ITERATIVE CARRIER SYNCHRONIZATION IN THE ABSENCE OF DISTRIBUTED PILOTS FOR LOW SNR APPLICATIONS

Giulio Colavolpe and Andrea Modenini
Department of Information Engineering, University of Parma, Italy.
mail: giulio@unipr.it, modenini@tlc.unipr.it

ABSTRACT

We consider the advanced modulation and coding schemes used in CCSDS (Consultative Committee for Space Data Systems) standards for deep space telemetry and telecommand. They are based on a powerful turbo or low-density parity check (LDPC) outer code and binary modulation formats that, for those schemes foreseen to be employed at the lowest baud rates, may contain an unsuppressed carrier to help synchronization. In this paper, we face the problem of carrier phase synchronization for these modulation and coding schemes.

I. INTRODUCTION

The need for *turbo* or *iterative detection/synchronization and decoding* is mainly due to the fact that, for the peculiar decoding process and typical operative signal-to-noise ratio of turbo and low-density parity-check (LDPC) codes, classical phase-tracking schemes may deliver, especially in the presence of a time-varying channel phase, a highly unreliable phase estimate or require a systematic use of pilot symbols to avoid tracking losses (see [1]–[8] and references therein). These algorithms for iterative detection/synchronization and decoding can be classified according to the way detection/synchronization is obtained.

A first family of algorithms can be applied to turbo codes and serially-concatenated convolutional codes but not to LDPC codes (unless an LDPC code is used as outer code in a serial concatenation). These algorithms modify the component decoders so that they can also compute an implicit (e.g., see [1], [2], [4], [9]) or explicit (e.g., see [7]–[9]) phase estimate. We have, in this case, *joint detection/synchronization and decoding*. These algorithms usually require to work on an expanded trellis and the adoption of techniques for complexity reduction becomes mandatory. A different approach, able to effectively reduce the computational complexity when the component encoders are rotationally invariant, is adopted in [7]. This latter approach can be also extended to continuous phase modulations (CPMs) [8] provided that the implicit rotational invariance of CPMs is not removed through a proper precoding [10], [11].

A second family is composed of algorithms that, on the contrary, leave the decoder unmodified and complement it with a *separate detector/synchronizer* whose aim is to estimate and compensate for the carrier phase and frequency uncertainties prior to decoding (e.g., see [5], [6], [12], [13]). Hence, they can be used for LDPC codes also. This separate detector/synchronizer operates in *soft-decision-directed* (SDD) mode in the sense that it employs the soft information provided by the decoder to refine the estimates at each iteration, using, to a larger extent, symbols with highest reliability. In other words, a SDD estimator is a sort of hybrid non-data-aided/data-aided (NDA/DA) estimator that starts in NDA mode and becomes DA as the iterations go ahead and the decoder provides reliable decisions. Within this family, the algorithms exhibiting the best trade-off between performance and complexity are those described in [5]. They are based on the *factor graphs/sum-product algorithm* (FG/SPA) [14] framework and employ a Bayesian approach, i.e., the channel parameters are modeled as stochastic processes with known statistics. In [5] a FG is built that includes both code constraints and channel statistics and in which channel parameters are explicitly represented. The SPA is then used to implement the MAP symbol detection strategy. Since the channel parameters, which are continuous random variables, are explicitly represented in the graph, the application of the SPA becomes impractical. To solve this problem, the method of canonical distributions [15] is adopted. By specializing the approach of [15] to particular channel phase statistics and canonical distributions, several algorithms for detection of LDPC or turbo codes in the presence of a Wiener phase noise have been proposed in [5] and extended to the case of presence of an uncompensated frequency offset in [12]. In particular, the algorithm with the lowest complexity describes the messages in the graph as Tikhonov (also known as Von mises) probability density functions and is the best candidate for the implementation in the receivers for DVB-S2 systems.

We consider here modulation and coding schemes used in the CCSDS (Consultative Committee for Space Data Systems) standards for deep space telemetry and telecommand [10], [11]. Since we are looking for algorithms that can be adopted for both turbo and LDPC codes, we consider algorithms from the second family. In particular, we consider the application/extension of the algorithm in [5] based on the Tikhonov canonical distribution to this scenario. A first modification will allow the algorithm to work in the presence of an unsuppressed carrier. We will then consider the modulation and coding formats with suppressed carrier. The algorithm in [5] has been conceived to work in the presence of distributed pilot symbols which allow to trigger the iterative detection/decoding process. These distributed pilot symbols, foreseen in many communication standards, are one of the possible techniques to be employed to solve the phase ambiguity problems related to the angle of symmetry of the employed constellation. As mentioned, another possible solution for these ambiguity problems is represented by the adoption of a rotationally invariant encoder.

Unfortunately, the CCSDS standards foresee neither the use of distributed pilot symbols nor the use of rotationally invariant encoders but only the presence of a known preamble which, as mentioned, is insufficient to make the algorithm in [5] work. We thus here propose a novel approach which entails only a minor complexity increase with respect to the algorithm in [5].

The paper is organized as follows. In the next section we review the detection algorithm described in [5]. Its extensions to the modulation formats in the CCSDS standards is addressed in Section III. Numerical results are presented in Section IV whereas conclusions are drawn in Section V.

II. REVIEW OF THE ALGORITHM BASED ON THE TIKHONOV CANONICAL DISTRIBUTION

We consider a transmission system in which a sequence of M-ary code symbols $\mathbf{c} = \{c_k\}_{k=0}^{K-1}$ is transmitted from epoch 0 to epoch $K-1$. These code symbols are obtained from the encoding, by means of a code C, of a sequence of information symbols $\mathbf{a} = \{a_k\}$. The encoding function mapping information sequences $\mathbf{a}$ into the codewords $\mathbf{c}$ will be denoted by η_C. This function will also include possible pilot symbols inserted in the sequence $\mathbf{c}$ to avoid phase ambiguity problems.

The sequence of code symbols is then linearly modulated and transmitted over the channel. Assuming Nyquist transmitted pulses, matched filtering, phase variations slow enough so as no intersymbol interference arises, the discrete-time baseband received signal is given by

$$r_k = c_k e^{j\theta_k} + w_k \tag{1}$$

where θ_k is an unknown stochastic and possibly time-varying phase and w_k is a discrete-time complex additive white Gaussian noise (AWGN) sample with each component of variance $\sigma^2 = N_0$. We will denote by $\mathbf{r} = \{r_k\}$ the discrete-time received sequence.

A common model for the phase noise process $\boldsymbol{\theta} = \{\theta_k\}$ is the random-walk (Wiener) model described by[1]

$$\theta_k = \theta_{k-1} + \Delta_k \tag{2}$$

where $\{\Delta_k\}$ is a real discrete-time white Gaussian process with mean zero and variance σ_Δ^2, and θ_0 is uniformly distributed in the interval $[0, 2\pi)$. Hence, it follows that

$$p(\theta_k|\theta_{k-1}, \theta_{k-2}, \ldots, \theta_0) = p(\theta_k|\theta_{k-1}) = p_\Delta(\theta_k - \theta_{k-1}) \tag{3}$$

where we define $p_\Delta(\varphi)$ as the probability density function (pdf) of the increment Δ_k mod 2π. The sequence of phase increments $\{\Delta_k\}$ is supposed unknown to both transmitter and receiver and statistically independent of $\mathbf{c}$ and $\mathbf{w} = \{w_k\}$.

The derivation of the detection algorithms described in [5] starts from the joint distribution of symbols and unknown parameters $p(\mathbf{a}, \mathbf{c}, \boldsymbol{\theta}|\mathbf{r})$,[2] and the corresponding FG. The SPA is then applied to this FG to compute the marginal pmfs $P(a_k|\mathbf{r})$ which are required for the implementation of the MAP symbol detection algorithm

The joint probability distribution function of symbols and channel parameters can be expressed as

$$\begin{aligned}
p(\mathbf{a}, \mathbf{c}, \boldsymbol{\theta}|\mathbf{r}) &\propto P(\mathbf{a})P(\mathbf{c}|\mathbf{a})p(\boldsymbol{\theta})p(\mathbf{r}|\boldsymbol{\theta}, \mathbf{c}) = P(\mathbf{a})\chi[\mathbf{c} = \eta_C(\mathbf{a})]p(\boldsymbol{\theta})p(\mathbf{r}|\boldsymbol{\theta}, \mathbf{c}) \\
&= P(\mathbf{a})\chi[\mathbf{c} = \eta_C(\mathbf{a})]p(\theta_0)\prod_{k=0}^{K-1} p(r_k|c_k, \theta_k)p(\theta_k|\theta_{k-1}) \\
&\propto P(\mathbf{a})\chi[\mathbf{c} = \eta_C(\mathbf{a})]p(\theta_0)\prod_{k=0}^{K-1} f_k(c_k, \theta_k)p(\theta_k|\theta_{k-1})
\end{aligned} \tag{4}$$

having defined

$$f_k(c_k, \theta_k) = \exp\left\{\frac{1}{\sigma^2}\Re[r_k c_k^* e^{-j\theta_k}] - \frac{|c_k|^2}{2\sigma^2}\right\} \propto \exp\left\{-\frac{1}{2\sigma^2}|r_k - c_k e^{-j\theta_k}|^2\right\} \tag{5}$$

whose corresponding FG is shown in Fig. 1.

The application of the SPA to this FG allows the exact (in the absence of cycles in the graph) or approximate (if cycles are present) computation of the marginal a posteriori probabilities $P(a_k|\mathbf{r})$ [14]. Since the channel parameters,

[1] This model will be employed in the design of the algorithms. As shown in [5], the resulting algorithms will work well also in the presence of a channel phase following different models.

[2] We use the term probability distribution function to denote a continuous pdf with some discrete probability masses. For a probability distribution function we still use the symbol $p(.)$.

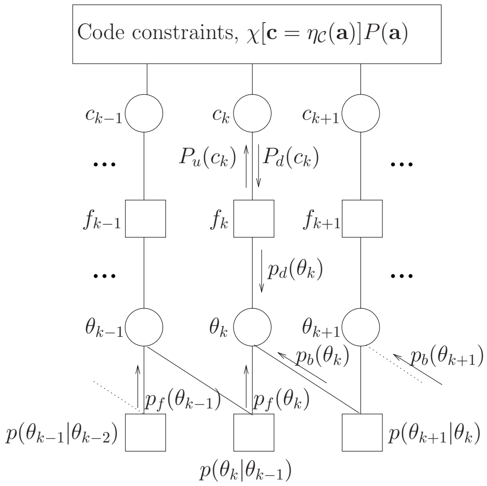

Fig. 1: Portion of the FG corresponding to (4).

which are continuous random variables, are explicitly represented in the graph, the application of the SPA becomes impractical since it involves integral computations. A solution for this problem is suggested in [15] and consists of the use of *canonical distributions*, i.e., the pdfs computed by the SPA are constrained to be in a certain "canonical" family, that admits a compact parametric representation. This representation can be exact or, more often, can involve some approximations. Hence, the SPA reduces to propagating and updating the parameters of the pdf rather than the pdf itself. Beyond this general idea, several different algorithms can be obtained depending of the choice of the canonical distribution family. These approximations of the SPA, albeit all derived from the same standard approach, offer different complexity and performance. Therefore, finding good canonical distribution parameterizations suited to the problem at hand is the key step in the algorithm design.

In the following, we concentrate on the message computation and exchange in the lower part of the graph since the SPA applied to the FG in the upper box, corresponding to the code constraints, consists of the decoding algorithm whose efficient implementation depends on the structure of the code and needs no details here. We stress that the messages the decoder exchanges with the detector are represented by an estimate of the code symbol a posteriori probabilities only. In other words, the detector operates without taking the code constraints into consideration. Omitting, for the sake of notational simplicity, the explicit reference to the current iteration, we will denote by $P_d(c_k)$ the message from variable node c_k to factor node f_k, and by $P_u(c_k)$ the message in the opposite direction (see Fig. 1).

With reference to the messages in Fig. 1, one obtains that the message $p_d(\theta_k)$ from factor node f_k to variable node θ_k can be expressed as

$$p_d(\theta_k) \propto \sum_{c_k} P_d(c_k) f_k(c_k, \theta_k) \,. \tag{6}$$

We also assume that in the lower part of the FG, a forward-backward node activation schedule is adopted. Therefore, the messages $p_f(\theta_k)$, from factor node $p(\theta_k|\theta_{k-1})$ to variable node θ_k, and $p_b(\theta_k)$, from factor node $p(\theta_{k+1}|\theta_k)$ to variable node θ_k, can be recursively computed as follows:

$$p_f(\theta_k) \propto \int_0^{2\pi} p_d(\theta_{k-1}) p_f(\theta_{k-1}) p(\theta_k|\theta_{k-1}) \, d\theta_{k-1} \tag{7}$$

$$p_b(\theta_k) \propto \int_0^{2\pi} p_d(\theta_{k+1}) p_b(\theta_{k+1}) p(\theta_{k+1}|\theta_k) \, d\theta_{k+1} \tag{8}$$

with uniform pdfs as initial conditions. Finally, the message $P_u(c_k)$ from f_k to c_k is given by

$$P_u(c_k) \propto \int_0^{2\pi} p_f(\theta_k) p_b(\theta_k) f_k(c_k, \theta_k) \, d\theta_k \,. \tag{9}$$

Different canonical distributions can now be adopted to approximately compute the messages in (7), (8), and (9), leading to algorithms with different performance and complexity. The first one is based on a discretization of the channel phase. This case corresponds to letting the canonical distribution be a weighted sum of impulses. The channel phase θ_k is assumed to take on the following L values: $\{0, 2\pi/L, \ldots, 2\pi(L-1)/L\}$. In [1], the authors found that for M-PSK signals, $L = 8M$ values are sufficient to have no performance loss. Obviously, this approach becomes "optimal" (in the sense that it approaches the performance of the exact SPA) for a sufficiently large number of discretization levels, at the expense of an increasing computational complexity and will be considered here as an unfeasible performance benchmark. In the numerical results, it will be denoted to as *discretized-phase algorithm*.

The second canonical distribution we consider is a Tikhonov pdf [5]. Let us consider (6). If the messages $P_d(c_k)$ were the exact probabilities of the code symbols, it would be

$$p_d(\theta_k) \propto \sum_{c_k} P_d(c_k) f_k(c_k, \theta_k) \propto p(r_k|\theta_k)\,. \tag{10}$$

We approximate $p(r_k|\theta_k)$ which, as a function of r_k is a Gaussian mixture, by the nearest Gaussian pdf in the sense of Kullbach-Leibler (KL) divergence [5]. This yields the Gaussian pdf with mean $\mathrm{E}[r_k|\theta_k]$ and variance $\mathrm{var}(r_k|\theta_k)$ [5]. Hence, letting α_k and β_k be the first and second-order moments of c_k, given by

$$\alpha_k = \sum_{c_k} c_k P_d(c_k)\,, \qquad \beta_k = \sum_{c_k} |c_k|^2 P_d(c_k) \tag{11}$$

we obtain[3]

$$\tilde{p}_d(\theta_k) \simeq \frac{1}{\pi(2\sigma^2+\beta_k-|\alpha_k|^2)} \exp\left\{-\frac{|r_k-\alpha_k e^{\jmath\theta_k}|^2}{2\sigma^2+\beta_k-|\alpha_k|^2}\right\} \propto \exp\left\{2\frac{\mathrm{Re}[r_k\alpha_k^* e^{-\jmath\theta_k}]}{2\sigma^2+\beta_k-|\alpha_k|^2}\right\}. \tag{12}$$

Substituting (12) in the forward recursion (7), we have

$$\tilde{p}_f(\theta_k) \simeq \int_0^{2\pi} \exp\left\{2\frac{\mathrm{Re}[r_{k-1}\alpha_{k-1}^* e^{-\jmath\theta_{k-1}}]}{2\sigma^2+\beta_{k-1}-|\alpha_{k-1}|^2}\right\} \tilde{p}_f(\theta_{k-1}) p(\theta_k|\theta_{k-1})\, d\theta_{k-1}\,. \tag{13}$$

When the channel phase is slowly-varying, i.e., for $\sigma_\Delta \to 0$, we have $p(\theta_k|\theta_{k-1}) = \delta(\theta_k - \theta_{k-1})$. In this case, the solution of the recursion given by (13) with uniform pdfs as initial conditions is a sequence of Tikhonov pdfs, i.e.,

$$\tilde{p}_f(\theta_k) = t(a_{f,k};\theta_k) \tag{14}$$

where

$$t(z;\theta) = \frac{e^{\Re[z e^{-\jmath\theta}]}}{2\pi \mathrm{I}_0(|z|)} \tag{15}$$

is the Tikhonov pdf with complex parameter z, and $\mathrm{I}_0(\cdot)$ is the zero-th order modified Bessel function of the first kind. Parameter $a_{f,k}$ can be recursively computed as

$$a_{f,k} = a_{f,k-1} + 2\frac{r_{k-1}\alpha_{k-1}^*}{2\sigma^2+\beta_{k-1}-|\alpha_{k-1}|^2} \tag{16}$$

with the initial condition $a_{f,0} = 0$. Similarly, the solution of the backward recursion (8) under the above approximations is the sequence of Tikhonov pdfs

$$\tilde{p}_b(\theta_k) = t(a_{b,k};\theta_k) \tag{17}$$

where $a_{b,k}$ can be recursively computed as

$$a_{b,k} = a_{b,k+1} + 2\frac{r_{k+1}\alpha_{k+1}^*}{2\sigma^2+\beta_{k+1}-|\alpha_{k+1}|^2} \tag{18}$$

with the initial condition $a_{b,K-1} = 0$. From (14), (17) and (9), we finally obtain

$$\tilde{P}_u(c_k) \propto \exp\left\{-\frac{|c_k|^2}{2\sigma^2}\right\} \mathrm{I}_0\left(\left|a_{f,k} + a_{b,k} + \frac{r_k c_k^*}{\sigma^2}\right|\right). \tag{19}$$

When the phase varies more rapidly, so that the approximation $p(\theta_k|\theta_{k-1}) \simeq \delta(\theta_k - \theta_{k-1})$ is no longer valid, it is shown in [5] that the distributions $p_f(\theta_k)$ and $p_b(\theta_k)$ are still approximately given in the form (14) and (17), where now the coefficients $a_{f,k}$ and $a_{b,k}$ are updated by properly modified forward and backward recursions [5].

This algorithm, can be summarized as follows. Given the code symbol a posteriori probabilities provided by the decoder, the first and second-order moments of symbols $\{c_k\}$ are first computed using (11). Two complex coefficients, one for each recursion, are then updated and used to compute the code symbol a posteriori probabilities to be passed to the decoder.

This algorithm, based on the Tikhonov canonical distribution and denoted to as *Tikhonov algorithm* in the numerical results, cannot work in the absence of distributed pilot symbols. This is due to the fact that, in the absence of a priori information on (some) code symbols, the detection algorithm, which does not exploit the code constraints, cannot bootstrap since it would result $\alpha_k = 0$, for all k. In other words, approximating $p_d(\theta_k)$ with $\tilde{p}_d(\theta_k)$ in (12) makes the algorithm unable to work in the absence of distributed pilots.

[3]In the following, we will denote by $\tilde{p}_d(\theta_k)$, $\tilde{p}_f(\theta_k)$, $\tilde{p}_b(\theta_k)$, and $\tilde{P}_u(c_k)$ the new messages (6), (7), (8), (9) resulting from the adopted approximations. Note that $\tilde{p}_d(\theta_k)$, seen as a function of θ_k and properly normalized, is a Tikhonov pdf.

III. APPLICATION TO THE CCSDS SCENARIO

A. *Modulation formats with unsuppressed carrier*

Without loss of generality, we consider two cases of binary modulations with an unsuppressed carrier. In the first case, a binary phase-shift keying (BPSK) with rectangular pulse of duration T, where T is the symbol time, is transmitted along with an unsuppressed carrier in quadrature. In the second case, a BPSK with Manchester encoding is transmitted along with an unsuppressed carrier in phase. In both case, we will denote by γ the ratio between the amplitude of the unsuppressed carrier and the amplitude of the information-bearing signal. In the first case, symbol-time samples after matched filtering can be expressed as

$$r_k = (c_k + \jmath\gamma)e^{\jmath\theta_k} + w_k \,. \tag{20}$$

In the second case, after a bank of two filters, the first one matched to the shaping pulse of the BPSK signal and the second one matched to a rectangular pulse of duration T, we obtain the following samples

$$r_k^{(1)} = c_k e^{\jmath\theta_k} + w_k^{(1)} \tag{21}$$
$$r_k^{(2)} = \gamma e^{\jmath\theta_k} + w_k^{(2)} \tag{22}$$

where $\{w_k^{(1)}\}$ and $\{w_k^{(2)}\}$ are independent discrete-time complex AWGN processes.

The extension of the Tikhonov algorithm to these two unsuppressed-carrier modulation formats is trivial. In fact, in the case of the received samples (20), it is sufficient to make the following substitutions: (i) in (16), substitute $r_{k-1}\alpha^*_{k-1}$ with $r_{k-1}(\alpha_{k-1} - \jmath\gamma)$, (ii) in (18), substitute $r_{k+1}\alpha^*_{k+1}$ with $r_{k+1}(\alpha_{k+1} - \jmath\gamma)$, and finally (iii) in (19) substitute $r_k c_k^*$ with $r_k(c_k - \jmath\gamma)$. Obviously, similar substitutions have to be made in the properly modified forward and backward recursions to be adopted in case of time-varying phase noise [5].

In the case of the received samples (21) and (22), (16), (18), and (19) become

$$a_{f,k} = a_{f,k-1} + 2\frac{r^{(1)}_{k-1}\alpha_{k-1}}{2\sigma^2 + \beta_{k-1} - \alpha^2_{k-1}} + \frac{r^{(2)}_{k-1}\gamma}{\sigma^2} \tag{23}$$

$$a_{b,k} = a_{b,k+1} + 2\frac{r^{(1)}_{k+1}\alpha_{k+1}}{2\sigma^2 + \beta_{k+1} - \alpha_{k+1}} + \frac{r^{(2)}_{k+1}\gamma}{\sigma^2} \tag{24}$$

$$\tilde{P}_u(c_k) \propto \exp\left\{-\frac{|c_k|^2}{2\sigma^2}\right\} \mathrm{I}_0\left(\left|a_{f,k} + a_{b,k} + \frac{r^{(1)}_k c_k}{\sigma^2} + \frac{r^{(2)}_k\gamma}{\sigma^2}\right|\right) . \tag{25}$$

B. *Modulation formats with suppressed carrier*

As mentioned, we approximated $p_d(\theta_k)$ in (6) with $\tilde{p}_d(\theta_k)$ which is a Tikhonov pdf. This aproximation makes the Tikhonov algorithm unable to work in the absence of distributed pilots. We will now address a possible technique to solve this problem with reference to the case of a BPSK modulation. The system model is thus that described by (1) and (2), where symbols $\{c_k\}$ are binary and belong to the alphabet $\{\pm 1\}$. The extension to other modulation formats of the standard, such as the offset quaternary phase-shift keying (OQPSK) or the Gaussian minimum-shift keying is straightforward. In this latter case, the use of the Laurent decomposition allows to approximate the GMSK signal as a linearly modulated signal [8].

The actual message $p_d(\theta_k)$ in (6), is a mixture of Tikhonov pdfs due the phase ambiguity of the constellation. As an example, in this case of a binary modulation, we have a mixture of two Tikhonov pdfs. For this reason, with the aim of improving the Tikhonov algorithm, Shayovitz and Raphaeli in [16] avoided any approximation on $p_d(\theta_k)$, whereas for messages $\tilde{p}_f(\theta_k)$ and $\tilde{p}_b(\theta_k)$ they adopted a canonical distribution made of a mixture of a given number of Tikhonov pdfs. Substituting in (7) and (8), at each step of the forward and the backward recursion a double (in this binary case) number of Tikhonov pdfs is obtained. This number is then reduced to keep a reasonable complexity by selecting and melting the mixture components more similar (in the sense of the KL divergence) to each other.

We adopt here a different approach based on *expectation propagation* (EP) [17], [18]. Let us consider the lower part of the graph in Fig. 1, i.e., without considering the code constraints, but assuming that the probabilities $\{P_d(c_k)\}$ are still available as a-priori information. This part of the graph does not contain cycles. When deriving the Tikhonov algorithm, the only introduced approximation is that of pdf $p_d(\theta_k)$ in (6) with a Tikhonov pdf. After this approximation, no further approximation is required, at least when $\sigma_\Delta \to 0$, and all remaining messages in the graph result to belong to the same family of Tikhonov pdfs. In addition, a natural forward-backward schedule results and, being the graph without cycles, the SPA has a natural termination. However, the introduced approximation is *ad hoc*, and no optimality, in any sense, can be claimed from this approach.

Instead of approximating in advance $p_d(\theta_k)$ with a Tikhonov pdf by using an *ad-hoc* approximation, EP constrains in advance all messages to belong to an exponential family because it is closed under product [17], [18]. This family includes Gaussian and Tikhonov pdfs as special cases and here we obviously adopt the latter pdfs. In particular, we still assume that $\tilde{p}_f(\theta_k) = t(a_{f,k}; \theta_k)$ and $\tilde{p}_b(\theta_k) = t(a_{b,k}; \theta_k)$. Let us now see how to approximate, according to EP, the pdf $p_d(\theta_k)$ with a Tikhonov pdf. From Fig. 1, it can be seen that the a-posteriori pdf of θ_k is given by

$$p(\theta_k|\mathbf{r}) = p_f(\theta_k)p_d(\theta_k)p_b(\theta_k) \simeq \tilde{p}_f(\theta_k)p_d(\theta_k)\tilde{p}_b(\theta_k)\,. \tag{26}$$

According to EP, we approximate this a-posteriori pdf to the closest Tikhonov pdf $t(v_k; \theta_k)$ in the KL divergence sense, i.e., with parameter v_k given by

$$v_k = \operatorname*{argmin}_{x\in\mathbb{C}} \mathrm{KL}\left(\tilde{p}_f(\theta_k)p_d(\theta_k)\tilde{p}_b(\theta_k)\|t(x;\theta_k)\right) \tag{27}$$

where $\mathrm{KL}(p(x)\|q(x))$ is the KL divergence defined as

$$\mathrm{KL}(p\|q) = \int p(x)\log_2\frac{p(x)}{q(x)}\,\mathrm{d}x\,.$$

The product $\tilde{p}_f(\theta_k)p_d(\theta_k)\tilde{p}_b(\theta_k)$ is a mixture of two Tikhonov pdfs and reads (after straightforward algebraic manipulations)

$$\tilde{p}_f(\theta_k)p_d(\theta_k)\tilde{p}_b(\theta_k) = \sum_{c_k\in\{\pm1\}} P_d(c_k)e^{-\frac{|c_k|^2}{2\sigma^2}}\mathrm{I}_0\left(\left|a_{f,k}+a_{b,k}+\frac{r_kc_k*}{\sigma^2}\right|\right)t\left(a_{f,k}+a_{b,k}+\frac{r_kc_k*}{\sigma^2};\theta_k\right)\,. \tag{28}$$

It can be easily shown that the KL divergence (27) is minimized when [17], [18]

$$\frac{\mathrm{I}_1(|v_k|)}{\mathrm{I}_0(|v_k|)}\exp\{\jmath\arg(v_k)\} = \sum_{c_k} P_d(c_k)e^{-\frac{|c_k|^2}{2\sigma^2}}\mathrm{I}_1\left(\left|a_{f,k}+a_{b,k}+\frac{r_kc_k*}{\sigma^2}\right|\right)\exp\left\{\jmath\arg(a_{f,k}+a_{b,k}+\frac{r_kc_k*}{\sigma^2})\right\}\,, \tag{29}$$

where $\mathrm{I}_1(\cdot)$ is the first order modified Bessel function of the first kind. Eqn. (29) cannot be solved in closed form. However, by adopting the approximation $\mathrm{I}_1(x)/\mathrm{I}_0(x) \simeq e^{-0.5/x}$ (which is good for $x \gg 0$ [19]), we have

$$v_k \simeq \frac{-0.5}{\log(|m_k|)}e^{\jmath\arg(m_k)}$$

where

$$m_k = \sum_{c_k} P_d(c_k)\exp\left\{-\frac{|c_k|^2}{2\sigma^2} - \frac{1}{2\left|a_{f,k}+a_{b,k}+\frac{r_kc_k*}{\sigma^2}\right|}\right\}\mathrm{I}_0\left(\left|a_{f,k}+a_{b,k}+\frac{r_kc_k*}{\sigma^2}\right|\right)\exp\left\{\jmath\arg(a_{f,k}+a_{b,k}+\frac{r_kc_k*}{\sigma^2})\right\} \tag{30}$$

Hence

$$\tilde{p}_d(\theta_k) \propto \frac{t(v_k;\theta_k)}{\tilde{p}_f(\theta_k)\tilde{p}_b(\theta_k)} \propto t(z_k;\theta_k)$$

is the approximation we are looking for, where

$$z_k = v_k - a_{f,k} - a_{b,k}\,. \tag{31}$$

This latter equation, provide the best Tikhonov approximation for $p_d(\theta_k)$ according to EP. Unfortunately, it involves $a_{f,k}$ and $a_{b,k}$, whereas in the original algorithm, described in Section II, it does not. We will solve this problem later. Assuming that z_k is available, the algorithm then proceeds as in Section II in the sense that $a_{f,k}$ and $a_{b,k}$ can be recursively computed as

$$a_{f,k} = a_{f,k-1} + z_{k-1} \tag{32}$$

$$a_{b,k} = a_{b,k+1} + z_{k+1} \tag{33}$$

and (19) still holds. Similarly, when the assumption $\sigma_\Delta \to 0$ is no longer valid, the recursive equations (32) and (33) have to be properly modified as described in [5].

As mentioned, the adoption of the EP technique introduces a dependence of z_k on $a_{f,k}$ and $a_{b,k}$. This is equivalent to have a computation over a FG with cycles. An approximated iterative computation thus results and a proper computation schedule has to be defined. This is typical of the EP method [17], [18]. We consider the following schedule that works well in the absence of distributed pilots. Once the turbo or LDPC decoder makes available an updated version of the probabilities $\{P_d(c_k)\}$, the following steps are executed:

1. z_k, $a_{f,k}$, and $a_{b,k}$ are set to zero $\forall k = 0, 1, \ldots, K-1$;
2. update z_{k-1} by using (31) and then $a_{f,k}$ by using (32), $\forall k = 1, 2, \ldots, K-1$;
3. update z_{k+1} by using (31) and then $a_{b,k}$ by using (32), $\forall k = K-2, \ldots, 1, 0$;
4. compute $\tilde{P}_u(c_k)$ by using (19), $\forall k = 1, 2, \ldots, K-1$.

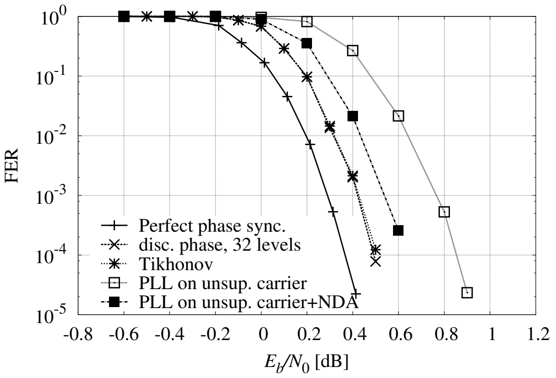

Fig. 2: FER preformance for a NRZ-L modulation transmitted at 4 Baud over the UHF bandwidth.

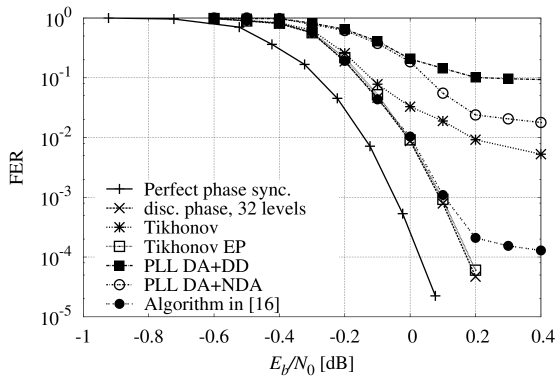

Fig. 3: FER performance for a BPSK modulation with suppressed carrier in the absence of distributed pilots.

Before executing step 4 and passing the updated symbol probabilities to the decoder, steps 2 and 3 could be executed again, although this is not done in our simulation results since, in any case, steps 1-4 are executed again after a few iterations of the turbo or LDPC code. However, a better performance can be obtained if, each time the probabilities $\{P_d(c_k)\}$ are made available by the decoder, we execute step 2 and then step 3 or viceversa, alternatively.

IV. NUMERICAL RESULTS

We now consider the application of the described algorithms to the modulation and coding formats of the CCSDS standards [10], [11]. The employed code is a turbo code with rate 1/6 and codeword length of 21,432 bits [10], [11]. In all cases, a maximum of 5 global iterations are allowed between detector and decoder, each with 10 inner iterations of the turbo decoder, for a total number of 50 decoder iterations. In the case of classical, non iterative, synchronization schemes whose performance is also shown for comparison, 50 iterations were performed directly at the decoder.

Fig. 2 shows the frame error ratio (FER) as a function of E_b/N_0, being E_b the energy per information bit, for a non-return-to-zero-level (NRZ-L) modulation transmitted at 4 Baud over the UHF bandwidth (2110–2120 MHz and 2290–2300 MHz) and with carrier suppression of 10 dB. The phase noise mask is that described in [11] . The figure shows the perfomance of the Tikhonov algorithm (extended to the case of presence of an unsuppressed carrier), of the discretized-phase algorithm with $L = 32$, and of a classical syncronization algorithm based on a phase-locked loop (PLL) whose phase error takes into account the presence of the unsuppressed carrier, possibly improved by also using the non-data-aided (NDA) Viterbi&Viterbi phase error [20]. The performance with perfect knowledge of the phase (perfect phase synchronization) is also shown for comparison. It can be seen that the Tikhonov algorithm has a performance very close to the optimal algorithm and outperforms classical synchronization schemes based on a PLL with optimized step-size.

Fig. 3 shows the FER for a BPSK modulation with suppressed carrier and with known symbols concentrated in a preamble and a postamble of 192 symbols each. No distributed pilots are inserted along the frame and the transmission is at 50 kBaud over the SHF bandwidth (8025 MHz - 8500 MHz). In Fig. 3, we report the performance of the standard Tikhonov algorithm in [5] and that of the Tikhonov EP algorithm derived in the previous section. For compararison the figure also shows the performance of the discretized-phase algorithm, of the algorithm proposed in [16], and of a classical synchronization scheme based on a PLL operating in DA mode over the preamble, and NDA or decision-direct (DD) mode over the coded symbols.

It can be seen that the Tikhonov algorithm does not work at all. As already mentioned, this is due to the fact that, at the first iteration, $\alpha_k = 0$ over coded symbols and therefore these symbols do not help synchronization. Hence, with the standard Tikhonov algorithm, after the preamble, the reliability of the phase estimation is high. However no further information is obtained after that, and the reliability decreases till the phase pdfs become flat in $[0, 2\pi)$, especially at the middle of the codeword, making the synchronization a hard task. The Tikhonov EP algorithm, instead, has a performance close to that of the optimal algorithm and outperforms also the algorithm in [16].

V. CONCLUSIONS

We considered the problem of phase synchronization in the absence of pilot symbols for low signal-to-noise ratio applications. In particular, we addressed the modulation and coding formats of CCSDS standards. The extension of the algorithm in [5] to the case of modulation with an unsuppressed carrier has shown to be trivial. On the other hand, for the modulation formats with suppressed carrier, since pilot symbols are not foreseen in the standard, proper extensions/modifications have been introduced. The proposed schemes, besides the excellent performance, have a very low complexity.

REFERENCES

[1] M. Peleg, S. Shamai (Shitz), and S. Galán, "Iterative decoding for coded noncoherent MPSK communications over phase-noisy AWGN channel," *IEE Proc. Commun.*, vol. 147, pp. 87–95, Apr. 2000.

[2] G. Colavolpe, G. Ferrari, and R. Raheli, "Noncoherent iterative (turbo) decoding," *IEEE Trans. Commun.*, vol. 48, pp. 1488–1498, Sept. 2000.

[3] A. Anastasopoulos and K. M. Chugg, "Adaptive iterative detection for phase tracking in turbo coded systems," *IEEE Trans. Commun.*, vol. 49, pp. 2135–2144, Dec. 2001.

[4] G. Ferrari, G. Colavolpe, and R. Raheli, *Detection Algorithms for Wireless Communications*. John Wiley & Sons, 2004.

[5] G. Colavolpe, A. Barbieri, and G. Caire, "Algorithms for iterative decoding in the presence of strong phase noise," *IEEE J. Select. Areas Commun.*, vol. 23, pp. 1748–1757, Sept. 2005.

[6] G. Colavolpe, "On LDPC codes over channels with memory," *IEEE Trans. Wireless Commun.*, vol. 5, pp. 1757–1766, July 2006.

[7] A. Barbieri and G. Colavolpe, "Soft-output decoding of rotationally invariant codes over channels with phase noise," *IEEE Trans. Commun.*, vol. 55, pp. 2125–2133, Nov. 2007.

[8] A. Barbieri and G. Colavolpe, "Simplified soft-output detection of CPM signals over coherent and phase noise channels," *IEEE Trans. Wireless Commun.*, vol. 6, pp. 2486–2496, July 2007.

[9] A. Anastasopoulos, K. M. Chugg, G. Colavolpe, G. Ferrari, and R. Raheli, "Iterative detection for channels with memory," *Proceedings of the IEEE, Special issue on Turbo Techniques: Algorithms & Applications*, vol. 95, pp. 1272–1294, June 2006.

[10] CCSDS 131.0-B-2, TM synchronization and channel coding, Issue 2, August 2011. Available at http://public.ccsds.org/publications/BlueBooks.aspx.

[11] CCSDS 401.0-B, Radio frequency and modulation systems, Part 1, July 2011. Available at http://public.ccsds.org/publications/BlueBooks.aspx.

[12] A. Barbieri, G. Colavolpe, and G. Caire, "Joint iterative detection and decoding in the presence of phase noise and frequency offset," *IEEE Trans. Commun.*, vol. 55, pp. 171–179, Jan. 2007.

[13] A. Barbieri and G. Colavolpe, "On pilot-symbol-assisted carrier synchronization for DVB-S2 systems," *IEEE Trans. Broadcast.*, vol. 53, pp. 685–692, Sept. 2007.

[14] F. R. Kschischang, B. J. Frey, and H.-A. Loeliger, "Factor graphs and the sum-product algorithm," *IEEE Trans. Inform. Theory*, vol. 47, pp. 498–519, Feb. 2001.

[15] A. P. Worthen and W. E. Stark, "Unified design of iterative receivers using factor graphs," *IEEE Trans. Inform. Theory*, vol. 47, pp. 843–849, Feb. 2001.

[16] S. Shayovitz and D. Raphaeli, "Multiple hypotheses iterative decoding of LDPC in the presence of strong phase noise," in *Proc. of the 27th IEEE Convention of Electrical and Electronics Engineers (IEEEI)*, (Israel), 2012.

[17] T. P. Minka, "Expectation propagation for approximate Bayesian inference," in *Proc. Conf. on Uncertainty in Artificial Intelligence (UAI)*, Aug. 2001.

[18] T. P. Minka, "Expectation propagation for approximate Bayesian inference," tech. rep., Microsoft Research Cambridge, 2005. MSR-TR-2005-173 (2005).

[19] M. Abramowitz and I. A. Stegun, eds., *Handbook of Mathematical Functions*. Dover, 1972.

[20] A. J. Viterbi and A. M. Viterbi, "Nonlinear estimation of PSK-modulated carrier phase with application to burst digital transmission," *IEEE Trans. Inform. Theory*, vol. 29, pp. 543–551, July 1983.