

# Fundamental limits of electronic dispersion compensation in optical communications with direct photodetection

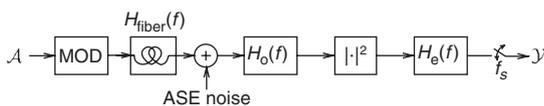
M. Franceschini, G. Ferrari, R. Raheli and G. Bongiorno

The information theoretical limits of electronic dispersion compensation (EDC) for on-off keying non-return-to-zero optical transmission systems with direct photodetection are investigated. The results obtained show that the optical signal-to-noise ratio penalty entailed by EDC can be limited to about 2 dB at most values of chromatic dispersion of interest in current applications.

**Introduction:** Owing to the decreasing cost of powerful integrated circuits, electronic signal processing in digital optical communication systems is receiving increasing scientific and industrial interest. In 10 Gbit/s optical links, such as those used in metropolitan and wide area networks, chromatic dispersion (CD) compensation plays an important role and affects significantly the communication system cost. Electronic dispersion compensation (EDC) techniques are appealing to replace costly optical dispersion compensation units and to guarantee robustness and adaptivity. Various EDC solutions are possible, such as electronic equalisation [1] and maximum likelihood sequence detection (MLSD) [2].

In this Letter, we evaluate the information rate (IR) of an optical fibre communication system with direct photodetection. The IR is the supremum of the achievable transmission rates, i.e. the rates at which reliable communication can be achieved [3, 4]. The IR is evaluated accounting for the modulation format, the fibre characteristics, and the receiver front end. The fibre is affected by CD and white Gaussian amplified spontaneous emission (ASE) noise. For a given value of CD, we compute a lower bound on the IR against optical signal-to-noise ratio (OSNR).

The OSNR required to achieve a desired value of IR is also derived against CD. In particular, the IR can be fixed at the rate of a typical error correction code (ECC). In the considered communication scenario, our results show that the theoretical OSNR penalty, with respect to a scenario with no dispersion (back-to-back), is at most 2 dB at CD values up to 6800 ps/nm. Moreover, our results suggest that the OSNR penalty flattens for increasing values of CD. The price to be paid for a limited OSNR penalty is an increase in EDC signal processing complexity with the amount of CD.



**Fig. 1** Considered direct photodetection optical communication system model

**Communication system model:** Fig. 1 shows the considered on-off keying (OOK) communication system model. At the input of the system, a sequence of independent and equally likely information bits  $\mathcal{A}$  is considered. The modulator (MOD) comprises a non-return-to-zero (NRZ) pulse shaping filter, an electrical lowpass filter and a Mach-Zehnder device. The modulated signal propagates through a linear optical fibre characterised by the following frequency response:

$$H_{\text{fibre}}(f) = e^{-jDz(\pi\lambda^2/c)f^2}$$

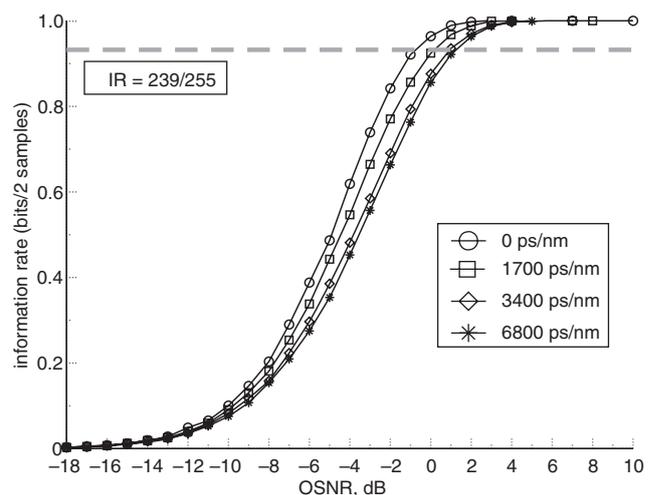
where  $D$  is the CD factor,  $z$  is the distance ( $Dz$  is the total amount of uncompensated CD),  $\lambda$  is the optical carrier wavelength, and  $c$  is the speed of light. The received optical signal is corrupted by wideband ASE noise, characterised by polarisation both parallel and orthogonal to that of the useful signal, filtered by an optical filter  $H_o(f)$ . The received optical signal is then converted to an electrical signal by a square-law photodetector. The signal obtained is filtered by an electrical filter  $H_e(f)$ , then sampled at frequency  $f_s$  corresponding to one or a few samples per bit interval. The discrete time process obtained, denoted as  $\mathcal{Y}$ , can be input to an EDC device. This device is outside the scope of this Letter since our focus is on the ultimate theoretical performance achievable by EDC.

**Information rate computation:** In this Section, for notational simplicity we consider an output process  $\mathcal{Y}$  based on one sample per bit interval: the extension to two or more samples per bit interval is straightforward. Under the assumption of joint ergodicity and stationarity of  $\mathcal{A}$  and  $\mathcal{Y}$ , the IR can be expressed by the Shannon-McMillan-Breiman theorem as [3]

$$I(\mathcal{A}; \mathcal{Y}) = \lim_{n \rightarrow \infty} \frac{1}{n} [-\log p(y_1, \dots, y_n) + \log p(y_1, \dots, y_n | a_1, \dots, a_n)]$$

where  $y_1, \dots, y_n$  are  $n$  received observables and  $a_1, \dots, a_n$  are the corresponding information bits. The IR can be computed using the Monte Carlo approach proposed in [4] and assuming a sufficiently large value of  $n$ . More precisely, the probability density function (PDF)  $p(y_1, \dots, y_n)$  is obtained by means of a recursive computation based on the received samples  $y_1, \dots, y_n$ . In [4] this computation is shown to be equivalent to the forward recursion of a BCJR algorithm [5]. In [4] it is also shown that if the received samples  $y_1, \dots, y_n$  are generated with the actual channel model and the PDFs  $p(y_1, \dots, y_n)$  and  $p(y_1, \dots, y_n | a_1, \dots, a_n)$  are computed using a different channel model, i.e. an 'auxiliary channel', the IR value obtained is a lower bound on the actual IR. As a consequence, these computations can be based on an auxiliary channel model which assumes *finite memory* and *conditional independence* of the observables given the data sequence. For the computation of the conditional PDF of a single observation  $y_k$  we use the exact method proposed in [6].

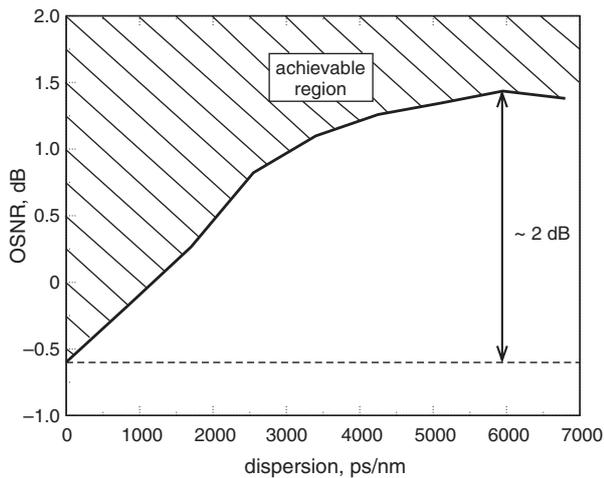
**Numerical results:** The IR has been evaluated by Monte Carlo simulations based on sequences of  $n = 10^6$  information bits, considering several dispersion and OSNR values. The OSNR is defined considering the noise power in a bandwidth of 0.5 nm at carrier wavelength equal to 1550 nm. The considered bit rate is 10 Gbit/s. The lowpass modulator electrical filter is a third-order Bessel filter of bandwidth 9.5 GHz, the modulation extinction ratio is 24.3 dB,  $H_o(f)$  is a third-order Bessel filter of bandwidth 32.5 GHz and  $H_e(f)$  is a fifth-order Bessel filter of bandwidth 7.7 GHz. The used sampling rate is two samples per bit interval since this rate guarantees accurate signal description. In Fig. 2 the IR is shown against OSNR for values of the CD from 0 to 6800 ps/nm (e.g.  $D = 17$  ps/nm/km and  $z$  from 0 to 400 km). As expected, the IR tends to 1 for increasing OSNR, i.e. reliable transmission of one bit per channel use is possible for sufficiently large OSNR. However, for a fixed value of IR, an increasing OSNR penalty is incurred for higher values of CD.



**Fig. 2** IR against OSNR at dispersion values 6800, 3400, 1700, 0 ps/nm

For a fixed IR each CD value can be associated with a specific OSNR value, and vice versa. In Fig. 3 the theoretical OSNR required to achieve IR equal to 239/255 is shown against CD. The value 239/255 corresponds to the rate of a standard Reed-Solomon code employed in optical communications. As can be seen, the OSNR penalty is limited to about 2 dB as CD increases. It should be noted that, since the IR curves in Fig. 2 are lower bounds on the actual IR, the OSNR curves are upper bounds on the actual OSNR. The curve in Fig. 3 represents the lower

limit of the *achievable region* in the OSNR-CD plane, for a given value of IR. Any couple of OSNR and CD values in this region can be achieved reliably (i.e. with error probability as low as desired) by using carefully designed and sufficiently complex EDC systems.



**Fig. 3** OSNR against dispersion at IR equal to 239/255

**Conclusion:** We have computed a lower bound on IR of an optical communication system with direct photodetection affected by uncompensated CD. In the considered communication system settings, this bound shows that OSNR penalties as low as 2 dB can be achieved by means of EDC techniques in the presence of high values of uncompensated CD.

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