. We then consider coded scenarios. Due to the absence of CSI at the transmitter, rate adaptation is not feasible and, hence, we limit our analysis to scenarios with *fixed* (source and/or channel) coding schemes at the sources. In the presence of “ideal” coding, it is analytically shown that DSC schemes (clearly suboptimal) are outperformed by JCD schemes. Motivated by this result, we focus on JCD schemes and, in the two source case, we consider an extrinsic information transfer (EXIT) chart-based approach to the design of optimized channel codes to be used at the sources. We consider both concatenated and low-density parity-check (LDPC) codes. The validity of the EXIT chart-based optimization framework is verified by simulation results in terms of probability of outage (PO) and BER.

## II. MULTIPLE ACCESS SCENARIO AND ACHIEVABLE TRANSMISSION RATES

Consider  $n$  spatially distributed nodes which detect (i.e., receive at their inputs) binary information sequences  $\mathbf{x}^{(k)} = [x_0^{(k)}, \dots, x_{L-1}^{(k)}]$ , where  $k = 1, \dots, n$  and  $L$  is the signals' length (the same for all sources). The information signals are assumed to be temporally white with  $P(x_i^{(k)} = 0) = P(x_i^{(k)} = 1) = 0.5$  and the following simple additive correlation model is considered [13]:

$$x_i^{(k)} = b_i \oplus z_i^{(k)} \quad i = 0, \dots, L-1 \quad k = 1, \dots, n$$

where  $\{b_i\}$  are independent and identically distributed (i.i.d.) binary random variables and  $\{z_i^{(k)}\}$  are i.i.d. binary random variables with probability  $\rho$  to be 0 (and  $1 - \rho$  to be 1). Obviously, if  $\rho = 0.5$  there is no correlation between the binary information signals  $\{\mathbf{x}^{(k)}\}_{k=1}^n$ , whereas if  $\rho = 1$  the information signals are identical. According to the chosen correlation model, the a-priori joint probability mass function (PMF) of the information signals at the inputs of the  $n$  sources at the  $i$ -th epoch ( $i \in \{0, \dots, L-1\}$ ) can be computed. After a few manipulations, one can show that [14]

$$\begin{aligned} p(\mathbf{x}_i) &= \sum_{b_i=0,1} p(\mathbf{x}_i|b_i)p(b_i) \\ &= \frac{1}{2} [\rho^{n_b} (1-\rho)^{n-n_b} + (1-\rho)^{n_b} \rho^{n-n_b}] \end{aligned} \quad (1)$$

where  $\mathbf{x}_i = (x_i^{(1)}, \dots, x_i^{(n)})$  and  $n_b = n_b(\mathbf{x}_i)$  ( $i = 0, \dots, L-1$ ) is the number of zeros in  $\mathbf{x}_i$ . Under the considered correlation model, it is straightforward to express the joint entropy  $H(n)$  of the  $n$ -dimensional vector  $\mathbf{x}_i$  emitted by the  $n$  sources at the  $i$ -th epoch as follows:

$$\begin{aligned} H(n) &= -\frac{1}{2} \sum_{n_b=0}^n \binom{n}{n_b} [\rho^{n_b} (1-\rho)^{n-n_b} + (1-\rho)^{n_b} \rho^{n-n_b}] \\ &\quad \cdot \log_2 \left\{ \frac{1}{2} [\rho^{n_b} (1-\rho)^{n-n_b} + (1-\rho)^{n_b} \rho^{n-n_b}] \right\}. \end{aligned} \quad (2)$$

In Fig. 1, the overall model for the multiple access scheme of interest is shown.

In the remainder of this work, we will assume the same transmitting rate  $r = L/N$  at all sources: however, the proposed

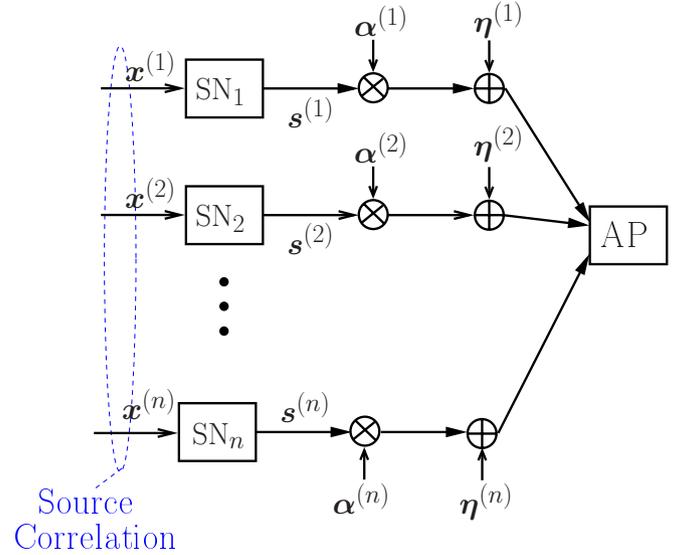


Fig. 1. Proposed multi-access communication scenario:  $n$  source nodes (SNs) communicate directly to the AP.

approach is general and can be applied also to scenarios where the transmitting rate varies from source to source. We also assume a block fading model for the communication links between the sources and the AP: more precisely, the fading coefficient is *constant* for the entire duration of a single packet transmission, i.e.,  $\alpha_i^{(k)} = \alpha^{(k)}$  for  $i = 0, \dots, N-1$ . The fading coefficients are assumed to be independent from link to link and, on a single link, between consecutive packet transmissions.<sup>2</sup>

Since we are considering a block fading model, assuming that the link gains can be perfectly estimated at the AP (e.g., using a short preamble with pilot symbols), after matched filtering and carrier-phase estimation the real observable at the AP, relative to a transmitted sample, can be expressed as

$$r_i^{(k)} = |\alpha^{(k)}| \sqrt{E_c^{(k)}} y_i^{(k)} + \eta_i^{(k)} \quad i = 0, \dots, N-1; k = 1, \dots, n \quad (3)$$

where  $\eta_i^{(k)}$  is an AWGN variable with zero mean and variance  $N_0/2$  and  $y_i^{(k)} \in \{\pm 1\}$  is the binary (modulated) coded bit, with energy  $E_c^{(k)}$ , transmitted by the  $k$ -th node.

In the scenario described above, DSC allows to reduce the amount of data to be transmitted to the AP without needing extra inter-sensor communications. In particular, the performance achievable by a DSC scheme is identical to that which could be achieved if the sources were jointly encoded. The SW theorem thus allows to determine the achievable rate region for the case of separate lossless encoding of correlated sources. Denoting by  $r_{s,k}$  the achievable compression rate for  $k$ -th transmitter, one obtains the following family of inequalities:

$$\sum_{m=1}^p r_{s,k_m} \geq H(x_i^{(k_1)}, x_i^{(k_2)}, \dots, x_i^{(k_p)} | x_i^{(j_1)}, x_i^{(j_2)}, \dots, x_i^{(j_{n-p})}) \quad (4)$$

where  $p \in \{1, \dots, n\}$ ,  $k_i \neq j_f$  ( $i \in \{1, \dots, p\}, f \in \{1, \dots, n-p\}$ ), and  $\{1, \dots, n\} = \{k_1, \dots, k_p\} \cup \{j_1, \dots, j_{n-p}\}$ . In other words,

<sup>2</sup>This fading model applies when the delay requirement is short with respect to the channel coherence time.

$H(x_i^{(k_1)}, x_i^{(k_2)}, \dots, x_i^{(k_p)} | x_i^{(j_1)}, x_i^{(j_2)}, \dots, x_i^{(j_{n-p})})$  is the conditional joint entropy of the group of  $p$  sources indexed by  $k_1, \dots, k_p$ , conditioned on the remaining  $n-p$  sources.

By exploiting the well known relation between joint and conditional entropies [7], one gets:

$$\begin{aligned} & H(x_i^{(k_1)}, x_i^{(k_2)}, \dots, x_i^{(k_p)} | x_i^{(j_1)}, x_i^{(j_2)}, \dots, x_i^{(j_{n-p})}) \\ &= H(x_i^{(1)}, \dots, x_i^{(n)}) - H(x_i^{(j_1)}, x_i^{(j_2)}, \dots, x_i^{(j_{n-p})}). \end{aligned} \quad (5)$$

The considered correlation model between the sources is such that the joint entropy depends only on the number of considered sources, as shown in (2). Therefore, the family of inequalities in (4) can be equivalently rewritten as follows:

$$\sum_{m=1}^p r_{s,k_m} \geq H(n) - H(n-p). \quad (6)$$

with the conventional assumption that  $H(0) = 0$ . By assuming that source coding is followed by channel coding, the actual channel code rates  $\{r_{c,k}\}_{k=1}^n$  may be expressed as

$$r_{c,k} = r_{s,k} \cdot r \quad (7)$$

where  $r$  is the (already introduced) transmission rate  $L/N$ . The channel code rates must satisfy the following Shannon bounds:

$$r_{c,k} \leq \lambda_k \quad k = 1, \dots, n \quad (8)$$

where  $\lambda_k$  is the capacity of the  $k$ -th link, i.e.,

$$\lambda_k \triangleq \frac{1}{2} \log_2(1 + \gamma_k) \quad (9)$$

and  $\gamma_k$  is the SNR, at the AP, relative to the  $k$ -th link, i.e.,

$$\gamma_k = \frac{|\alpha^{(k)}|^2 E_c^{(k)}}{N_0}. \quad (10)$$

As discussed in Section I, compressing each source up to the SW limit and then utilizing independent capacity-achieving channel codes allows to reach the ultimate performance limits. Therefore, combining (6), (9), and (10), the link capacities  $\{\lambda_k\}_{k=1}^n$  have to satisfy the following inequalities:

$$\sum_{m=1}^p \lambda_{k_m} \geq r [H(n) - H(n-p)] \quad (11)$$

for  $p \in \{1, \dots, n\}$  and  $\{k_1, \dots, k_p\} \subseteq \{1, \dots, n\}$ . From (11), using (9) it follows directly that the feasible  $n$ -dimensional SNR region of the considered multiple access scenario is characterized by the link SNRs  $\{\gamma_k\}_{k=1}^n$  such that, for any chosen value of  $p \in \{1, \dots, n\}$ , the following inequalities are satisfied:

$$\sum_{m=1}^p \log_2(1 + \gamma_{k_m}) \geq 2r \cdot [H(n) - H(n-p)] \quad (12)$$

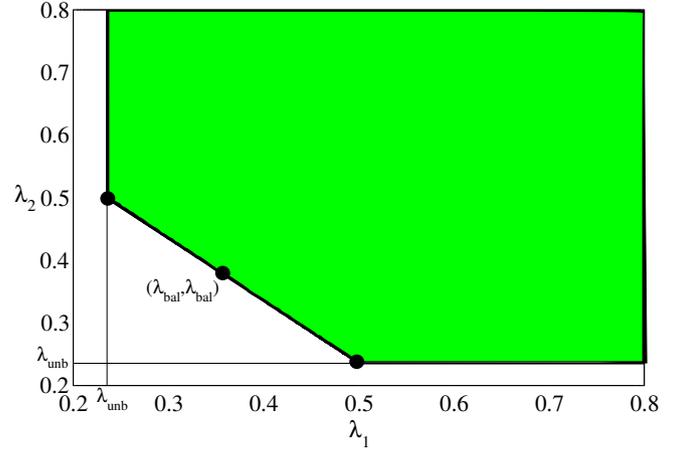
$\forall \{k_1, \dots, k_p\} \subseteq \{1, \dots, n\}$ .

In a scenario with  $n = 2$  sources, inequalities (11) and (12) reduce to the following:

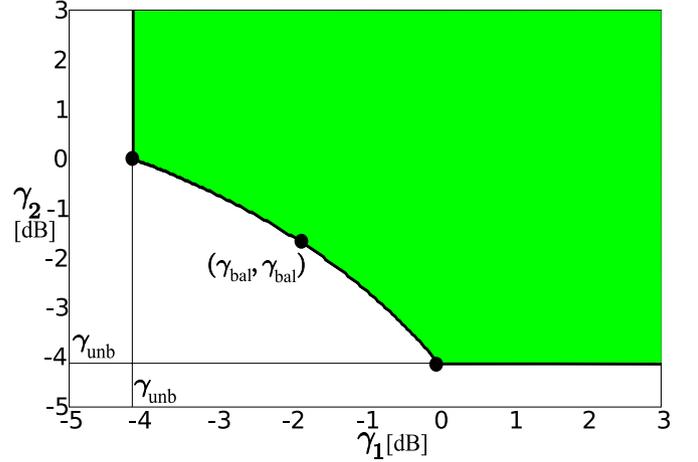
$$\lambda_1, \lambda_2 \geq r[H(2) - 1] \quad \lambda_1 + \lambda_2 \geq rH(2) \quad (13)$$

$$\gamma_1, \gamma_2 \geq 2^{2r[H(2)-1]} \quad (1 + \gamma_1)(1 + \gamma_2) \geq 2^{2rH(2)}. \quad (14)$$

Taking into account that  $H(2)$  is a function of  $\rho$ , in the case with  $\rho = 0.95$  and  $r = 1/2$ , the two-dimensional feasible capacity (identified by (13)) and SNR (identified by (14)) regions



(a)



(b)

Fig. 2. Feasible (a) capacity and (b) SNR regions, in a scenario with  $n = 2$  sources,  $\rho = 0.95$ , and fixed information rate  $r = 1/2$  at each source.

are shown in Fig. 2 (a) and (b), respectively. In Fig. 2 (a),  $\lambda_{\text{unb}}$  and  $\lambda_{\text{bal}}$  correspond to  $r[H(2) - 1]$  and  $rH(2)/2$ , respectively, whereas in Fig. 2 (b),  $\gamma_{\text{unb}}$  and  $\gamma_{\text{bal}}$  correspond to  $2^{2r[H(2)-1]}$  and  $2^{2rH(2)}/2$ , respectively.

### III. UNCODED TRANSMISSIONS

A scheme with neither source nor channel coding at the sources may be of interest in WSN scenarios, where sensor nodes are battery equipped and need to save as much energy as possible. In this case, channel coding/decoding techniques may require an unavailable processing power and thus lead to a waste of energy [1]. Moreover, in the de-facto IEEE 802.15.4 standard for WSNs neither source nor channel coding is considered [15].

In an uncoded scenario, the optimal bit-by-bit detector at the AP exploits the source correlation by means of JCD. Hence, one can concentrate on a single bit and set  $L = 1$ . Denoting by  $\mathbf{x} = (x^{(1)}, x^{(2)}, \dots, x^{(n)})$  and  $\hat{\mathbf{x}} = (\hat{x}^{(1)}, \hat{x}^{(2)}, \dots, \hat{x}^{(n)})$  the transmitted and estimated sequences (of  $L \cdot n = 1 \cdot n = n$  bits), respectively, one can easily derive the pairwise bit error probability in the case of optimal maximum a posteriori

(MAP) detection [16]:

$$P_e(\{\mathbf{x}, \tilde{\mathbf{x}}\}) = \frac{1}{2} \operatorname{erfc} \left\{ \sqrt{\gamma_{\text{eq}}(\mathbf{x}, \tilde{\mathbf{x}})} + \frac{1}{4\sqrt{\gamma_{\text{eq}}(\mathbf{x}, \tilde{\mathbf{x}})}} \ln \left[ \frac{P(\mathbf{x})}{P(\tilde{\mathbf{x}})} \right] \right\} \quad (15)$$

where  $P(\mathbf{x})$  and  $P(\tilde{\mathbf{x}})$  can be derived from (1) and  $\gamma_{\text{eq}}(\mathbf{x}, \tilde{\mathbf{x}}) = \sum_{k=1}^n (x^{(k)} \oplus \tilde{x}^{(k)}) \gamma_k$  is the equivalent energy per bit to spectral noise density ratio. Denoting by  $N_e(\mathbf{x}, \tilde{\mathbf{x}}) = \sum_{k=1}^n (x^{(k)} \oplus \tilde{x}^{(k)})$  the number of bits in error, it is straightforward to derive an upper bound (union bound) on the BER by averaging over all the pairwise error probabilities, i.e.:

$$P_b^{(\text{JCD})} \leq \sum_{\mathbf{x}} \sum_{\tilde{\mathbf{x}}} \frac{N_e(\mathbf{x}, \tilde{\mathbf{x}})}{2n} \operatorname{erfc} \left\{ \sqrt{\gamma_{\text{eq}}(\mathbf{x}, \tilde{\mathbf{x}})} + \frac{1}{4\sqrt{\gamma_{\text{eq}}(\mathbf{x}, \tilde{\mathbf{x}})}} \ln \left[ \frac{P(\mathbf{x})}{P(\tilde{\mathbf{x}})} \right] \right\} P(\mathbf{x}). \quad (16)$$

In the presence of fading, the term  $\gamma_{\text{eq}}$  in (16) is a random variable and a further upper bound on the BER can be derived by averaging the bound (16) over the fading statistical distribution. Assume, for simplicity, that all links are characterized by the same average SNR, i.e.,  $\Gamma \triangleq \mathbb{E}[\gamma_k]$ ,  $\forall k$ . In the considered scenarios with Rayleigh fading, the SNR has an exponential distribution with parameter  $1/\Gamma$  [17]. Owing to the independence between the communication links, one obtains

$$f_{\gamma_{\text{eq}}(\mathbf{x}, \tilde{\mathbf{x}})}(t) = \frac{t^{N_e(\mathbf{x}, \tilde{\mathbf{x}})-1}}{[N_e(\mathbf{x}, \tilde{\mathbf{x}}) - 1]!} \times \frac{\exp(-\frac{t}{\Gamma})}{\Gamma^{N_e(\mathbf{x}, \tilde{\mathbf{x}})}} U(t)$$

where  $U(t)$  is the unit step function. By denoting, for the sake of conciseness,  $N_e = N_e(\mathbf{x}, \tilde{\mathbf{x}})$  and  $\gamma_{\text{eq}} = \gamma_{\text{eq}}(\mathbf{x}, \tilde{\mathbf{x}})$ , the upper bound (16) can be rewritten as

$$P_b^{(\text{JCD})} \leq \sum_{\mathbf{x}} P(\mathbf{x}) \sum_{\tilde{\mathbf{x}}} \int_0^{\infty} \frac{N_e}{2n} \operatorname{erfc} \left\{ \sqrt{t} + \frac{1}{4\sqrt{t}} \ln \left[ \frac{P(\mathbf{x})}{P(\tilde{\mathbf{x}})} \right] \right\} f_{\gamma_{\text{eq}}}(t) dt. \quad (17)$$

We now focus on a scenario with  $n = 2$  sources, providing the reader with more intuition on the obtained results. Denote by  $p \triangleq P\{x^{(1)} = x^{(2)}\}$  the probability that the corresponding (time-wise) bits transmitted by the two sources are identical. From the definition of  $\rho$  given in Section II, it follows that  $p = \rho^2 + (1 - \rho)^2$ . Introducing the log-likelihood ratio  $L_p \triangleq \ln \left( \frac{p}{1-p} \right)$ , it is then straightforward to rearrange (16) as

$$P_b^{(\text{JCD})} \leq \operatorname{erfc} \left\{ \sqrt{\gamma_1 + \gamma_2} \right\} + \sum_{k=1}^2 \frac{p}{4} \times \operatorname{erfc} \left\{ \sqrt{\gamma_k} + \frac{1}{4\sqrt{\gamma_k}} L_p \right\} + \frac{1-p}{4} \times \operatorname{erfc} \left\{ \sqrt{\gamma_k} - \frac{1}{4\sqrt{\gamma_k}} L_p \right\}. \quad (18)$$

Consider now the following inequality:

$$1 \pm \frac{1}{2\gamma} L_p \leq \left( 1 \pm \frac{L_p}{4\gamma} \right)^2. \quad (19)$$

It can be shown that inequality (19) is tight for large values of the SNR  $\gamma$ , in particular for  $\gamma \gg L_p/4$ . We now show that this is verified in normal operative conditions. For example, if  $\rho = 0.99$  then  $p \cong 0.98$  and  $L_p/4 \cong 1$ . Hence, inequality (19) is tight for  $\gamma \gg 0$  dB, as is the case for practical low values of the BER. For lower values of  $\rho$ , it follows that  $L_p/4 < 1$  and, therefore, the condition  $\gamma \gg L_p/4$  is verified even more

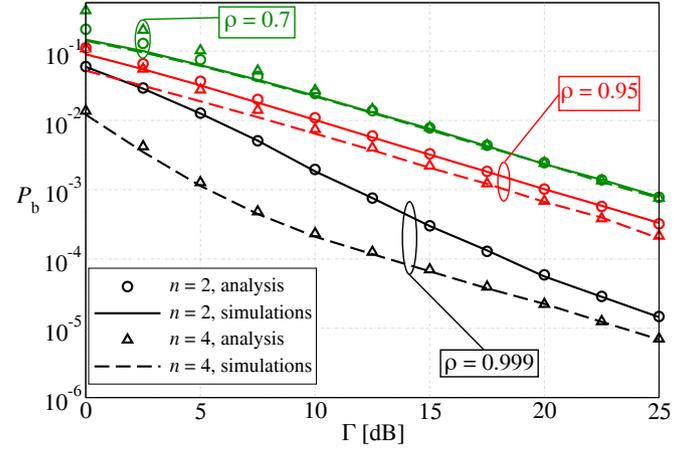


Fig. 3. BER, as a function of the SNR, in the uncoded case. The number of users  $n$  is fixed either to 2 or 4. In both cases, various values of  $\rho$  are considered and analytical results (symbols) are presented together with simulation results (lines).

accurately. It can then be concluded that inequality (19) is very tight in all situations of practical interest. Using (19) into (18), one obtains:

$$P_b^{(\text{JCD})} \leq \operatorname{erfc} \left\{ \sqrt{\gamma_1 + \gamma_2} \right\} + \sum_{k=1}^2 \frac{p}{4} \times \operatorname{erfc} \left\{ \sqrt{\gamma_k + \frac{L_p}{2}} \right\} + \frac{1-p}{4} \times \operatorname{erfc} \left\{ \sqrt{\gamma_k - \frac{L_p}{2}} \right\} \quad (20)$$

Moreover, by considering the Chernoff-Rubin bound for the error function, i.e.,  $\operatorname{erfc}(x) \leq 2e^{-x^2}$ , with simple passages we can write from (20):

$$P_b^{(\text{JCD})} \leq 2e^{-(\gamma_1 + \gamma_2)} + (e^{-\gamma_1} + e^{-\gamma_2}) \cdot \sqrt{p(1-p)}. \quad (21)$$

Consider now the presence of independent Rayleigh faded links. Assuming that the two links are characterized by the same average SNR  $\Gamma$  and denoting by  $\gamma_{b,s} = \gamma_1 + \gamma_2$  the sum of the SNRs, owing to the independence between the communication links one obtains  $f_{\gamma_{b,s}}(t) = (t/\Gamma^2) \exp(-t/\Gamma) U(t)$ . The following upper bound on the BER can be finally derived by averaging over the distribution of the fading terms appearing in (21):

$$P_b^{(\text{JCD})} \leq \frac{2}{(\Gamma+1)^2} + \left( \frac{2}{\Gamma+1} \right) \cdot \sqrt{p(1-p)}. \quad (22)$$

The upper bound (22), although not very tight at low SNRs, allows to clearly separate the effects of the SNR and the correlation between the two sources. As an example, if  $p = 1$ , i.e., the two sources are identical, the system is equivalent to a classical transmission diversity system where the slope of the BER curve is two. On the other hand, for  $p = 0.5$ , if we neglect the term which depends on the squared SNR, the slope of the BER curve is unitary, as in systems with no diversity.

In Fig. 3, the BER is shown as a function of the SNR, for two values of  $n$ : 2 and 4. In both cases, various values of  $\rho$  are considered. Both analytical (using the bound (17) and denoted by symbols) and simulation (denoted by lines) results are shown. It can be observed that the proposed upper bound is very tight. Note that for high values of  $\rho$ , e.g.,  $\rho = 0.999$ ,

the BER curves are characterized, at low-to-medium SNRs, by higher slopes, and this is more pronounced for a large number of sources. This behavior can be interpreted as a “correlation-induced diversity gain,” due to the fact that the message transmitted through a strongly faded link can be partially recovered by other received more reliable messages transmitted through less faded links. In the case with  $n = 2$ , the rationale behind this behavior can be explained as follows. As one can see from (22), the upper bound on the BER is given by two distinct additive terms: the first term is inversely proportional to the square SNR and, therefore, the slope of the BER curve is two, as in systems with diversity; the second term is instead characterized by a unitary slope, as in systems with no diversity. However, for high values of  $p$  (i.e., high values of  $\rho$ ) the second term is very small and becomes significant only for high SNRs. This also justifies the fact that the slopes of all BER curves, regardless of the value of  $\rho$ , tend to approach one for increasing SNRs, as can be seen by the results in Fig. 3. However, when the number of sources increases (e.g., from  $n = 2$  to  $n = 4$ ), the BER also decreases, since it is more likely that at least the message sent by one of the sources is correctly received at the AP and can be used as reliable a priori information in detecting the messages transmitted by the other sources. It can also be observed that for  $n = 4$  the slope of the BER curve is steeper for low SNRs, i.e., the asymptotic slope is reached faster, for increasing SNR, than in the case with  $n = 2$ .

#### IV. CODED TRANSMISSIONS

##### A. An Analytical Framework for Ideal JCD and symmetric DSC Schemes with Fixed Coding Rate

Let us first consider JCD schemes. In this case, since correlation is fully exploited at the receiver, no distributed compression is performed, i.e.,  $r_{s,k} = 1$ ,  $k = 1, \dots, n$  (channel rates are equal to transmission rates). Accordingly,  $r_{c,k} = L/N$ ,  $k = 1, \dots, n$ , and the PO can be computed, considering the inequalities (12), as follows:

$$P_o^{(\text{JCD})} = 1 - P \left\{ \sum_{m=1}^p \log_2(1 + \gamma_{k_m}) \geq 2r \cdot [H(n) - H(n-p)] \right\}, \quad (23)$$

$$\forall \{k_1, \dots, k_p\} \subseteq \{1, \dots, n\}.$$

Consider first the case with  $n = 2$ . Since the link SNR  $\gamma$  has an exponential distribution, the following distribution for the capacity  $\lambda = \log_2(1 + \gamma)/2$  can be obtained:

$$f_\lambda(t) = \frac{\ln(2)}{\Gamma} \cdot 2^{2t+1} \exp\left(-\frac{2^{2t}-1}{\Gamma}\right) U(t).$$

The PO in (23) can then be evaluated as follows:

$$P_o^{(\text{JCD})} = 1 - \int_{r^{H(2)-1}}^r f_\lambda(x) dx \int_{H(2)r-x}^\infty f_\lambda(y) dy - \int_r^\infty f_\lambda(x) dx \int_{r^{H(2)-1}}^\infty f_\lambda(y) dy$$

$$= 1 - \int_{r^{H(2)-1}}^r \exp\left(\frac{-2^{2rH(2)-2x}+1}{\Gamma}\right) f_\lambda(x) dx$$

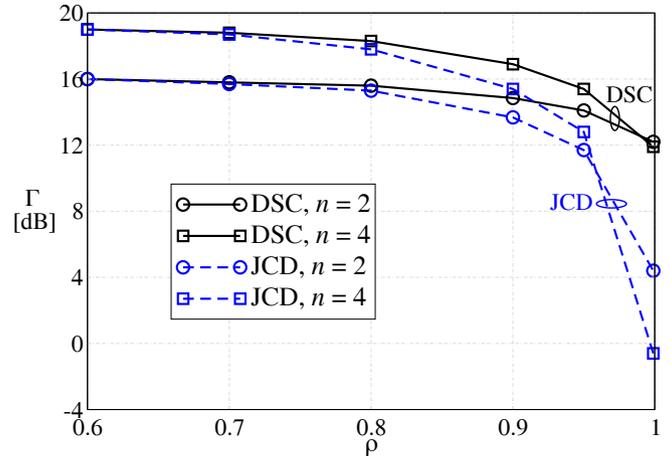


Fig. 4. SNR, as a function of  $\rho$ , required to achieve a PO (at the AP) equal to  $5 \cdot 10^{-2}$ . Scenarios with  $n = 2$  and  $n = 4$  sources are considered. For each value of  $n$ , the performance of the JCD scheme (with channel coding rate  $r = 1/2$ ) is compared with that of the DSC scheme (with compression rate  $H(n)/n$  and channel coding rate  $r/[H(n)/n]$ ).

$$- \exp\left(\frac{-2^{2r}+1}{\Gamma}\right) \exp\left(\frac{-2^{2r(H(2)-1)}+1}{\Gamma}\right) \quad (24)$$

For  $n > 2$ , (23) yields a very cumbersome  $(n-1)$ -dimensional integration. However, one can estimate  $P_o^{(\text{JCD})}$  through the following sampling-based approach. After a large number of link SNRs  $\{\gamma_k\}$  are independently generated according to the channel model introduced in Section II,  $P_o^{(\text{JCD})}$  can then be evaluated as the ratio between (i) the number of times that the SNRs  $\{\gamma_k\}$  do not satisfy the constraints (14) and (ii) the number of generations.

In the DSC case, under the assumption of the same fixed coding strategy at the sources, the transmitters set the source coding rate to  $r_s = H(n)/n$ , which is the minimum source coding rate achievable by symmetric DSC codes [18]. Accordingly, from (9)-(12) it follows that the feasible SNR region is characterized by the following inequalities:

$$\gamma_k \geq 2^{2rH(n)/n} - 1 \quad k = 1, \dots, n. \quad (25)$$

It is straightforward to observe that the feasible SNR region identified by the inequalities (25) is rectangular and smaller than that of the JCD case (identified by the inequalities (12)). Therefore, the following PO of the DSC scheme is higher than that in the JCD case:

$$P_o^{(\text{DSC})} = 1 - \left( \int_{r^{H(n)/n}}^\infty f_\lambda(x) dx \right)^n = 1 - \left[ \exp\left(-\frac{2^{2rH(n)/n}-1}{\Gamma}\right) \right]^n. \quad (26)$$

In Fig. 4, the SNR required to obtain a PO equal to  $5 \cdot 10^{-2}$  is shown, as a function of the correlation coefficient  $\rho$ , considering two values for  $n$ , namely 2 and 4. The information rate  $r$  is set to  $1/2$  in all cases. The performance of JCD schemes (based on (24)) is compared with that of DSC schemes (based on (26)). In all cases, it can be observed that the SNR gain, with respect to a scenario with uncorrelated sources, becomes relevant for values of  $\rho$  higher than 0.8. Moreover, only at large values of  $\rho$ , i.e., when a priori

information becomes very accurate, a scenario with  $n = 4$  sources is to be preferred with respect to a scenario with  $n = 2$  sources. On the other hand, for low-to-medium values of  $\rho$  the PO is lower in the case with  $n = 2$ , as an outage (at least one bit in error in at least one of the detected messages) becomes more likely for higher values of  $n$ —however, the BER reduces, regardless of the value of  $\rho$ , when  $n$  increases, as will be shown in Fig. 7 considering practical coding schemes. It can also be seen that JCD schemes significantly outperform DSC schemes for high values of  $\rho$  ( $\rho \geq 0.95$ ). In fact, in the presence of ideal channel coding (with rate  $r/[H(n)/n]$ ) and in the absence of CSI at the transmitters, the considered DSC approach with compression rate equal to  $H(n)/n$  is suboptimal [19]. Rate adaptation should be considered, but this would require feedback communications from the AP to the sources. In fact, in the presence of channel errors and in the absence of CSI at the sources, fixed DSC schemes are very vulnerable to channel fading and residual redundancy can significantly help, as clearly shown, for the single source case, in [20]–[22]. We also remark that practical DSC schemes (e.g., based syndrome coding) incur a performance loss with respect to the theoretically predicted performance, as clearly shown in [23], and that they become computationally heavy when the number of sources is larger than two. Therefore, in the remainder of this section we focus on JCD schemes and propose an optimization framework for the design of good channel codes.

### B. EXIT chart-based Performance Analysis of JCD Schemes with Two Sources

In a JCD scenario with  $n = 2$  sources, the feasible two-dimensional SNR region associated to a specific channel coding strategy at the sources can be obtained by applying the EXIT chart-based analytical framework proposed in [24]. In particular, it is possible to determine if an SNR pair  $\{\gamma_1, \gamma_2\}$  (or, equivalently, the corresponding rates<sup>3</sup>  $\{\lambda_1, \lambda_2\}$ ) allows to achieve zero BER at the output of the joint channel decoder at the AP. In order to simplify the derivation of the PO of a practical (channel coded) JCD scheme, let us assume that its feasible rate region has the same shape of the feasible capacity region, i.e., that shown in Fig. 2 (a), the only difference being the values of  $\lambda_{\text{unb}}$  and  $\lambda_{\text{bal}}$ . More precisely, we assume that the feasible rate region is still characterized by the inequalities  $\lambda_1, \lambda_2 \geq \lambda_{\text{unb}}$  and  $\lambda_1 + \lambda_2 \geq 2\lambda_{\text{bal}}$ , where, in a practical JCD scheme

$$\lambda_{\text{unb}} \triangleq \min_{\lambda} \{ \lambda : (\lambda, \infty), (\infty, \lambda) \text{ are in the feasible rate region} \} \quad (27)$$

$$\lambda_{\text{bal}} \triangleq \min_{\lambda} \{ \lambda : (\lambda, \lambda) \text{ is in the feasible rate region} \}. \quad (28)$$

Therefore, expression (24) for the PO in the ideal case can be directly extended by considering proper integration extremes:

$$P_o^{(\text{JCD})} = 1 - \int_{\lambda_{\text{unb}}}^{2\lambda_{\text{bal}} - \lambda_{\text{unb}}} f_{\lambda}(x) dx \int_{2\lambda_{\text{bal}} - \lambda_{\text{unb}}}^{\infty} f_{\lambda}(y) dy$$

<sup>3</sup>In Section II,  $\{\lambda_k\}$  are capacities. Here, they are the feasible rates for the considered JCD schemes. The same notation is kept for the sake of notational simplicity.

$$\begin{aligned} & - \int_{2\lambda_{\text{bal}} - \lambda_{\text{unb}}}^{\infty} f_{\lambda}(x) dx \int_{\lambda_{\text{unb}}}^{\infty} f_{\lambda}(y) dy \\ & = 1 - \int_{\lambda_{\text{unb}}}^{2\lambda_{\text{bal}} - \lambda_{\text{unb}}} \exp\left(\frac{-2^{4\lambda_{\text{bal}} - 2x} + 1}{\Gamma}\right) f_{\lambda}(x) dx \\ & - \exp\left(\frac{-2^{4\lambda_{\text{bal}} - 2\lambda_{\text{unb}}} + 1}{\Gamma}\right) \exp\left(\frac{-2^{2\lambda_{\text{unb}}} + 1}{\Gamma}\right). \end{aligned} \quad (29)$$

We remark that the values of  $\lambda_{\text{unb}}$  and  $\lambda_{\text{bal}}$  in (27) and (28) are obtained, as mentioned at the beginning of this subsection, with the approach proposed in [24]. More precisely, a multi-dimensional EXIT chart-based approach is applied to the JCD iterative decoder, by deriving an EXIT chart where the two EXIT curves are associated with the corresponding component decoders (either concatenated or LDPC decoders in the scenarios considered in the remainder of this section). This EXIT chart-based approach allows to numerically determine the values of  $\lambda_{\text{unb}}$  and  $\lambda_{\text{bal}}$ . For each component iterative (either concatenated or LDPC) decoder, the corresponding EXIT curve is obtained by considering density evolution. For further details on this approach, the interested reader is referred to [24].

### C. Case Study: Optimized Rate-1/2 Codes

On the basis of the EXIT chart-based framework introduced in Subsection IV-B, we now derive optimized structures of rate-1/2 concatenated and LDPC codes. More precisely, we propose an optimization procedure which leads to the identification of channel codes with lowest PO, i.e., channel codes with largest feasible rate regions. We remark that the optimization is carried out in the scenario with  $n = 2$  sources and the resulting optimized codes are used also in the scenario with  $n = 4$  sources. Optimized channel codes for a scenario with  $n = 4$  sources should be determined with proper optimization techniques. However, this is a non-trivial optimization problem and goes beyond the scope of this paper.

1) *Concatenated Codes*: We consider two specific classes of rate-1/2 concatenated codes: parallel concatenated convolutional codes (PCCCs) and serially concatenated convolutional codes (SCCCs). In the PCCC case, we consider classical schemes [25], [26] where the constituent codes are given by the same rate-1/2 recursive systematic code (RSC) with the following generator matrix:

$$G_{\text{comp}}(D) = \begin{bmatrix} 1 & \frac{G_{\text{PCCC}}(D)}{H_{\text{PCCC}}(D)} \end{bmatrix} \quad (30)$$

where the degrees of  $G_{\text{PCCC}}(D)$  and  $H_{\text{PCCC}}(D)$  are at most 3, i.e., the RSC code has at most 8 states.

The overall code rate equal to 1/2 is obtained via classical puncturing, by selecting coded bits alternately from the two component encoders. In the SCCC case, we consider various rate-1/2 SCCC configurations, obtained by properly puncturing the outputs of two component rate-1/2 convolutional codes (CCs) with at most 8 states. In the SCCC case, we optimize the generator matrices of outer and inner codes, with the following

general forms:

$$G_{\text{outer}}(D) = \left[ \begin{array}{c} \frac{G_{1-o}(D)}{H_{\text{SCCC-o}}(D)} \\ \frac{G_{2-o}(D)}{H_{\text{SCCC-o}}(D)} \end{array} \right] \quad (31)$$

$$G_{\text{inner}}(D) = \left[ \begin{array}{c} \frac{G_{1-i}(D)}{H_{\text{SCCC-i}}(D)} \\ \frac{G_{2-i}(D)}{H_{\text{SCCC-i}}(D)} \end{array} \right] \quad (32)$$

where the degrees of each of the polynomials  $G_{1-o}(D)$ ,  $G_{2-o}(D)$ ,  $H_{\text{SCCC-o}}(D)$ ,  $G_{1-i}(D)$ ,  $G_{2-i}(D)$ , and  $H_{\text{SCCC-i}}(D)$  is at most 3, i.e., the component inner and outer CCs have at most 8 states. At this point, an overall rate equal to 1/2 is obtained considering the following inner and outer CC rates ( $r_{\text{inner}}$  and  $r_{\text{outer}}$ ), with corresponding puncturing matrices ( $P_{\text{inner}}$  and  $P_{\text{outer}}$ ):

- $r_{\text{inner}} = 2/3$ ,  $r_{\text{outer}} = 3/4$ ,  $P_{\text{inner}} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ ,  $P_{\text{outer}} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$ ;
- $r_{\text{inner}} = 3/4$ ,  $r_{\text{outer}} = 2/3$ ,  $P_{\text{inner}} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$ ,  $P_{\text{outer}} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ ;
- $r_{\text{inner}} = 1$  (the generator matrix has the form  $G_{1-i}(D)/H_{\text{SCCC-o}}(D)$ ),  $r_{\text{outer}} = 1/2$  (no puncturing);
- $r_{\text{inner}} = 1/2$  (no puncturing),  $r_{\text{outer}} = 1$  (the generator matrix has the form  $G_{1-o}(D)/H_{\text{SCCC-o}}(D)$ ).

In the case of PCCCs, the optimized PCCC structure is obtained by carrying out an exhaustive search over all possible generator matrices (30). In the case of SCCCs, an exhaustive search over (31) and (32), for each of the four puncturing patterns, would require a search over almost a million configurations, making it completely unfeasible. By preliminary investigations, our results suggest that the best performance is approached when the feedback polynomials  $H_{\text{SCCC-o}}(D)$  and  $H_{\text{SCCC-i}}(D)$  belong the following set:  $\{1, 1+D, 1+D+D^2, 1+D+D^2+D^3\}$ . Therefore, we have limited our search over all possible configurations where  $H_{\text{SCCC-o}}(D)$  and  $H_{\text{SCCC-i}}(D)$  are constrained to belong to the indicated small set. We have then considered about 1000 configurations of the polynomials  $G_{1-o}(D)$ ,  $G_{2-o}(D)$ ,  $G_{1-i}(D)$ , and  $G_{2-i}(D)$  per coding rate. We believe that our heuristic strategy has led to the selection of good SCCCs: however, we cannot exclude, a priori, that there may be other SCCCs with better performance. In Table I, we show the performance of different concatenated coding structures, considering  $\rho = 0.95$ —similar (relative) results hold also for larger values of  $\rho$ . More precisely: PCCC<sup>(1)</sup> is the PCCC proposed in [11]; PCCC<sup>(opt)</sup> is the PCCC determined with our optimization procedure; SCCC<sup>(1)</sup> has an outer non-recursive CC and is optimized for point-to-point transmission over an AWGN channel [27]; SCCC<sup>(2)</sup> is the SCCC proposed in [8] (note that in this case the outer CC has 32 states, i.e., it is more complex than the one fixed for our optimization); SCCC<sup>(opt)</sup>, SCCC<sub>2</sub><sup>(opt)</sup>, SCCC<sub>3</sub><sup>(opt)</sup>, and SCCC<sub>4</sub><sup>(opt)</sup> are the best SCCCs predicted by our optimization procedure in the cases with the following pairs  $r_{\text{inner}} - r_{\text{outer}}$ , respectively: 2/3 – 3/4, 3/4 – 2/3, 1 – 1/2, and 1/2 – 1.

The obtained results lead to the following conclusions. The optimized PCCC in the current block faded scenario, i.e., PCCC<sup>(opt)</sup>, is the optimal one also in the AWGN case considered in [24]. In the SCCC case, the overall best

code is SCCC<sup>(opt)</sup> ( $r_{\text{inner}} = 2/3$  and  $r_{\text{outer}} = 3/4$ ). Unlike the PCCC case, the optimized SCCC requires the use of an outer *recursive* non systematic convolutional (RNSC) code. This goes against classical results on optimized SCCC for transmission over a single AWGN channel [27]. However, it can be observed that, as in the case of transmission over an AWGN channel, the SCCC with inner code rate equal to 1, i.e., SCCC<sub>3</sub><sup>(opt)</sup>, guarantees a very good performance as well: more precisely, it guarantees the best performance in the balanced SNR case. Overall, the performance, in terms of PO and BER, of SCCC<sup>(opt)</sup> is slightly better than that with SCCC<sub>3</sub><sup>(opt)</sup>. This improvement increases for higher values of  $\rho$  (e.g., 0.999)—the results are not shown here for lack of space—and this is due to the fact that the “stronger” inner CC of SCCC<sup>(opt)</sup> exploits the a priori information more efficiently than the rate-1 inner CC of SCCC<sub>3</sub><sup>(opt)</sup>. We also remark that the optimized structure of SCCC<sup>(opt)</sup> found for the considered faded scenario is also different from that of the SCCC proposed in [8] for a scenario with correlated sources and AWGN links. The accuracy of the proposed EXIT chart-based optimization will be validated, through simulation results, in Subsubsection IV-C3. For the sake of conciseness, only the performance results of SCCC<sup>(opt)</sup> will be shown.

2) *LDPC Codes*: In [28], the EXIT chart-based approach proposed introduced in [24] is applied to LDPC codes. As in the case of concatenated codes, using (29) it is possible to predict the feasible rate region, i.e., the rates  $\lambda_{\text{unbal}}$  and  $\lambda_{\text{bal}}$ , and, therefore, to evaluate the PO. By running an exhaustive search over several LDPC code (variable and check) node degree distributions, denoted as  $\lambda_{\text{LDPC}}(x)$  and  $\rho_{\text{LDPC}}(x)$ , respectively, the performance of several LDPC codes, with  $n = 2$  sources and  $\rho = 0.95$ , is summarized in Table I—similar results also hold for larger values of  $\rho$ . We remark that while in the case of SCCCs an exhaustive search over an overall *finite* set of possible outer CCs has been carried out, in the case of LDPC codes the set of all possible (continuous) degree distributions is *infinite* and, therefore, an exhaustive search is not feasible. We thus limit ourselves to investigate the performance of various regular and irregular LDPC codes. The LDPC codes which guarantee the best performance, evaluated using the same EXIT chart-based approach considered for the concatenated coded case, were already proposed in the literature. Design of optimized LDPC code degree distributions is currently a subject of our research activity, even though our results suggest that “standard” LDPC codes might allow to reach a good performance only in the balanced SNR case. We also remark, however, that optimized degree distributions might not be associated with a feasible LDPC code.

As one can see, the best code is the one denoted as “Irregular 3,” which slightly outperforms the best code proposed in [28], labelled “Irregular 2.” However, the structure of the best codes is typically very complicated, e.g., the maximum degree of variable node distribution is very large. Moreover, the performance improvement with respect to the simpler LDPC codes proposed in [28] is very limited (on the order of 0.1 dB). Therefore, it is possible to conclude that LDPC codes optimized for AWGN channel guarantee a good performance also in Rayleigh faded scenarios.

TABLE I

 MAIN POINTS ON THE PERIMETER OF THE FEASIBLE RATE REGION IN FIG. 2 FOR A FEW POSSIBLE CONCATENATED AND LDPC CODES IN A SCENARIO WITH  $n = 2$ ,  $\rho = 0.95$ , AND  $r = 1/2$  (PUNCTURING IS CONSIDERED IN THE CASE OF CONCATENATED CODES).

CODE NAME		GENERATOR MATRIX OR DEGREE DISTRIBUTIONS	$\lambda_{\text{bal}}$	$\gamma_{\text{bal}}$ [dB]	$\lambda_{\text{unb}}$	$\gamma_{\text{unb}}$ [dB]
CONCATENATED CODES	PCCC <sup>(1)</sup> ([11])	$G_{\text{comp}}^{(1)}(D) = \begin{bmatrix} 1 & \frac{1+D+D^2+D^3}{1+D^2+D^3} \end{bmatrix}$	0.41	-1.06	0.39	-1.37
	PCCC <sup>(opt)</sup>	$G_{\text{comp}}^{(\text{opt})}(D) = \begin{bmatrix} 1 & \frac{1+D^2+D^3}{1+D+D^3} \end{bmatrix}$	0.418	-1.05	0.393	-1.4
	SCCC <sup>(1)</sup> ([27])	$G_{\text{outer}}^{(1)}(D) = \begin{bmatrix} 1+D^2+D^3 & 1+D+D^2+D^3 \end{bmatrix}$	0.44	-0.69	0.34	-2.25
	SCCC <sup>(2)</sup> ([8])	$G_{\text{inner}}^{(2)}(D) = \begin{bmatrix} 1 & \frac{1+D^2}{1+D+D^2} \end{bmatrix}; G_{\text{outer}}^{(2)}(D) = \begin{bmatrix} 1 & \frac{1+D+D^2+D^3}{1+D+D^2} \end{bmatrix}$	0.48	-0.24	0.34	-2.25
	SCCC <sup>(opt)</sup>	$G_{\text{inner}}^{(2)}(D) = \begin{bmatrix} \frac{1+D^2+D^3}{1+D+D^2+D^3} & \frac{1}{1+D+D^2+D^3} \end{bmatrix}; G_{\text{outer}}^{(\text{opt})}(D) = \begin{bmatrix} 1+D^2+D^3 & 1+D^2 \end{bmatrix}$ $r_{\text{inner}} = 2/3; r_{\text{outer}} = 3/4$	0.4646	-0.43	0.2607	-3.61
	SCCC <sub>2</sub> <sup>(opt)</sup>	$G_{\text{inner}}^{(2)}(D) = \begin{bmatrix} 1 & \frac{1+D^2}{1+D+D^2} \end{bmatrix}; G_{\text{outer}}^{(\text{opt})}(D) = \begin{bmatrix} \frac{1+D^2+D^3}{1+D+D^2+D^3} & \frac{1+D^2}{1+D+D^2+D^3} \end{bmatrix}$ $r_{\text{inner}} = 3/4; r_{\text{outer}} = 2/3$	0.4846	-0.18	0.2801	-3.23
	SCCC <sub>3</sub> <sup>(opt)</sup>	$G_{\text{inner}}(D) = \begin{bmatrix} \frac{1+D^2}{1+D+D^2+D^3} \end{bmatrix}; G_{\text{outer}}^{(\text{opt})}(D) = \begin{bmatrix} \frac{1+D^2}{1+D+D^2} & \frac{1}{1+D+D^2} \end{bmatrix}$ $r_{\text{inner}} = 1; r_{\text{outer}} = 1/2$	0.4413	-0.73	0.2638	-3.55
	SCCC <sub>4</sub> <sup>(opt)</sup>	$G_{\text{inner}}^{(2)}(D) = \begin{bmatrix} 1 & \frac{1+D^2+D^3}{1+D+D^2+D^3} \end{bmatrix}; G_{\text{outer}}^{(\text{opt})}(D) = \begin{bmatrix} \frac{1+D^2}{1+D+D^2} \end{bmatrix}$ $r_{\text{inner}} = 1/2; r_{\text{outer}} = 1$	0.9665	4.5	0.2638	-3.55
LDPC CODES	Regular	$\rho_{\text{LDPC}}(x) = x^5$ $\lambda_{\text{LDPC}}(x) = x^2$	0.482	-0.22	0.4473	-0.66
	Irregular DD ([10])	$\rho_{\text{LDPC}}(x) = x^5$ $\lambda_{\text{LDPC}}(x) = 0.333x + 0.667x^3$	0.4503	-0.62	0.4144	-1.1
	Irregular 1 ([28, Table I])	$\rho_{\text{LDPC}}(x) = x^5$ $\lambda_{\text{LDPC}}(x) = 0.355844x + 0.288313x^2 + 0.355844x^5$	0.4262	-0.94	0.4	-1.3
	Irregular 2 ([28, Table I])	$\rho_{\text{LDPC}}(x) = 0.69x^3 + 0.31x^6$ $\lambda_{\text{LDPC}}(x) = 0.338002x + 0.12878x^2 + 0.533215x^5$	0.4291	-0.9	0.382	-1.56
	Irregular 3 ([29, Table II])	$\rho_{\text{LDPC}}(x) = 0.00749x^7 + 0.99101x^8 + 0.00150x^9$ $\lambda_{\text{LDPC}}(x) = 0.19606x + 0.24039x^2 + 0.00228x^5 + 0.05516x^6$ $+ 0.16602x^7 + 0.04088x^8 + 0.01064x^9 + 0.00221x^{27} + 0.28636x^{29}$	0.4144	-1.1	0.3567	-1.94
	Irregular 4 ([30, p. 133])	$\rho_{\text{LDPC}}(x) = 0.7x^6 + 0.3x^7$ $\lambda_{\text{LDPC}}(x) = 0.272536x + 0.237552x^2 + 0.070380x^3 + 0.419532x^9$	0.4173	-1.06	0.3699	-1.74

3) *Performance Analysis*: In order to further investigate, through simulations, the performance of JCD schemes, we consider realistic scenarios where some of the codes introduced above are used. In call cases, each source node transmits packets of length  $L = 1000$ . In both cases, the correlation is exploited at the AP by using a proper iterative algorithm between the two component decoders. In the following, we concisely recall the basics of this algorithm. However, more details can be found in [10], [11], [24], [28].

The total log-likelihood ratio (LLR) relative to the  $i$ -th observable at the input of the  $k$ -th subdecoder can be expressed as follows:

$$\mathcal{L}_{i,\text{in}}^{(k)} = \begin{cases} \mathcal{L}_{i,\text{ch}}^{(k)} + \mathcal{L}_{i,\text{ap}}^{(k)} & i = 0, \dots, L-1 \\ \mathcal{L}_{i,\text{ch}}^{(k)} & i = L, \dots, N-1. \end{cases}$$

In other words, the LLR of the  $i$ -th observable associated with an information bit ( $i = 0, \dots, L-1$ ) includes, besides the channel reliability value given by  $\mathcal{L}_{i,\text{ch}}^{(k)}$ , the ‘‘suggestion’’ (represented by the soft reliability value  $\mathcal{L}_{i,\text{ap}}^{(k)}$ ) obtained from a posteriori reliability values output by the other decoders. In particular, the a priori component of the LLR at the input of

the  $k$ -th decoder can be written as

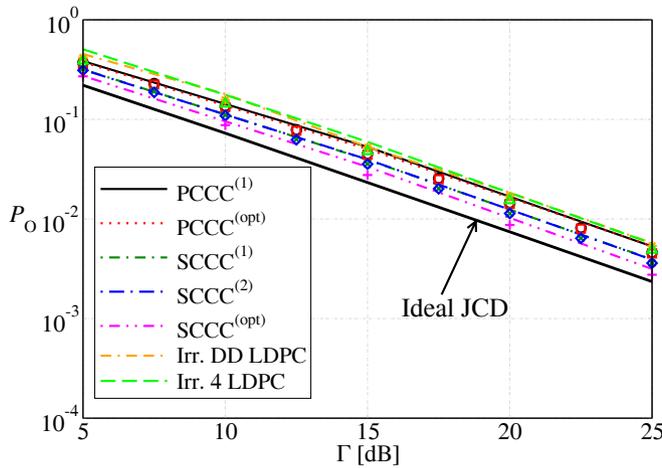
$$\mathcal{L}_{i,\text{ap}}^{(k)} = \ln \frac{P(y_i^{(k)} = 1)}{P(y_i^{(k)} = -1)} \quad i = 0, \dots, L-1$$

where  $\{P(y_i^{(k)} = +1), P(y_i^{(k)} = -1)\}$  are derived from the soft-output values generated by the other decoders. In [31], it is shown that

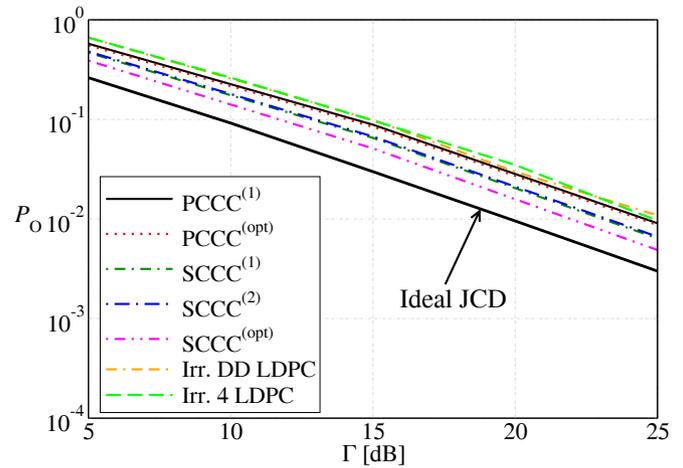
$$P(y_i^{(k)}) \simeq \frac{2}{n-1} \sum_{\ell \neq k} \sum_{y_i^{(\ell)} = \pm 1} \underbrace{\hat{P}(y_i^{(\ell)})}_{[\text{from decoder } \ell]} \cdot \underbrace{P(y_i^{(\ell)}, y_i^{(k)})}_{[\text{a priori source correl.}]} \quad (33)$$

In Fig. 5, we compare the PO of various JCD schemes. The number of sources is  $n = 2$  and the correlation coefficient  $\rho$  is fixed to 0.95 in (a) and 0.999 in (b). Analytical (based on the computation of (29) and indicated as symbols) and simulation (indicated as lines) results are presented. For the sake of comparison, the PO in the presence of ideal JCD (given by (24)) is also shown. As one can see, for  $\rho = 0.95$  the PO of the considered schemes is relatively similar and close to the ideal one. For  $\rho = 0.999$ , the performance gaps between different coding schemes widens, as well as the gap from the ideal JCD performance.

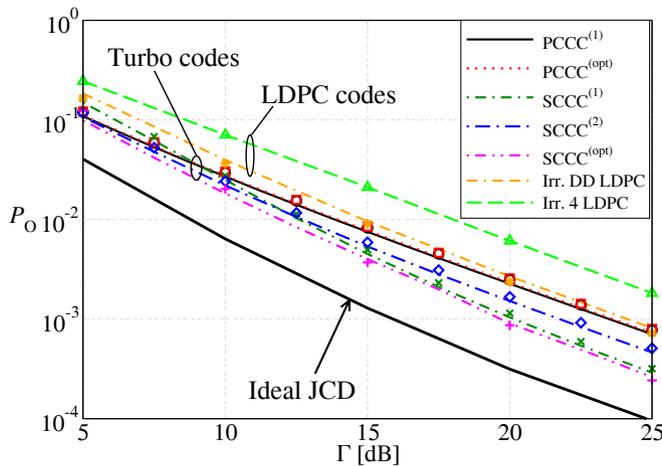
In Fig. 6, the same scenario of Fig. 5 is considered, the only difference being the number of sources, which is now set to 4. In this case, the performance of all JCD schemes is



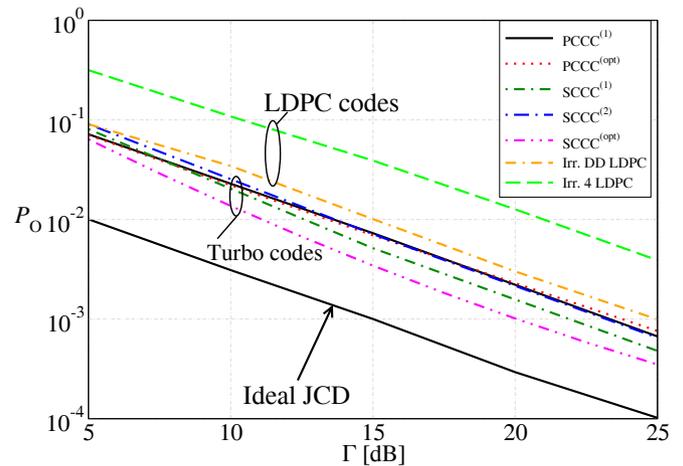
(a)



(a)



(b)



(b)

Fig. 5. PO, as a function of the SNR, in JCD channel coded scenarios with  $n = 2$  sources. The correlation coefficient  $\rho$  is fixed to 0.95 in (a) and 0.999 in (b). Simulation results (lines, as labelled in the figure) for rate-1/2 concatenated and LDPC codes are presented, together with analytical results ( $\circ$  for  $PCCC^{(1)}$ ,  $\square$  for  $PCCC^{(opt)}$ ,  $\times$  for  $SCCC^{(1)}$ ,  $\diamond$  for  $SCCC^{(2)}$ ,  $+$  for  $SCCC^{(opt)}$ ,  $*$  for Irregular DD LDPC code, and  $\triangle$  for Irregular 4 LDPC code).

analyzed through simulations. First, for given values of  $\rho$  and of the SNR, it can be observed that the performance of the ideal JCD scheme with  $n = 4$  sources is better than that of a scheme with  $n = 2$  sources. However, it can be observed that the PO of practical JCD schemes degrades when the number of sources increases from 2 to 4. This is due to the fact that with a larger number of sources an outage (at least one bit in error in at least one of the detected messages) becomes more likely, unless  $\Gamma$  becomes extremely large. On the other hand, the BER improves when the number of sources increases, as shown by the results in Fig. 7 (discussed below). Moreover, for large values of  $\rho$  (i.e., 0.999), comparing Fig. 5 (b) with Fig. 6 (b) it can be observed that at large values of the SNR the PO tends to be the same, regardless of the value of  $n$ . It can also be observed that the performance gain of the optimized SCCC, with respect to that of the best LDPC code, increases for increasing values of  $\rho$  and is slightly larger for  $n = 2$ .

In Fig. 7, the BER is shown, as a function of the SNR, considering two values of  $n$ : (a) 2 and (b) 4. The correlation

Fig. 6. PO, as a function of the SNR, in JCD channel coded scenarios with  $n = 4$  sources. The correlation coefficient  $\rho$  is fixed to 0.95 in (a) and 0.999 in (b). Simulation results for rate-1/2 concatenated and LDPC codes are presented.

coefficient  $\rho$  is fixed to 0.95. Simulation results for the same rate-1/2 concatenated and LDPC codes considered in Fig. 5 and Fig. 6 are presented. First, one can observe that, regardless of the number of sources, concatenated codes have better performance than LDPC codes. While in the case with  $n = 2$  all concatenated codes have approximately the same performance and  $SCCC^{(opt)}$  is to be preferred for values of BER below  $10^{-3}$ , in the case with  $n = 4$   $SCCC^{(opt)}$  has the lowest BER for all values of  $\Gamma$ . This confirms the validity of the code optimization strategy proposed in Subsection IV-C.

## V. CONCLUSIONS

In this paper, we have analyzed the performance of multiple access schemes where  $n$  correlated sources communicate to an AP, in the absence of cooperation, through noisy separated block-faded links. Using an information-theoretic approach, we have first determined, applying the SW theorem, the feasible capacity and SNR regions. In the uncoded case, we have derived a novel and tight upper bound on the BER of JCD schemes. Considering coded schemes with fixed information rate at the sources, it has been shown that JCD schemes outperform DSC schemes, which, in the absence of rate adaptation,

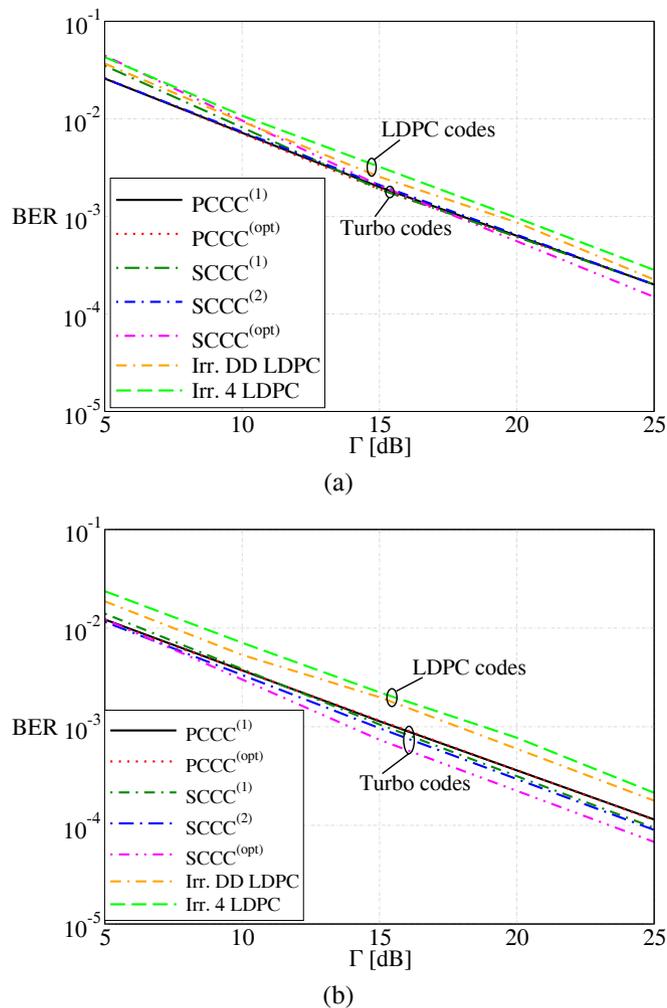


Fig. 7. BER, as a function of the SNR, in JCD channel coded scenarios with two values for the number  $n$  of sources: (a) 2 and (b) 4. The correlation coefficient  $\rho$  is fixed to 0.95. Simulation results for rate-1/2 concatenated and LDPC codes are presented.

are suboptimal in the block-faded scenario at hand. Therefore, we have focused on JCD schemes and proposed a EXIT chart-based framework to design optimized concatenated and LDPC codes, considering, as optimization criterion, the minimization of the PO. The best channel code turns out to be a SCCC. The prediction of the analytical optimization framework are then verified, through simulations, in terms of PO and BER.

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