Decentralized Detection in Sensor Networks with Noisy Communication Links*

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Abstract. This paper presents a general approach to distributed detection with multiple sensors in network scenarios with noisy communication links between the sensors and the fusion center (or access point, AP). The sensors are independent and observe a common phenomenon. While in most of the literature the performance metrics usually considered are missed detection and false alarm probabilities, in this paper we follow a Bayesian approach for the evaluation of the probability of *decision error* at the AP. We first derive an optimized fusion rule at the AP in a scenario with ideal communication links. Then, we consider the presence of noisy links and model them as binary symmetric channels (BSCs). This assumption leads to a simple, yet meaningful, performance analysis. Under this assumption, we show, both analytically and through simulations, that if the noise intensity is above a critical level (i.e., the cross-over probability of the BSC is above a critical value), the lowest probability of decision error at the AP is obtained if the AP selectively discards the information transmitted by the sensors with noisy links.
Key words: Decentralized detection, sensor networks, noisy communication links, multiple observations, cross-layer design.

1 Introduction

Distributed detection has been an active research field for a long time [19]. In particular, several approaches have been proposed to study this problem, in the realms of information theory [10], target recognition [16, 17], and several other areas. The increasing interest, over the last decade, for sensor networks, has spurred a significant research activity on distributed detection techniques in this context [21, 4, 7, 9].

In recent years, wireless sensor networks are becoming more common, as, for example, in terrain monitoring applications [18]. In a wireless communication scenario, links between sensors and the access point (AP) are likely to be faded [15, 6]. In this case, most of the results proposed in the literature are not

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immediately applicable, since they are based on the assumption that communication links between sensors and AP are "ideal," i.e., the information transmitted by sensors is received correctly by the AP. The characteristics (in terms of capacity) of the radio multiple access channel in wireless sensor networks are taken into account in [5], where optimal configurations for decentralized detection are analyzed. Study of decentralized detection taking into account realistic communication constraints is also considered in [1, 12].

In this paper, we first revisit the basic principles of distributed detection with binary decisions at the sensors. In order to model a scenario where some of the links between sensors and AP are non-ideal, we assume that a link can be modeled as a binary symmetric channel (BSC) [14]. We show that selective elimination of noisy links may lead to a performance improvement when the *cross-over* (or *bit-flipping*) probability of the BSC increases. In particular, for each value of the common signal-to-noise ratio (SNR) at the sensors we determine a critical bit-flipping probability which discriminates between two network operating regimes: for values of the bit-flipping probability above the critical value, the best performance is obtained when the AP excludes the sensors with noisy links. In particular, selective exclusion of sensors with noisy links could be obtained, for instance, by using a clever medium access control (MAC) protocol at the AP. Therefore, our results suggest that the use of a *cross-layer approach* to the design of sensor networks with unreliable communication links (e.g., wireless sensor networks) is the best choice.

This paper is structured as follows. In Section 2, we provide the reader with preliminaries on distributed detection principles, referring to a classical distributed detection scheme with *parallel* schedule. In Section 3, the presence of noisy links, modeled as BSCs, is considered, and the corresponding sensor network performance is analyzed. Conclusions and future research directions are presented in Section 4.

2 Preliminaries on Distributed Detection

We consider a classical sensor network scenario where all sensors are connected to a single AP [21]. Two main approaches for combining the information gathered by multiple sensors have been proposed.

- The first approach is referred to as *centralized*: all sensors observations are transmitted to a central processor that performs a global decision.
- The second approach is referred to as *decentralized*: each sensor makes a local decision and a fusion processor, i.e., the AP, makes the final decision, by applying a suitable *fusion rule*.

In this paper, all sensors make an observation of a common binary phenomenon. In other words, we consider the binary hypothesis signal detection problem [13], with statistically independent observations from sensor to sensor. We will refer to the two hypotheses as H_1 and H_0, respectively. The true hypothesis will be simply denoted as H. We will assume that the two hypotheses are equally likely. The extension of this work to the case of correlated sensors [3] is currently under investigation.

Suppose that there are N sensors and that they observe the same phenomenon at a given point in time (for notational simplicity, we do not explicitly consider the time instant of the observation). The discrete-time observation at the i-th sensor can be expressed as

$$r_i = y_i + n_i \tag{1}$$

where

$$y_i \triangleq \begin{cases} 0 & \text{if} \quad H_0 \\ s & \text{if} \quad H_1 \end{cases}$$

with $i = 1, 2, \ldots, N$. Assuming that the noise samples $\{n_i\}$ are independent and identically distributed with the same Gaussian distribution $\mathcal{N}(0, \sigma^2)$, the *common* signal-to-noise ratio (SNR) at each sensor can be defined as follows:

$$\text{SNR}_{\text{sensor}} \triangleq \frac{[\text{E}\{y_i|H_1\} - \text{E}\{y_i|H_0\}]^2}{\sigma^2} = \frac{s^2}{\sigma^2}. \tag{2}$$

For the sake of notational simplicity, we assume that $\sigma = 1$, so that $\text{SNR}_{\text{sensor}} = s^2$. We also assume that the SNR is the same at all sensors, i.e., the sensors are equivalent.

In a classical distributed detection scheme with parallel schedule, each sensor makes an observation of the common phenomenon, decides for one of the two hypotheses, and then sends its binary decision, denoted as u_i, to the AP. In general, the decision rule at each sensor (common for all sensors) can be written as $u_i = \gamma(r_i)$, where $\gamma(\cdot)$ is a suitable *decision function*. Usually, the communication link between each sensor and the AP is *ideal*, i.e., the AP receives correctly the bit transmitted by each sensor. In order to make a decision, the i-th sensor compares the observation r_i with a threshold value τ and computes its binary decision as follows:

$$u_i = \gamma(r_i) = \begin{cases} 1 & \text{if} \quad r_i < \tau \\ 0 & \text{if} \quad r_i > \tau. \end{cases} \tag{3}$$

Equivalently, one can write $\gamma(r_i) = U(r_i - \tau)$, where $U(\cdot)$ is the unitary step function. It is possible to show that this decision rule is equivalent to a local likelihood ratio test [11]. In [20], it is shown that selecting the same value of τ for all sensors is an asymptotically (for large values of N) optimal choice for minimizing the probability of incorrect decision. Moreover, in [20] the author shows also that selecting the same value of τ also for a relatively small number N of sensors leads a negligible performance loss with respect to an optimal threshold selection among the sensors. Motivated by this observation, in the remainder of this paper we will assume that the threshold value τ for local decision is the same for all sensors.

Once all sensors have made their local decisions $\{u_i\}$, the AP receives an array of N binary values, and makes a final decision u_0 according to a *fusion rule* $u_0 = \mathit{\Gamma}(u_1, \ldots, u_N)$. As shown in the literature, the fusion rule must be based on

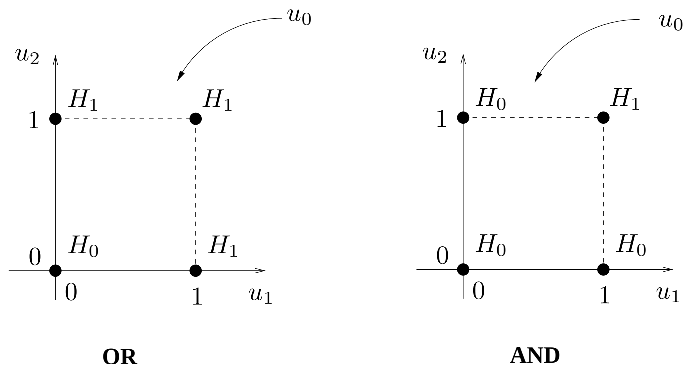

Fig. 1. Decision regions for majority-like fusion rules in the case with $N = 2$ sensors: OR (left) and AND (right) rules. In the axes there are the local decisions (denoted as "0" and "1") at the two sensors, while within the diagram there is the final decision at the AP.

a binary monotonic increasing function of the decisions array of length N [21]. Given N, even if there are 2^{2^N} possible fusion rules, one can limit herself/himself at investigating only binary monotonic increasing functions [16, 21]. Under the assumption that the SNR is the same at all sensors, these fusion rules can be given the following general *majority-like* expression:

$$\Gamma(u_1, \ldots, u_N) = \begin{cases} 1 & \text{if} \quad \sum_{i=1}^{N} u_i \geq k \\ 0 & \text{if} \quad \sum_{i=1}^{N} u_i < k \end{cases} \tag{4}$$

where $k = 1, \ldots, N$. In general, if $k = 1$ the OR fusion rule is obtained, while if $k = N$ the AND fusion rule is obtained. In a network with $N = 2$ sensors, only the OR and AND fusion rules are possible and a pictorial description of these rules is shown in Fig. 1.

Provided that the fusion rule is in the form given by (4), the key problem consists in determining the value of k that minimizes the probability of error under a Bayesian criterion, defined as

$$P_{\mathrm{e}} \triangleq P\{u_0 \neq H\}.$$

Based on our assumption of equally likely hypotheses ($P(H_0) = P(H_1) = 1/2$), the probability of error can be written as

$$P_{\mathrm{e}} = \frac{1}{2} P(u_0 = H_0 | H_1) + \frac{1}{2} P(u_0 = H_1 | H_0). \tag{5}$$

In the general case with $N \geq 2$ sensors, the two terms at the right side of (5) can be evaluated as follows:

$$P(u_0 = H_0 | H_1) = P\{\text{less than } k \text{ sensors decide for } H_1 | H_1\}$$

$$
\begin{aligned}
&= \sum_{i=0}^{k-1} \binom{N}{i} P(u_i = H_1|H_1)^i P(u_i = H_0|H_1)^{N-i} \\
&= \sum_{i=0}^{k-1} \binom{N}{i} [1 - \Phi(\tau - s)]^i \Phi^{N-i}(\tau - s) \qquad (6)
\end{aligned}
$$

$$
\begin{aligned}
P(u_0 = H_1|H_0) &= P\{\text{at least } k \text{ sensors decide for} H_1|H_0\} \\
&= \sum_{i=k}^{N} \binom{N}{i} P(u_i = H_1|H_0)^i P(u_i = H_0|H_0)^{N-i} \\
&= \sum_{i=k}^{N} \binom{N}{i} [1 - \Phi(\tau)]^i \Phi^{(N-i)}(\tau) \qquad (7)
\end{aligned}
$$

where $\Phi(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{y^2}{2}} \, \mathrm{d}y$. Therefore, using (6) and (7) into (5), one obtains

$$
\begin{aligned}
P_\mathrm{e} &= \frac{1}{2} P(u_0 = H_0|H_1) + \frac{1}{2} P(u_0 = H_1|H_0) \\
&= \frac{1}{2} \sum_{i=0}^{k-1} \binom{N}{i} [1 - \Phi(\tau - s)]^i \Phi^{N-i}(\tau - s) + \frac{1}{2} \sum_{i=k}^{N} \binom{N}{i} [1 - \Phi(\tau)]^i \Phi^{N-i}(\tau).
\end{aligned} \qquad (8)
$$

The behavior of the probability of error, as a function of the threshold value τ, is shown in Fig. 2, in the case with $\mathrm{SNR}_{\mathrm{sensor}} = s^2 = 0\,\mathrm{dB}$. As one can observe from Fig. 2, for each decision rule the probability of error is a quasi-convex function of τ and has an absolute minimum. Numerically, one can characterize the absolute minimum depending on the value of N.

- N odd: the optimal value of τ is $s/2$ and the best fusion rule is the *majority rule*, i.e., $k = \lfloor N/2 \rfloor + 1$.
- N even: between the optimal value for the threshold τ and $s/2$ there is an offset that, in general, depends on (i) the number of sensors N, (ii) the sensor SNR s^2, and (iii) the fusion rule. In particular, the best fusion rules are obtained selecting $k = N/2 + 1$ (i.e., adopting a majority rule) or $k = N/2$. For both fusion rules, by properly selecting the threshold value τ the probability of decision error is the same.

As intuitively expected, increasing the number N of sensors and choosing the corresponding optimal fusion rule, the performance (in terms of P_e) improves dramatically.

3 Sensor Networks with Noisy Communication Links

While all previous results apply to a sensor network scenario where the communication links between sensors and AP are ideal, in a realistic scenario (e.g., a

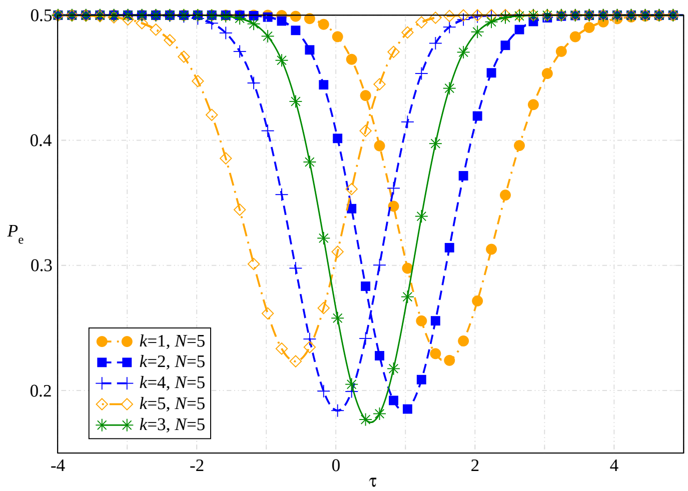

Fig. 2. Probability of error, as a function of the threshold value τ, in a scenario with $N = 5$ sensors and $\text{SNR}_{\text{sensor}} = 0$ dB. Various values of k, corresponding to different fusion rules, are shown.

wireless sensor network) it might happen that these links are noisy (e.g., they are *faded* [6]). Studying such a scenario is difficult, since the presence of fading might also create correlations among the sensors [15]. The analysis and optimization of wireless sensor networks is, therefore, a complicated problem. In [6], the authors propose fusion algorithms that take into account channel fading statistics. In order to derive significant insights into the problem of decentralized detection in sensor networks with realistic communication links, we now consider a simplified model for a noisy communication link. More precisely, a noisy link between a sensor and the AP is modeled as a BSC with parameter p, corresponding to the channel cross-over probability[1] [8]. In other words, the bit transmitted by the sensor has a probability p of being "flipped." The parameter p will depend on the specific characteristics of the sensors-AP communication links (e.g., modulation format, presence of channel coding, presence of fading, detection strategy at the AP, etc.). Assuming binary hard decision at each sensor, if u_i is the decision sent by the i-th sensor, the AP will receive the following information:

$$u_i^{\text{received}} = \begin{cases} u_i & \text{with probability } 1-p \\ 1-u_i & \text{with probability } p. \end{cases}$$

[1] We remark that the sensor SNR, i.e., $\text{SNR}_{\text{sensor}} = \sqrt{s}$, is the SNR at each sensor relative to the local detection of the common phenomenon (or state of nature). A realistic communication link between a sensor and the AP could be characterized by an SNR at the AP. In this paper, however, we do not explicitly consider the communication link SNR, since we concisely describe the communication link as a BSC, which is completely characterized by the single parameter p.

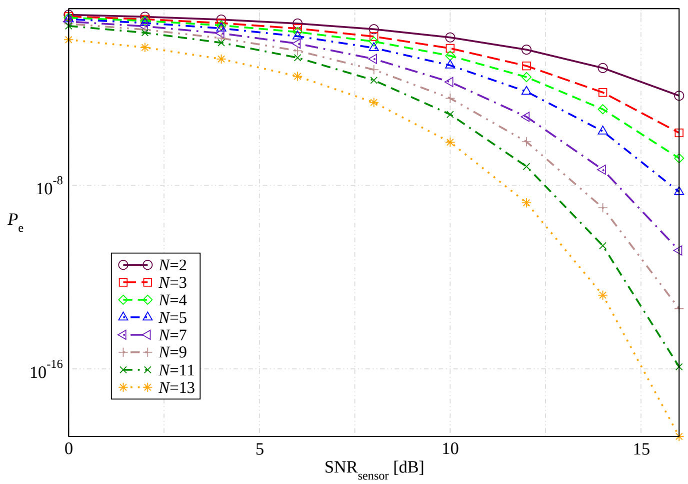

Fig. 3. Probability of error for various values of the number of sensors N. In each case, the optimal fusion rule is considered.

We extend the derivation of the probability of error proposed in Section 2 in order to encompass the presence of noisy links. More precisely, we want to evaluate the final probability of error (5) in a sensor network with noisy links. We consider a majority-like fusion rule at the AP, as described in Section 2, with optimized values of k and τ. We first consider a scenario where all N links are noisy. Then, we generalize the obtained results to the case where $d \leq N$ links are noisy. For example, this scenario could correspond to a wireless sensor network where some of the sensors do not have, temporarily, a "clear" communication path to the AP. Note that the proposed approach could be extended to a scenario where the noise intensity is not the same in all noisy links.

3.1 Sensor Networks with All Noisy Communication Links

After proper algebraic manipulations, it is possible to show that the first conditional probability in (5) can be written as

$$P(u_0 = H_0|H_1) = P\{i < k \text{ sensors for } H_1|H_1\} = \sum_{i=0}^{k-1} \binom{N}{i} P_{\text{c1}}^i P_{\text{e1}}^{N-i} \qquad (9)$$

where

$$P_{\text{c1}} = (1-p)P(s+n > \tau) + pP(s+n < \tau) = (1-p)[1 - \Phi(\tau - s)] + p\Phi(\tau - s)$$

and $P_{\text{e1}} = 1 - P_{\text{c1}}$. Similarly, the second conditional probability in (5) can be written as

$$P(u_0 = H_1|H_0) = P\{i \geq k \text{ sensors for } H_1|H_0\} = \sum_{i=k}^{N} \binom{N}{i} P_{\text{e2}}^i P_{\text{c2}}^{N-i} \qquad (10)$$

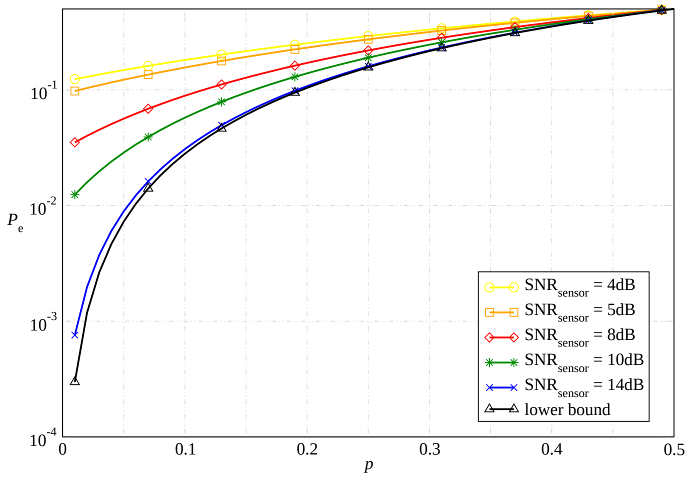

Fig. 4. Probability of error, as a function of the cross-over probability p, for different values of the sensor SNR. The number of sensors is $N = 3$. The curve labeled "lower bound" corresponds to the theoretical limit with $\mathrm{SNR}_{\mathrm{sensor}} = \infty$.

where

$$P_{\mathrm{e}2} = (1-p)P(n > \tau) + pP(n < \tau) = (1-p)[1 - \Phi(\tau)] + p\Phi(\tau)$$

and $P_{\mathrm{c}2} = 1 - P_{\mathrm{e}2}$.

The probability of error (5) can then be evaluated numerically, by using the derived expressions (9) and (10). In particular, the probability of error depends on (i) the decision threshold value τ at the sensors, (ii) the sensor SNR s^2, and (iii) the cross-over probability p.

In Fig. 4, the probability of error is shown as a function of the cross-over probability p, for various values of $\mathrm{SNR}_{\mathrm{sensor}}$, in a scenario with $N = 3$ sensors. As one can observe, regardless of the sensor SNR, for increasing values of p the probability of error becomes unacceptable. The *lower bound* corresponds to a theoretical case where the sensor SNR is infinite. This lower bound, denoted as $P_{\mathrm{e-lb}}(p)$ (to underline its dependence on the cross-over probability p), can be given the following analytical expression:

$$P_{\mathrm{e-lb}}(p) = \lim_{s \to \infty} P_{\mathrm{e}} = \frac{1}{2}\left[\sum_{i=0}^{k-1} \binom{N}{i} (1-p)^i p^{N-i} + \sum_{i=k}^{N} \binom{N}{i} p^i (1-p)^{N-i}\right].$$

From the results shown in Fig. 4, one can conclude that, for any value of p, increasing the sensor SNR beyond a critical threshold does not lead to any significant performance improvement. This might have practical implications on the design of sensors, in terms of their detection accuracy. In fact, one should not increase the sensor sensitivity without limit, but, rather, should find the critical sensitivity at which the ultimate theoretical performance is practically obtained.

Table 1. Analytic expressions of $P(u_0 = H_1|H_0)$ in the following cases: (a) $d \geq k$, $N-d \geq k$, (b) $d \geq k$, $N-d < k$, (c) $d < k$, $N-d < k$ and (d) $d < k$, $N-d \geq k$.

Case	$P(u_0 = H_1\|H_0)$
(a)	$\sum_{d_e=0}^{k} \left[\binom{d}{d_e} P_{\mathrm{e}2}^{d_e} P_{\mathrm{c}2}^{d-d_e} \sum_{i_e=k-d_e}^{N-d} \binom{N-d}{i_e} P_{\mathrm{eH_0}}^{i_e} P_{\mathrm{cH_0}}^{N-d-i_e} \right] + \sum_{d_e=k+1}^{d} \binom{d}{d_e} P_{\mathrm{e}2}^{d_e} P_{\mathrm{c}2}^{d-d_e} \cdot U(d-k-1)$
(b)	$\sum_{d_e=k+d-N}^{k} \left[\binom{d}{d_e} P_{\mathrm{e}2}^{d_e} P_{\mathrm{c}2}^{d-d_e} \sum_{i_e=k-d_e}^{N-d} \binom{N-d}{i_e} P_{\mathrm{eH_0}}^{i_e} P_{\mathrm{cH_0}}^{N-d-i_e} \right] + \sum_{d_e=k+1}^{d} \binom{d}{d_e} P_{\mathrm{e}2}^{d_e} P_{\mathrm{c}2}^{d-d_e} \cdot U(d-k-1)$
(c)	$\sum_{d_e=k+d-N}^{k} \left[\binom{d}{d_e} P_{\mathrm{e}2}^{d_e} P_{\mathrm{c}2}^{d-d_e} \sum_{i_e=k-d_e}^{N-d} \binom{N-d}{i_e} P_{\mathrm{eH_0}}^{i_e} P_{\mathrm{cH_0}}^{N-d-i_e} \right]$
(d)	$\sum_{d_e=0}^{k} \left[\binom{d}{d_e} P_{\mathrm{e}2}^{d_e} P_{\mathrm{c}2}^{d-d_e} \sum_{i_e=k-d_e}^{N-d} \binom{N-d}{i_e} P_{\mathrm{eH_0}}^{i_e} P_{\mathrm{cH_0}}^{N-d-i_e} \right]$

3.2 Sensor Networks with a Generic Number of Noisy Links

We now extend the previous analysis to encompass the case with a generic number $d \leq N$ of noisy links—and, consequently, $N-d$ ideal links. The fusion rule is the majority-like rule given in (4), with an optimized value of k.

In order to evaluate the probability of error, we first compute the conditional probability $P(u_0 = H_1|H_0)$ at the right-hand side of (5). Let us denote by $d_e \leq d$ the number of noisy links associated to sensors in error, i.e., sensors which decide for H_1 when H_0 has happened, and by $i_e \leq N-d$ the number of ideal links associated to sensors in error, i.e., sensors which decide for H_1 when H_0 has happened. With these definitions, the AP *might make*[2] a final erroneous decision if $d_e + i_e \geq k$, with $d_e \in \{0, \ldots, d\}$ and $i_e \in \{0, \ldots, N-d\}$. Depending on the relations between the integers N, k and d, one can distinguish the following four cases, respectively: (a) $d \geq k$, $N-d \geq k$, (b) $d \geq k$, $N-d < k$, (c) $d < k$, $N-d < k$ and (d) $d < k$, $N-d \geq k$. After tedious manipulations, the final expressions for $P(u_0 = H_1|H_0)$, in the four considered cases, are shown in Table 1, where

$$P_{\mathrm{eH_0}} \triangleq P(u_0 = 1|H_0, p = 0) = 1 - \Phi(\tau)$$

and $P_{\mathrm{cH_0}} = 1 - P_{\mathrm{eH_0}}$.

We now consider the second conditional probability at the right-hand side of (5), i.e., $P(u_0 = H_0|H_1)$. In this case, the AP makes a final decision error when $n \leq k-1$ sensors decide for H_1. Let us denote by d_c and i_c the number of sensors in errors (i.e., they decide for H_0 even if H_1 has happened) connected with noisy and ideal links to the AP, respectively. A final decision error *might happen* if $d_c + i_c \leq k-1$, with $d_c \in \{0, \ldots, d\}$ and $i_c \in \{0, \ldots, N-d\}$. As for the computation of $P(u_0 = H_1|H_0)$, four possible cases can be distinguished, depending on the values of N, k and d: (a) $d \leq k-1$, $N-d \leq k-1$, (b)

[2] The reader should observe that if a sensor is in error and the bit transmitted to the AP is flipped, then the bit actually received by the AP is correct.

Table 2. Analytic expressions of $P(u_0 = H_0|H_1)$ in the four cases corresponding to (a) $d \leq k-1$, $N-d \leq k-1$, (b) $d \leq k-1$, $N-d > k-1$, (c) $d > k-1$, $N-d > k-1$ and (d) $d > k-1$, $N-d \leq k-1$.

Case	$P(u_0 = H_0\|H_1)$
(a)	$\sum_{d_c=k+d-N}^{d} \left[\binom{d}{d_e} P_{\mathrm{c}_1}^{d_c} P_{\mathrm{e}_1}^{d-d_c} \sum_{i_c=0}^{k-1-d_c} \binom{N-d}{i_c} P_{\mathrm{cH}_1}^{i_c} P_{\mathrm{eH}_1}^{N-d-i_c}\right] + \sum_{d_c=0}^{k-1+d-N} \binom{d}{d_c} P_{\mathrm{c}_1}^{d_c} P_{\mathrm{e}_1}^{d-d_c}$
(b)	$\sum_{d_c=0}^{d} \left[\binom{d}{d_e} P_{\mathrm{c}_1}^{d_c} P_{\mathrm{e}_1}^{d-d_c} \sum_{i_c=0}^{k-1-d_c} \binom{N-d}{i_c} P_{\mathrm{cH}_1}^{i_c} P_{\mathrm{eH}_1}^{N-d-i_c}\right]$
(c)	$\sum_{d_c=0}^{k-1} \left[\binom{d}{d_e} P_{\mathrm{c}_1}^{d_c} P_{\mathrm{e}_1}^{d-d_c} \sum_{i_c=0}^{k-1-d_c} \binom{N-d}{i_c} P_{\mathrm{cH}_1}^{i_c} P_{\mathrm{eH}_1}^{N-d-i_c}\right]$
(d)	$\sum_{d_c=k+d-N}^{k-1} \left[\binom{d}{d_e} P_{\mathrm{c}_1}^{d_c} P_{\mathrm{e}_1}^{d-d_c} \sum_{i_c=0}^{k-1-d_c} \binom{N-d}{i_c} P_{\mathrm{cH}_1}^{i_c} P_{\mathrm{eH}_1}^{N-d-i_c}\right] \cdot U(N-d-1) + \sum_{d_c=0}^{k-1+d-N} \binom{d}{d_c} P_{\mathrm{c}_1}^{d_c} P_{\mathrm{e}_1}^{d-d_c}$

$d \leq k-1$, $N-d > k-1$, (c) $d > k-1$, $N-d > k-1$ and (d) $d > k-1$, $N-d \leq k-1$, respectively. Reasoning as before, one obtains the expressions for $P(u_0 = H_0|H_1)$ shown in Table 2, where

$$P_{\mathrm{eH}_1} \triangleq P(u_0 = H_0|H_1) = \Phi(\tau - s)$$

and $P_{\mathrm{cH}_1} = 1 - P_{\mathrm{eH}_1}$.

In Fig. 5, the probability of decision error is shown for $N = 3$ sensors. All possible values (from 0 to N) of the number d of noisy links are considered. Obviously, for increasing number of noisy links the performance degrades, and this degradation is more pronounced (relatively) for high values of the sensor SNR. This means that when sensors are very reliable, i.e., the sensor SNR is high, the impact of noisy communication links is (proportionally) higher.

3.3 Selective Exclusion of Sensors with Noisy Links

As we have observed from the results in Fig. 5, for increasing number of noisy links, the sensor network performance (in terms of probability of decision error at the AP) degrades rapidly. At this point, one might consider an "intelligent" AP, which neglects the decisions of a sensor if the link is noisy. For example, in a wireless sensor network, each sensor could send a pilot symbol to the AP, which, consequently, could determine the status of the corresponding link. Obviously, if some sensors are excluded, there is a loss of information. Therefore, selective elimination of the noisy links will lead to a performance improvement depending on the value of p, i.e., on the noise intensity in noisy links.

In order to understand *when* exclusion of noisy links leads to a performance improvement, we evaluate the probability of decision error, as a function of the bit-flipping probability p, for a given value of the sensor SNR. In Fig. 6, the probability of decision error is shown in a scenario with $N = 5$ sensors and $\mathrm{SNR}_{\mathrm{sensor}} = 12$ dB, for different values of the number of noisy links $d \geq 1$. In a scenario with $N = 3$ sensors and no noisy link ($d = 0$), from the results in Fig. 5

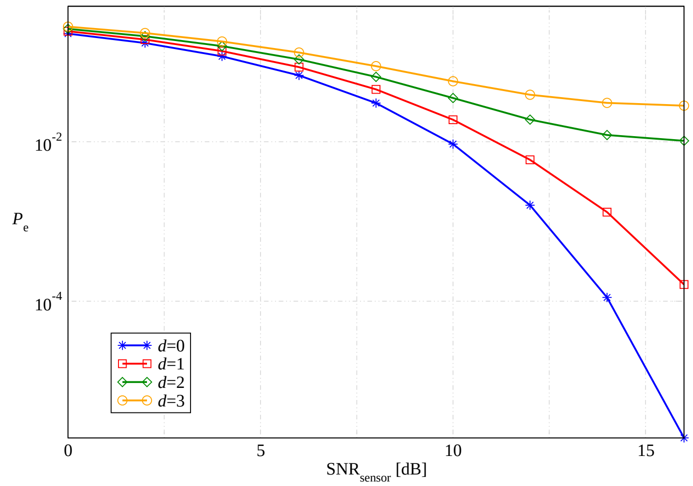

Fig. 5. Probability of error, as a function of the sensor SNR, in a scenario with $N = 3$ sensors.

one concludes that the probability of error is 1.6×10^{-3}. Considering, in Fig. 6, the curve relative to the case with $d = 2$ noisy links out of $N = 5$, it follows that $P_\mathrm{e} = 1.6 \times 10^{-3}$ corresponds to a value $p = 0.12$. Therefore, one can distinguish the following two network operating regions (depending on the value of p).

– If $p < 0.12$, the probability of decision error in a sensor network with $N = 5$ sensors and $d = 2$ noisy links is *lower* than that of a sensor network with $N = 3$ sensors and ideal links. Therefore, using the local decisions of all sensors (even if $d = 2$ communication links are noisy) is the best strategy.
– If $p > 0.12$, the probability of decision error in a sensor network with $N = 5$ sensors and $d = 2$ noisy links is *higher* than that of a sensor network with $N = 3$ sensors and ideal links. In this case, the AP should neglect the information originated by the sensors corresponding to noisy links, and use only the bits coming from the sensors with ideal links.

In a scenario with $d = 3$ noisy links, the critical value of the bit-flipping probability which discriminates between use of all sensors or selection of the subset of sensors with ideal links is (obviously) lower than the critical value in a scenario with $d = 2$ links.

In general, given a particular sensor network structure (N sensors and d noisy links), for each value of the sensor SNR it is possible to determine the critical bit-flipping probability which discriminates between (i) using all sensors or (ii) using only the subset of sensors with ideal links. In the example previously considered with $N = 5$ sensors and $d = 2$ noisy links, the critical bit flipping probability is shown, as a function of the sensor SNR, in Fig. 7. The diagram has

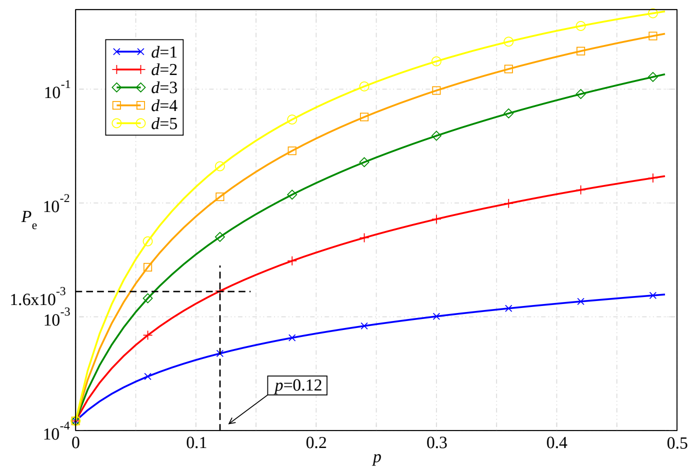

Fig. 6. Probability of error, as a function of cross-over probability p, in a sensor network with $N = 5$ sensors and $\mathrm{SNR}_{\mathrm{sensor}} = 12$ dB.

to be interpreted as follows. Given a particular sensor network scenario with a particular sensor SNR and a cross-over probability p (which will depend on the characteristics of the channel between the sensor and the AP), one can determine the $(\mathrm{SNR}_{\mathrm{sensor}}, p)$ network operating point: if this point falls above the critical curve, then the AP should neglect the sensors with noisy links; otherwise, if this point falls below the critical curve, then the AP should use all sensors. For ease of understanding, we have also indicated the critical $(\mathrm{SNR}_{\mathrm{sensor}}, p)$ operating points corresponding to the probabilities of error between 10^{-2} and 10^{-6}. For example, consider the sensor SNR corresponding to $P_{\mathrm{e}} = 10^{-3}$: if $p < 0.16$, then using all sensors will lead to a probability of error lower than 10^{-3}; for $p \geq 0.16$, the lowest possible probability of error (equal to 10^{-3}) is obtained by using only the sensors with ideal links.

Finally, we remark that the results in Fig. 6 show that the critical bit-flipping probability decreases for increasing values of the sensor SNR. In other words, whenever sensors are very sensitive (i.e., the sensor SNR is high), then the presence of even a limited link noise has a significant impact on the network performance—in fact, the best operating regime is the one corresponding to selective exclusion of the sensors with noisy links. On the constructive side, sensors which are selectively excluded could be temporarily turned off (e.g., by properly estimating the fade duration in a wireless communication scenario), prolonging the sensor network lifetime. The analysis of this network performance metric, i.e., the network lifetime, is the subject of on-going research activity.

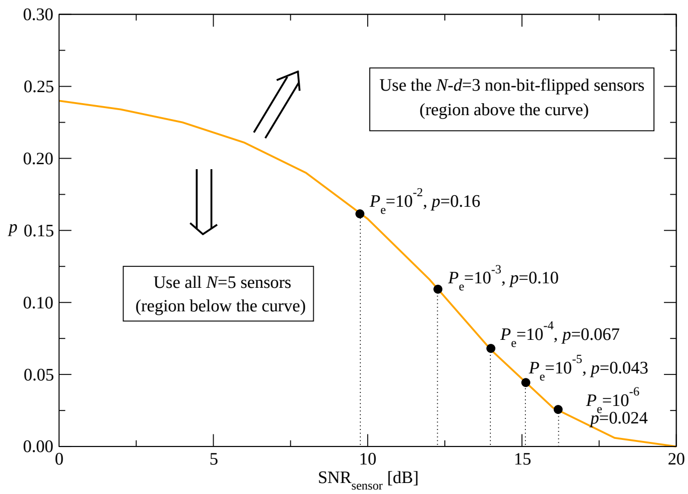

Fig. 7. Critical cross-over probability p as a function of the sensor SNR, relative to a sensor network with $N = 5$ sensors and $d = 2$ noisy links. The curve divides two regions: in the upper region the best performance is obtained by selecting only the $N - d = 3$ sensors with ideal links, whereas in the lower region the best performance is obtained using all $N = 5$ sensors.

3.4 Multiple Observations at the Sensors

In [2], it has been shown that the use of multiple consecutive and independent observations of the same phenomenon at each sensor has a beneficial effect on the performance, i.e., it reduces the probability of decision error at the AP. While in [2] multiple observations have been considered for sensor networks with *ideal* communication links, we now evaluate the effect of multiple observations in sensor networks with *noisy* communication links. After tedious manipulations (not reported for lack of space), it is possible to extend the previous analysis (carried out in a scenario with single observations at the sensors) and derive analytical expressions for the probability of decision error at the AP. More precisely, in a sensor network scenario with N sensors and d noisy communication links, given a number M of multiple observations, it is possible to evaluate the critical bit flipping probability which discriminates between (a) using all sensors and (b) discarding the sensors with noisy communication links.

In a scenario with $M = 2$ observations at each sensor, the critical bit flipping probability curve is shown in Fig. 8. In the same figure, for the sake of comparison, the critical bit flipping probability curve of Fig. 7 (relative to a scenario with $M = 1$ observation per sensor) is also shown. It is immediate to observe that the critical bit flipping probability increases when $M = 2$ observations per sensor are used (roughly speaking, it doubles). In the same figure, both analysis and simulation results are shown: as one can see, there is excellent agreement.

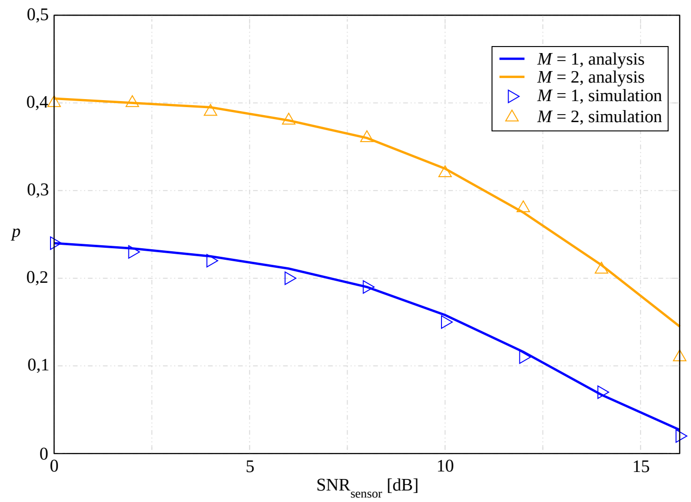

Fig. 8. Critical bit-flipping probability p as a function of the sensor SNR, relative to a sensor network with $N = 5$ sensors and $d = 2$ noisy links with $M = 1$ and $M = 2$ observations per sensor, respectively.

In order to consider a higher number of observations per sensor, for numerical reasons we have reduced the number of sensors to $N = 3$. The obtained analytical results, for various numbers of observations, are shown in Fig. 9. As expected, increasing the number of observations has a beneficial effect on the performance in terms of probability of decision error. In other words, our results suggest that use of multiple observations (which comes at the cost of (i) increased delay in the final decision and (ii) increased energy consumption at the sensors and AP) makes the sensor network more robust against impairments in the sensor-AP communication links.

From the results in Fig. 9, it is also interesting to observe that the improvement brought by the use of 2 observations instead of 1 is higher than the improvement obtained by considering 4 observations instead of 3. More precisely, since the optimal fusion rule is a majority-like rule, (i) significant performance improvements are obtained for larger *odd* values of M and (ii), considering the next even value (i.e., $M + 1$), the relative improvement becomes negligible for increasing values of M. In other words, the number of observations, if sufficiently high, should be *odd*.

4 Conclusions and Future Work

In this paper, we have considered the problem of distributed detection in sensor networks where some of the communication links between the sensors and the AP may be noisy. First, we have revisited basic principles of distributed detection with binary decisions at the sensors, discussing optimal fusion rules at the

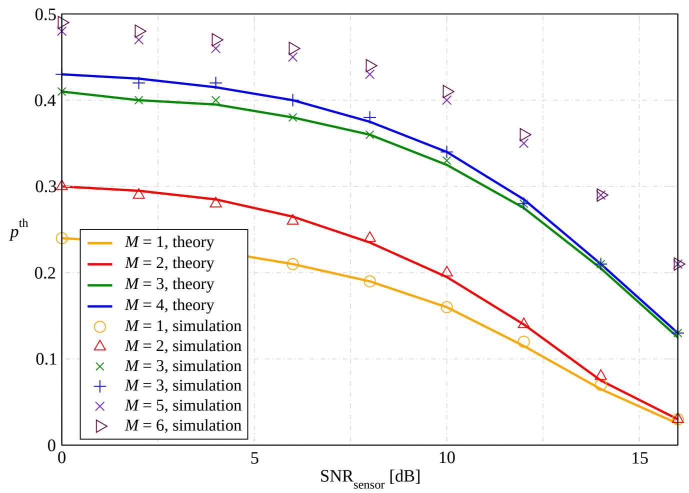

Fig. 9. Critical bit-flipping probability p as a function of the sensor SNR, relative to a sensor network with $N = 3$ sensors and $d = 2$ noisy links with multiple observations, compared to the performance of one sensor with ideal link.

AP. Then, we have introduced a simple BSC model for noisy communication links between sensors and AP, and we have analyzed the corresponding network performance, in terms of probability of decision error at the AP. For each value of the sensor SNR, we have shown the existence of a *critical bit-flipping probability*: for values of p *higher* than this critical value, network performance is optimized by discarding the decisions coming from sensors with noisy links; for values of p *lower* than this critical value, network performance is optimized by using the decisions from all sensors. Our results show that the critical bit-flipping probability is a monotonically decreasing function of the sensor SNR.

The different sensor network operating regimes, depending on the number of noisy links and the noise intensity over such links, could be forced by the use of a suitable MAC protocol (with channel sensing) at the AP and we are currently working on its design. We are also extending our approach to encompass the presence of quantization at the sensors.

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