Decentralized Binary Detection with Noisy Communication Links

This correspondence presents a Bayesian framework for distributed detection in sensor networks with noisy communication links between the sensors and the fusion center (or access point (AP)). Noisy links are modeled as binary symmetric channels (BSCs), but the proposed framework can be extended to other communication link models. To improve the system robustness against observation and communication noises, we propose schemes with 1) *multiple observations* and a single AP and 2) single observations and *multiple APs*. By using the De Moivre-Laplace approximation, we derive simple and accurate expressions for the probability of decision error in scenarios with a *large* number of nodes.

I. INTRODUCTION

Distributed detection has been an active research field for a long time [1, 2]. In particular, several approaches have been proposed to study this problem, in the realms of information theory [3], target recognition [4, 5], and several other areas. The increasing interest, over the last decade, for sensor networks, has spurred a significant research activity burst on distributed detection techniques in this context [6–9].

In recent years, wireless sensor networks are becoming more common in various application scenarios, such as, for example, terrain monitoring [10]. In a wireless communication scenario, links between sensors and the access point (AP) are likely to be faded [11, 12]. The problem of distributed detection with faulty processors is considered in [13]. In [14], the problem of decentralized detection with local quantization at the sensors in a multiple-access network with noise and interchannel interference is considered. The characteristics (in terms of capacity) of the radio multiple access channel in wireless sensor networks are taken into account in [15], where optimal configurations for decentralized detection are studied. The problem of decentralized detection with nonideal communication links, with networking delays and noise-induced errors, is considered in [16], where a Neyman-Pearson approach is used. The impact of communication constraints, e.g., limited bandwidth and presence of noise, is considered in [17], where a randomization paradigm for decentralized detection is proposed to overcome the communication bottleneck. In [18], the authors consider the problem of decentralized detection in wireless sensor networks where communication links are affected by fading. This approach is further extended in [19], where the local decision strategy in sensor networks is optimized taking into account the presence of fading, and in [20], where the authors propose a decentralized detection strategy based on censoring sensors, which transmit only when their local likelihood ratios are sufficiently large. The problem of decentralized detection in sensor networks with faded communication links is also considered in [21] and [22]. Sensor networks with censoring nodes are considered in [23] and [24], where robust and locally-optimum solutions are proposed, and measurement and transmission "costs" are taken into account for the design of a sensor network. The design of "universal" decentralized detectors for sensor networks where the communication links are bandwidth-constrained is studied in [25].

In this correspondence, we first revisit the basic principles of distributed binary detection, assuming that the sensors are independent and detect the same binary phenomenon. In order to model a scenario where some of the links between sensors and AP are nonideal, we assume that a link can be modeled as a binary symmetric channel (BSC) [16, 19, 26]. We show that selective elimination of noisy links may lead to a performance improvement when the cross-over probability of the BSC increases. We refer to this concept as *selective decentralized detection*. In order to make the system more robust against the presence of noise in the communication links, we first consider the use of multiple observations at the sensors in networks with a single AP. Various multiple observation-based system configurations, with different energy efficiencies, are considered. Then, instead of increasing the number of observations at each sensor, we consider the use of an intermediate layer of APs, between the sensor layer and the final AP—for instance, these intermediate APs could be "enhanced sensors." This leads to the concept of *multi-layer AP decentralized detection*. Based on the use of the De Moivre-Laplace approximation, we find simple and accurate analytical expressions for the probability of decision error in networks with a

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large number of sensors, both with ideal and/or noisy communication links.

II. SENSOR NETWORKS WITH IDEAL COMMUNICATION LINKS

We consider a classical sensor network scenario where all sensors are connected to a single AP [6]. We focus on binary decentralized detection, in the sense that the observed phenomenon can assume two possible values. We denote these two hypotheses as H_0 and H_1, respectively. Each sensor makes a local binary decision and transmits it to a fusion processor, i.e., the AP, which makes the final decision by applying a suitable fusion rule.

The observation at the ith sensor, at a given time instant,[1] can be expressed as

$$r_i = c_{\rm E} + n_i, \qquad i = 1,2,\ldots,N \tag{1}$$

where

$$c_{\rm E} \triangleq \begin{cases} 0 & \text{if} \quad H_0 \\ s & \text{if} \quad H_1. \end{cases}$$

Assuming that the noise samples $\{n_i\}$ are independent and identically distributed with the same Gaussian distribution $\mathcal{N}(0,\sigma^2)$, the common signal-to-noise ratio (SNR) at each sensor can be defined as follows [27]:

$$\text{SNR}_{\text{sensor}} \triangleq \frac{[\text{E}\{c_{\rm E} \mid H_1\} - \text{E}\{c_{\rm E} \mid H_0\}]^2}{\sigma^2} = \frac{s^2}{\sigma^2}. \tag{2}$$

For the sake of notational simplicity, in the remainder of this correspondence we assume that $\sigma^2 = 1$, so that $\text{SNR}_{\text{sensor}} = s^2$. We also assume that the SNR is the same at all sensors, i.e., the sensors are equivalent from an observation viewpoint.

In order to make a decision, the ith sensor compares the observation r_i with a threshold value τ_i and computes a binary decision, denoted by $u_i = U(r_i - \tau_i)$, where $U(\cdot)$ is the unit step function. It is possible to show that this decision rule is equivalent to a local likelihood ratio test [28]. In [29], in a scenario with ideal links it is shown that selecting the same threshold value τ for all sensors is an asymptotically (for large values of N) and practically (for relatively small values of N) optimal choice for minimizing the probability of incorrect decision. Motivated by this result, in the remainder of this correspondence we assume that the threshold value for local decision is the same for all sensors. In all considered scenarios, the local decision threshold τ is optimized in order to minimize the probability of decision error (more details on this issue are given in the following).

Once all sensors have made their local decisions $\{u_i\}$, the AP receives an array of N binary values, and makes a final decision u_0 according to a fusion rule $u_0 = \Gamma(u_1,\ldots,u_N)$. Under the assumption that the SNR is the same at all sensors, these fusion rules can be given the following general majority-like form [30]:

$$u_0 = \Gamma(u_1,\ldots,u_N) = \begin{cases} 1 & \text{if} \quad \sum_{i=1}^{N} u_i \geq k \\ 0 & \text{if} \quad \sum_{i=1}^{N} u_i < k \end{cases} \tag{3}$$

where k can assume a value in $\{1,\ldots,N\}$. Provided that the fusion rule is in the form given by (3) and denoting by H the true hypothesis, the key problem for system optimization consists in determining the value of k that minimizes the following probability of decision error:

$$P_{\rm e} \triangleq P(u_0 \neq H) = P(u_0 = H_0 \mid H_1)P(H_1) + P(u_0 = H_1 \mid H_0)P(H_0) \tag{4}$$

where Bayes rule is used in the second equality.

We now consider a classical scheme with ideal, i.e., error-free, communication links between the sensors and the AP. In the general case with $N \geq 2$ sensors, the two conditional probabilities at the right-hand side of (4) can be written as

$$P(u_0 = H_0 \mid H_1) = P\{\text{less than } k \text{ sensors decide for } H_1 \mid H_1\} = \sum_{i=0}^{k-1} \binom{N}{i} [1 - \Phi(\tau - s)]^i [\Phi(\tau - s)]^{N-i} \tag{5}$$

$$P(u_0 = H_1 \mid H_0) = P\{\text{at least } k \text{ sensors decide for } H_1 \mid H_0\} = \sum_{i=k}^{N} \binom{N}{i} [1 - \Phi(\tau)]^i [\Phi(\tau)]^{N-i} \tag{6}$$

where $\Phi(x) \triangleq (1/\sqrt{2\pi}) \int_{-\infty}^{x} e^{-y^2/2} \mathrm{d}y$. Therefore, the probability of decision error becomes

$$P_{\rm e} = P(H_1) \sum_{i=0}^{k-1} \binom{N}{i} [1 - \Phi(\tau - s)]^i [\Phi(\tau - s)]^{N-i} + P(H_0) \sum_{i=k}^{N} \binom{N}{i} [1 - \Phi(\tau)]^i [\Phi(\tau)]^{N-i}. \tag{7}$$

Numerically, one can show that the best fusion rule is the *majority* rule, i.e., $k = \lfloor N/2 \rfloor + 1$, and the corresponding optimal value[2] for the threshold τ is $s/2$ (regardless of the value of N) [31].

In order to investigate the behavior of sensor networks with a large number of nodes, i.e., $N \gg 1$,

[1] For ease of notational conciseness, we do not explicitly indicate the time instant of the observations. However, we are assuming that the sensors simultaneously observe the common phenomenon.

[2] More precisely, our results show that the optimal threshold at the sensors is exactly $s/2$ if N is odd. If N is even, for increasing values of N the optimized threshold converges rapidly to the case with odd N, i.e., $s/2$.

it is possible to use the De Moivre-Laplace approximation [32] to evaluate the sums of binomial terms which appear in (7). After simple calculations, one can approximate the exact probability of decision error in (7) as follows:

$$P_e \simeq P(H_1)\Phi\left(\frac{k-1-\eta_1}{\sigma_1}\right) + P(H_0)\left[1-\Phi\left(\frac{k-1-\eta_2}{\sigma_2}\right)\right] \tag{8}$$

where

$$\eta_1 \triangleq N[1-\Phi(\tau-s)]$$

$$\sigma_1 \triangleq \sqrt{N[1-\Phi(\tau-s)]\Phi(\tau-s)}$$

$$\eta_2 \triangleq N[1-\Phi(\tau)]$$

$$\sigma_2 \triangleq \sqrt{N[1-\Phi(\tau)]\Phi(\tau)}.$$

In a scenario with $k \simeq N/2$ (majority decoding at the AP) and $\tau = s/2$, recalling that $\Phi(x) \simeq 1 - (1/\sqrt{2\pi}x)\exp(-x^2/2)$, $x \gg 0$, one can further simplify (8) as

$$P_e \simeq \frac{1}{\sqrt{2\pi[\alpha(s)]^2 N}} \exp\left\{-\frac{[\alpha(s)]^2 N}{2}\right\} \tag{9}$$

where

$$\alpha(s) \triangleq \frac{\Phi\left(\frac{s}{2}\right) - \frac{1}{2}}{\sqrt{\Phi\left(\frac{s}{2}\right)\left[1-\Phi\left(\frac{s}{2}\right)\right]}}.$$

Interestingly, the approximate asymptotic (for large values of N) expression (9) *does not* depend any longer on the a priori probabilities of the phenomenon under observation, but only on the sensor SNR (through s) and the statistics of the observation noise (in this case Gaussian). Moreover, since $\lim_{s\to\infty} \alpha(s) = \infty$, one can conclude that for *any* number of sensors, the probability of decision error can be made arbitrarily small provided that the sensors are sufficiently sensitive, i.e., the sensor SNR is sufficiently high.

In Fig. 1, the probability of decision error is shown, as a function of the sensor SNR, for various values of the number of sensors N (between 2 and 151). Obviously, for increasing values of the number of sensors N, the performance improves drastically. Expression (9) also leads to the conclusion that in sensor networks with 1) a sufficiently large number of nodes and 2) sufficiently large sensor SNR, the probability of decision error in the considered decentralized detection scheme is $\Theta(e^{-N}/\sqrt{N})$, where the notation $f(n) = \Theta(g(n))$ means that there exists an n_0 such that for $n \geq n_0$, $\exists c_1 \in (0,1), c_2 > 1$ such that $c_1 g(n) \leq f(n) \leq c_2 g(n)$ [33].

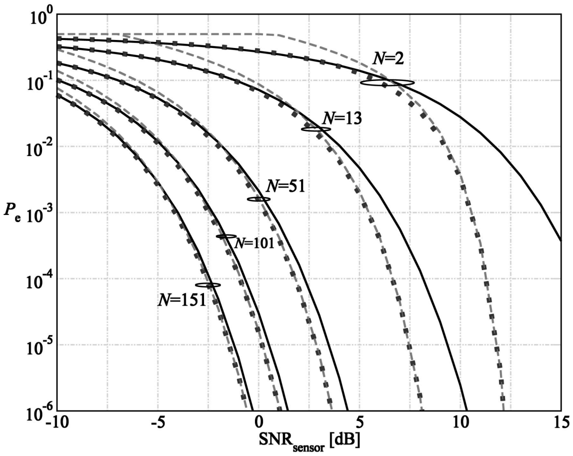

Fig. 1. Probability of decision error, as a function of the sensor SNR, for different values of N. In each case, the exact probability of decision error given by (7) (solid line), and the approximate expressions (8) (dotted line) and (9) (dashed line), are shown.

III. SENSOR NETWORKS WITH NOISY COMMUNICATION LINKS

While all previous results apply to a sensor network scenario where the communication links between sensors and AP are ideal, in a realistic scenario it might happen that these links are noisy. The analysis and optimization of wireless sensor networks is, therefore, a complicated problem. In order to derive significant insights into this problem, we model a noisy link between a sensor and the AP as a BSC with cross-over probability[3] p [16, 34]. According to the BSC model for a communication link, a bit transmitted by the sensor has a probability p of being "flipped." We now extend the derivation of the probability of decision error proposed in Section II in order to encompass the possible presence of bit-flipping. More precisely, we want to evaluate the final probability of decision error (4) in a sensor network with noisy communication links. We consider a majority-like fusion rule at the AP as described in Section II, with optimized values of k and τ. This does not necessarily mean that a k-over-N rule is the "optimal" fusion rule for this problem, i.e., decentralized detection in sensor networks with noisy communication links. In fact, a better detection strategy could consist in weighing sensor decisions according to the quality of the corresponding links, as considered in [18] and [19].

We consider a general scenario with a number $d \leq N$ of noisy links and, consequently, $N - d$ ideal links. The fusion rule is the majority-like rule given in (3), with optimized value of k. As observed in Section II, in a scenario with ideal communication

[3]We remark that the sensor SNR, i.e., $\text{SNR}_{\text{sensor}} = s^2$, is the SNR at each sensor relative to the local detection of the common phenomenon (or state of nature). Each communication link between a sensor and the AP can be characterized by an SNR at the AP.

TABLE I
Analytic Expressions of $P(u_0 = H_1 \mid H_0)$ in the Following Four Cases: (a) $d \geq k$, $N-d \geq k$, (b) $d \geq k$, $N-d < k$, (c) $d < k$, $N-d < k$ and (d) $d < k$, $N-d \geq k$

Case	$P(u_0 = H_1 \mid H_0)$
(a)	$\sum_{d_e=0}^{k}\left[\binom{d}{d_e}P_{e_2}^{d_e}P_{c_2}^{d-d_e}\sum_{i_e=k-d_e}^{N-d}\binom{N-d}{i_e}P_{eH_0}^{i_e}P_{cH_0}^{N-d-i_e}\right]+\sum_{d_e=k+1}^{d}\binom{d}{d_e}P_{e_2}^{d_e}P_{c_2}^{d-d_e}\cdot U(d-k-1)$
(b)	$\sum_{d_e=k+d-N}^{k}\left[\binom{d}{d_e}P_{e_2}^{d_e}P_{c_2}^{d-d_e}\sum_{i_e=k-d_e}^{N-d}\binom{N-d}{i_e}P_{eH_0}^{i_e}P_{cH_0}^{N-d-i_e}\right]+\sum_{d_e=k+1}^{d}\binom{d}{d_e}P_{e_2}^{d_e}P_{c_2}^{d-d_e}\cdot U(d-k-1)$
(c)	$\sum_{d_e=k+d-N}^{d}\left[\binom{d}{d_e}P_{e_2}^{d_e}P_{c_2}^{d-d_e}\sum_{i_e=k-d_e}^{N-d}\binom{N-d}{i_e}P_{eH_0}^{i_e}P_{cH_0}^{N-d-i_e}\right]$
(d)	$\sum_{d_e=0}^{d}\left[\binom{d}{d_e}P_{e_2}^{d_e}P_{c_2}^{d-d_e}\sum_{i_e=k-d_e}^{N-d}\binom{N-d}{i_e}P_{eH_0}^{i_e}P_{cH_0}^{N-d-i_e}\right]$

links the local sensor threshold depends, in general, on the number of sensors N, the fusion rule (i.e., the value of k), and the sensor SNR. In a scenario with noisy links, the optimized value of the threshold should depend also on 1) the number d of noisy links and 2) the intensity of the noise, i.e., the value of the cross-over probability p. We point out that using the same threshold value τ at all sensors might not be the best choice in a scenario with noisy communication links. In this case, a *joint* optimization of local decision thresholds and fusion rule at the AP should be carried out.

In order to evaluate the probability of decision error, we now compute the second term at the right-hand side of (4), i.e., $P(u_0 = H_1 \mid H_0)$. Let us denote by $d_e \leq d$ the number of erroneous *received* decisions[4] at the end of noisy links and by $i_e \leq N-d$ the number of ideal links associated with sensors in error (the received decisions from these sensors are, obviously, erroneous). With these definitions, the AP makes a final erroneous decision if $d_e + i_e \geq k$, with $d_e \in \{0,\ldots,d\}$ and $i_e \in \{0,\ldots,N-d\}$. Depending on the relations between the integers N, k, and d, one can distinguish the following four cases, respectively: (a) $d \geq k$, $N-d \geq k$; (b) $d \geq k$, $N-d < k$; (c) $d < k$, $N-d < k$; and (d) $d < k$, $N-d \geq k$. The expressions for $P(u_0 = H_1 \mid H_0)$ in the four considered cases are shown in Table I, where $P_{eH_0} \triangleq P(u_0 = H_1 \mid H_0, p=0) = 1-\Phi(\tau)$ and $P_{cH_0} \triangleq 1 - P_{eH_0}$. Similar expressions can be derived also for the first term at the right-hand side of (4), i.e., $P(u_0 = H_0 \mid H_1)$ [35]. We point out that the final expressions for the probability of decision error based on the results in Table I can be simplified by applying the De Moivre-Laplace approximation, as considered in Section II in a scenario with ideal links. In the following, we consider the application of the De Moivre-Laplace approximation in a scenario with large sensor SNR.

For high values of d (when $d \geq k = 3$), it can be shown that the probability of decision error flattens, i.e., there is a floor for increasing values of the sensor SNR [35]. This is due to the fact that the quality of the noisy communication links, described by p, is independent of the observation phase at the sensors and, therefore, of the sensor SNR. Thus, if $d \geq k$, even if the sensors decide perfectly (this would happen for very large values of the sensor SNR), there is a non-zero probability that a number (larger than k) of decisions received by the AP are wrong (because of bit-flipping in the noisy links). The limiting probability of decision error, i.e., the floor, corresponds exactly to this non-zero probability, which, in the case with $d \geq k$, has the following expression:

$$
\begin{aligned}
P_{e-lb}(p,d) &= \lim_{s\to\infty} P_e \\
&= P\{\text{at most } k-1-N+d \text{ noisy link transmissions are not bit-flipped } \mid H_1\}P(H_1) \\
&\quad + P\{\text{at least } k \text{ noisy link transmissions are bit-flipped } \mid H_0\}P(H_0) \\
&= P(H_1)\sum_{i=0}^{k-1-N+d}\binom{d}{i}(1-p)^i p^{d-i} \\
&\quad + P(H_0)\sum_{i=k}^{d}\binom{d}{i}p^i(1-p)^{d-i}. \qquad (10)
\end{aligned}
$$

A particular case of (10) is obtained when $d = N$, i.e., all links are noisy. As considered at the end of Section II in a scenario with all ideal communication

[4]The reader should observe that we refer to *received* decisions, rather than *sensor* decisions. In fact, while in a scenario with ideal links the decisions made by the sensors arrive correctly at the AP, in a scenario with noisy communication links the decisions received by the AP might be different from those transmitted by the sensors, because of the errors introduced by the noise present in the communication links.

links, the probability of decision error in a scenario with all noisy communication links and a large number of nodes ($N \gg 1$) can be approximated, through the application of the De Moivre-Laplace approximation, as follows:

$$P_e \simeq P(H_1)\Phi\left(\frac{k-1-N(1-p)}{\sqrt{Np(1-p)}}\right) + P(H_0)\left[1-\Phi\left(\frac{k-Np}{\sqrt{Np(1-p)}}\right)\right]. \quad (11)$$

As considered in Section II, expression (11) can be further simplified. Provided that $p < 1/2$ and recalling that $\Phi(x) \simeq 1-(1/\sqrt{2\pi}x)\exp(-x^2/2)$, $x \gg 0$, after a few manipulations from (11) one obtains

$$P_e \simeq \frac{\sqrt{p(1-p)}}{\sqrt{2\pi}(\frac{1}{2}-p)\sqrt{N}}\exp\left\{-\frac{(\frac{1}{2}-p)^2N}{2p(1-p)}\right\}. \quad (12)$$

As observed in a scenario with ideal communication links, in this case as well the asymptotic (for large number of sensors) probability of decision error *does not* depend on the a priori probabilities of the phenomenon and is $\Theta(e^{-N}/\sqrt{N})$ (as in the scenario with ideal communication links). However, unlike the scenario with ideal communication links, expression (12) corresponds to a scenario with very large sensor SNR. In fact, as expected, even if the sensors are made infinitely sensitive ($\mathrm{SNR}_{\mathrm{sensor}} \to \infty$), the ultimate performance is dictated by the noise level in the communication links, i.e., p. Note that for $p \to 0.5$, expression (12) diverges. Therefore, it is expected that it will be valid for sufficiently low values of p—this is, on the other hand, the operative regime for a practical sensor network.

In Fig. 2, the asymptotic ($\mathrm{SNR}_{\mathrm{sensor}} \to \infty$) probability of decision error (10) is shown, together with its approximate expressions (11) and (12), for various values of the number N of sensors (5, 31, and 151, respectively). As observed in a scenario with ideal links, the approximate expression (11) is a lower bound for the exact probability of decision error, whereas the approximate expression (12) is valid only for sufficiently low values of P_e. As one can see, the approximate expression (12) for the probability of decision error becomes very accurate for increasing values of N.

IV. SELECTIVE DECENTRALIZED DETECTION

In this section, our goal is to characterize the considered decentralized detection schemes in terms of their robustness against the noise. Since two sources of noise affect the system, i.e., *observation* and *communication*, our goal is to study the relative impacts of these two types of noise. The key concept for evaluating the impact of these noises and quantify the network robustness against them is the concept

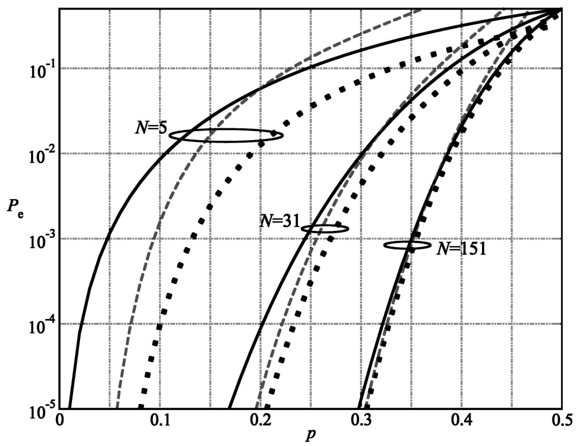

Fig. 2. Probability of decision error, as a function of p, in a scenario with infinite sensor SNR. Various values of the number N of sensors are considered and all links are noisy, i.e., $d = N$. For any number of sensors, three curves are shown: the exact expression (10) (solid line), the approximate expression (11) (dotted line) and the approximate expression (12) (dashed line).

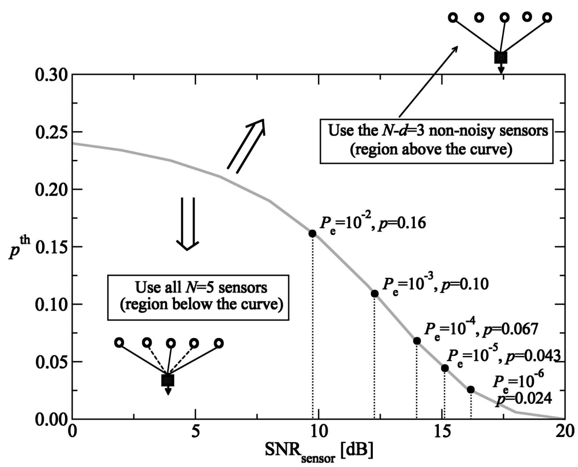

Fig. 3. Critical cross-over probability pth as a function of the sensor SNR, relative to a sensor network with $N = 5$ sensors and $d = 2$ noisy links. The curve divides two regions: in the region above the critical cross-over probability curve the best performance is obtained by selecting only the $N - d = 3$ sensors with ideal links (solid lines in the sketched 5-sensor network), whereas in the region below the critical cross-over probability curve the sensor network operates at its best using all $N = 5$ sensors (with ideal and noisy links). The two network configurations are shown on the side for ease of understanding.

of selective decentralized detection, described in the following.

In general, given a particular sensor network structure (N sensors and d noisy links), for each value of the sensor SNR it is possible to determine the *critical* cross-over probability pth which discriminates between 1) using all sensors or 2) using only the subset of sensors with ideal links. In an illustrative scenario with $N = 5$ sensors and $d = 2$ noisy links, the critical cross-over probability is shown, as a function of the sensor SNR, in Fig. 3. The results in Fig. 3 have to be interpreted as follows. Given a sensor network scenario with specific values of sensor SNR and p (which will depend on the channel between the sensor and the AP), one can determine the $(\mathrm{SNR}_{\mathrm{sensor}}, p)$ network operating point: if this point falls above the critical cross-over probability curve, then the AP should neglect the sensors with noisy links; otherwise, i.e., if this point falls below the critical cross-over probability curve, the AP should use all sensors. For ease of understanding, we have also indicated the critical $(\mathrm{SNR}_{\mathrm{sensor}}, p)$ operating points corresponding to probabilities of error between 10^{-2} and 10^{-6}.

We underline that no specific *operative strategy* is suggested here to perform selective decentralized detection. More precisely, in order to do this, the AP should recover the statuses of the communication links . The proposed selective decentralized detection approach could be applicable in scenarios where the links become noisy for a priori known events (e.g., regular link obstruction during parts of the day). An interesting research direction, however, consists in extending our framework to design schemes where the AP does not need any information on the statuses of the communication links. This extension could lead to sensor networks with an *adaptive* structure.

Finally, we remark that the results in Fig. 3 show that the critical cross-over probability decreases for increasing values of the sensor SNR. In other words, whenever sensors are very sensitive (i.e., the sensor SNR is high), then even the presence of a limited noise in the communication links has a significant impact on the sensor network performance—in fact, the best operating regime is the one corresponding to selective elimination of the sensors with noisy links. On the constructive side, sensors which are selectively excluded could be temporarily turned off, prolonging the network lifetime. For example, selective exclusion of sensors with noisy links could be obtained by using a clever medium access control (MAC) protocol at the AP [36, 37].

A. Multiple Observations at the Sensors

In Section III, we have proposed an analytical framework to characterize the performance, in terms of probability of decision error at the AP, in sensor networks with an arbitrary number of noisy links (modeled as BSCs). We now extend our model in order to encompass the possibility that each sensor makes more than one observation of the same phenomenon [38, 39]—note that this is equivalent to binary integration in radar systems [40, p. 499].

At some point in time, the sensors make an observation of a common phenomenon, and transmit their binary decisions to the AP, through communication links which can be either ideal or noisy. The AP makes a *preliminary* binary decision $u_{0,1}$ based on the received N binary decisions. At

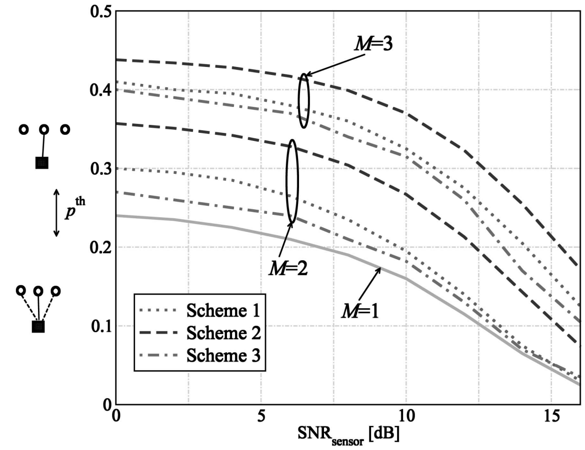

Fig. 4. Critical cross-over probability pth as a function of the sensor SNR, in a sensor network with $N = 3$ sensors and $d = 2$ noisy links. Two sets of curves, relative to various multiple observations schemes and associated with scenarios with $M = 2$ and $M = 3$ observations, are shown. For comparison, the curve associated with a scenario with $M = 1$ observation is also shown.

this point, the sensors make another observation of the same phenomenon (independent of the previous observations), and send their new decisions to the AP. After M sets of N received decisions, the AP has generated a set of M preliminary decisions $\{u_{0,1}, \ldots, u_{0,M}\}$. The final decision u_0 is obtained by applying a k'-over-M decision rule over the M preliminary decisions. It is then possible to extend the previous analysis (carried out in a scenario with single observations at the sensors) and derive analytical expressions for the probability of decision error at the AP [35].

In Fig. 4, the critical cross-over probability in a scenario with $N = 3$ sensors and $d = 2$ noisy links (i.e., the critical cross-over probability which discriminates between using all 3 sensors or only the sensor with an ideal communication link, as depicted on the side of Fig. 4) is shown, as a function of the sensor SNR, for various values of the number of observations. More precisely, besides the reference curve (solid line) corresponding to a single observation ($M = 1$), there are 2 sets of curves, associated with $M = 2$ and $M = 3$ observations, respectively. Each set is composed by 3 curves, referred to as “scheme 1” (dotted line), “scheme 2” (dashed line), and “scheme 3” (dot-dashed line), which are characterized as follows.

1) Scheme 1 refers to the multiple observation scheme introduced at the beginning of this subsection.

2) In scheme 2, each sensor makes M consecutive and independent decisions of the same phenomenon and sends these decisions, through M consecutive transmission acts, to the AP. Rather than making preliminary decisions based on the consecutive sets of N single observations, the AP collects all multiple decisions from all sensors, i.e., $N \times M$

decisions, and then makes a single majority decision. Since consecutive observations at each sensor are independent, this scheme is equivalent to a sensor network with $N \times M$ sensors, $d \times M$ noisy communication links, and single observations.

3) In scheme 3, each sensor makes M consecutive and independent decisions of the same phenomenon. Unlike scheme 1, each sensor fuses *locally* its M consecutive decisions, and sends the final decision to the AP. The AP then makes a final majority decision over the N received decisions.

As expected, the results in Fig. 4 show that the highest robustness is guaranteed by scheme 2 (this is intuitive, since in this case a single fusion operation is carried out and no information is lost in other preliminary fusion operations). The use of scheme 3, however, guarantees a performance slightly worse than that of scheme 1. Since scheme 3 is more energy efficient than scheme 1 (only 1 transmission act per sensor, rather than M, is required), this makes scheme 3 very attractive from an implementation viewpoint. Our results also show that considering more than 3 observations leads to minor (in relative terms) improvements [41].

B. Multi-Layer AP Detection

We now consider a new scheme to perform decentralized detection in a sensor network with noisy communication links. More precisely, we assume that there is an intermediate layer of L APs between the N sensors and the final fusion processor, i.e., the final AP. In particular, each sensor is connected to all intermediate APs, which are then connected to the final fusion processor. Assuming that part of the links between the sensors and the intermediate APs are noisy, it is possible to extend the previous analytical approach to this scenario, in order to compute the probability of decision error at the final AP.[5] Each intermediate AP makes a preliminary decision (according to a majority rule), and then all intermediate APs send their decisions to the fusion processor, which makes the final decision. We refer to this decentralized detection scheme as *multi-layer AP detection scheme*.

In order to compare the performance of a multi-layer AP detection scheme with that of a standard sensor network (with a single AP), we assume that all communication links from the sensors are noisy. Obviously, each intermediate AP in a multi-layer scheme has the same probability of decision error than the single AP in a standard scenario. It is reasonable to expect that the performance of a multi-layer AP scheme will be better than that of a regular scheme with a single observation at each sensor. Therefore, it is interesting to compare the performance of a multi-layer AP scheme with that of a standard (single AP) scheme with multiple observations.

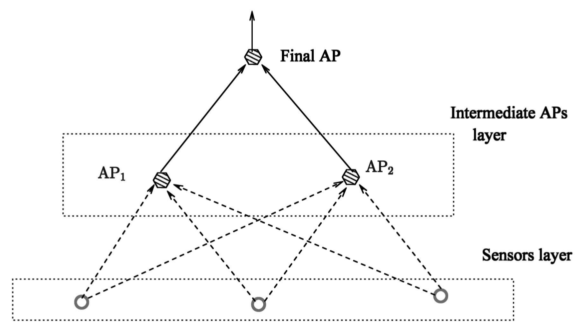

Fig. 5. An example of multi-layer AP detection scheme, with 3 intermediate APs, and a single final AP.

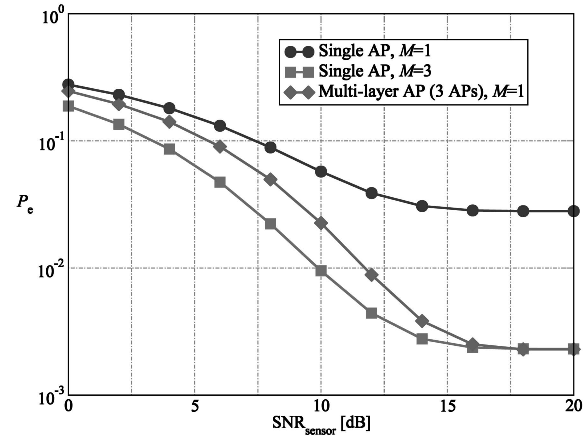

Fig. 6. Probability of decision error, as a function of the sensor SNR, in a network with $N = 3$ sensors and all noisy links from the sensors ($p = 0.1$). A multi-layer AP scheme with $L = 3$ intermediate APs and a standard scheme with 1 AP are considered. In the multi-layer AP scheme only the case with $M = 1$ observation per sensor is considered, whereas in the standard (single AP) scheme both cases with $M = 1$ observation and $M = 3$ observations are considered.

We consider the multi-layer AP scenario shown in Fig. 5, where there are $N = 3$ sensors, each of which is connected to the $L = 2$ APs through noisy communication links. First, all $N = 3$ sensors make binary decisions based on $M = 1$ observation of the common phenomenon. Afterwards, each of them transmits its decision to the $L = 2$ intermediate APs. The intermediate APs decide, based on a majority fusion rule, and send their decisions to the fusion processor, which, in turns, performs a final majority decision. The probability of decision error at the final AP, as a function of the sensor SNR, is shown in Fig. 6. In the same figure, for comparison, the performance of a standard scheme, in the cases with $M = 1$ observation and $M = 3$ observations is also shown. It can be observed that the multi-layer AP scheme offers a performance better than that of the

[5]We are implicitly assuming that the links between the intermediate APs and the final AP are ideal.

standard scheme with $M = 1$ observation (single AP). However, the probability of decision error of a standard scheme with $M = 3$ observations is lower than that of a multi-layer AP scheme for low values of the sensor SNR, while it converges to the same value for increasing sensor SNR.

As expected, the multi-layer AP scheme guarantees a better performance than a standard scheme with the same number of sensors and $M = 1$ observation per sensor. This comes at the price of the introduction of $L = 2$ new APs. However, the asymptotic probability of decision error becomes the same if the number of observations per sensor in a standard scheme is equal to the number of intermediate APs of a multi-layer AP scheme. From an energy consumption viewpoint, while in a standard scheme with $M = 3$ observations per sensor, a total of 9 sensor transmission acts are required (3 per sensor), in a multi-layer AP scheme only 3 sensor transmission acts are required (assuming that a signal transmitted by a sensor is simultaneously received by the L intermediate APs) and 2 transmission acts from the APs. Therefore, use of multi-layer AP architectures could lead to lower energy consumption at the sensors, i.e., it could prolong the sensor network lifetime.

As a final comment, we point out that in our analysis, the dependence between the sensors has been neglected, in order to keep the derivation tractable. Should the sensors be dependent, the overall decentralized detection scheme should be properly optimized taking into account this characteristic.

C. Phenomena with Unequal A Priori Probabilities

In order to investigate the impact of unequal a priori probabilities in a scenario with noisy communication links, we consider a standard scheme (single AP) with $N = 5$ sensors and $d = 2$ noisy communication links, in the case where $P(H_0) = 10P(H_1)$. The obtained results are shown in Fig. 7. In a scenario with unequal a priori probabilities of the phenomenon, there is an "optimal" sensor SNR which maximizes the critical cross-over probability. Note that this does not mean that the maximizing sensor SNR minimizes the probability of decision error. In fact, the maximizing sensor SNR is such that the robustness of the sensor network when all N sensors are used (rather than the $N - d$ with ideal links) is the highest possible. In other words, if the sensor network cannot make use of selective decentralized detection techniques and there are unequal a priori probabilities for the phenomenon under observation, then the sensor SNR should be carefully tuned in order to maximize the system robustness against noisy communication links.

In the example above, where the a priori probabilities are unbalanced, i.e., $P(H_0) \gg P(H_1)$, the event H_1 is highly unlikely. Therefore, it is

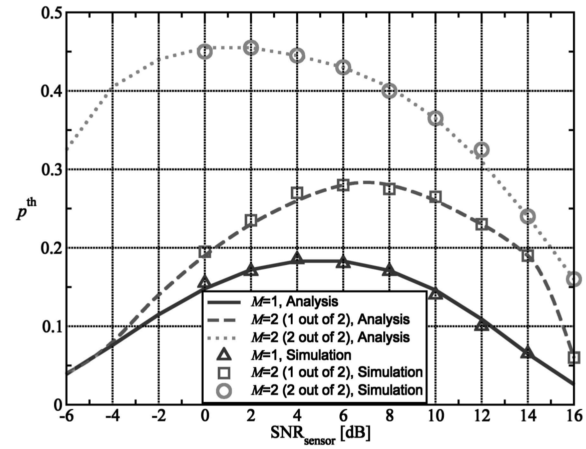

Fig. 7. Critical cross-over probability p as a function of the sensor SNR, in a sensor network with $N = 5$ sensors and $d = 2$ noisy links, with $M = 1$ observation and $M = 2$ observations, respectively. The a priori probabilities are $P(H_0) = 1/11 \simeq 0.09$ and $P(H_1) = 10/11 \simeq 0.91$. Note that in the case with $M = 2$ observations, two possible fusion rules for the preliminary decisions at the AP are considered: 1 out of 2 and 2 out of 2.

intuitive that the probability of false alarm (given by $P(u_0 = H_1 \mid H_0)$) is relatively high. In a scenario with $M = 2$ observations, one could assume that the AP decides for H_1 only if *both* preliminary decisions are in favor of H_1. The corresponding critical cross-over probability, given by the curves in Fig. 7 labeled as "$M = 2$ (2 out of 2)," is significantly higher than that in the case with $M = 2$ observations and fusion rule "1 out of 2" at the AP. This confirms our intuition. More generally, the obtained results show that the optimized fusion rule at the AP should also depend on the a priori probabilities of the phenomenon under observation.

V. CONCLUSIONS

In this correspondence, we have proposed a general framework for decentralized detection in sensor networks where the communication links between the sensors and the AP may be noisy. First, we have revisited basic principles of distributed detection with binary decisions at the sensors. Then, we have introduced a simple BSC model for noisy communication links between sensors and AP, and we have analyzed the corresponding network performance, in terms of probability of decision error at the AP. We have shown that there exists a critical noise level, in the communication links, beyond which the system performance is optimized by excluding the sensors with noisy links. By suitable application of the De Moivre-Laplace approximation, we have derived simple and accurate expressions for the probability of decision error in networks with a large number of sensors, in scenarios with ideal and/or noisy communication links. Possible techniques to improve the system robustness against communication noise have been proposed: 1) use of *multiple observations*, and 2) use of a *multi-layer AP* detection scheme, where a supplementary layer of intermediate APs is used between the sensors and the final fusion processor. Our results also show that the fusion rule should be optimized taking into account the statistics of the observed phenomenon.

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