Joint Decoding of TCM and Detection of PTS Side Information: a Multiple Trellis Solution

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Abstract—One of the major drawbacks of orthogonal frequency-division multiplexing (OFDM) systems is the large Peak-to-Average power ratio (PAR) of the transmit signal. High levels of PAR may be a limiting factor for power line communication (PLC) where regulatory bodies have fixed the maximum amount of transmit power.

Partial transmit sequence (PTS) is a well-known solution for PAR reduction, but its effectiveness depends on an accurate, and possibly error-free, exchange of side information (SI) between the transmitter and the receiver. In this article, a new maximum likelihood (ML) algorithm for joint decoding of trellis code modulation (TCM) and detection of PTS is presented. The proposed algorithm exploits the redundancy introduced by the TCM code to estimate the correct PTS, and represents a practical solution to avoid the explicit insertion of SI into the OFDM signal. The proposed algorithm has been assessed by bit-error rate (BER) simulation, showing a good performance over additive white Gaussian noise (AWGN) channels and amplifier nonlinearities.

Keywords—Orthogonal frequency-division multiplexing (OFDM), partial transmit sequences (PTS), peak-to-average power ratio (PAR), joint maximum likelihood sequence detection and decoding, side information.

I. INTRODUCTION

Orthogonal frequency-division multiplexing (OFDM) has been widely recognized as an effective modulation scheme for high speed data transmission over frequency selective channels [1]. One of the major drawbacks of OFDM is that the peak transmitted power can be substantially larger than the average power. As a consequence, the high OFDM peak-to-average power ratio (PAR) poses challenges for power amplification in the transmitter, especially in a scenario like power line communication (PLC) where regulatory bodies have fixed the maximum amount of transmit power [2]. In order to cope with the high PAR, nonlinear amplifiers must operate with very high power backoffs to avoid significant out-of-band radiation and intermodulation among OFDM carriers, leading to power-inefficient amplification.

In the literature, many PAR reduction schemes have been studied, such as block coding [3]-[5], clipping [6]-[8], trellis shaping [9], [10] and probabilistic techniques [11]-[14]. Selected mapping (SLM) [11] and partial transmit sequences (PTSs) [12], which belong to the probabilistic class, reduce the PAR by generating a multiple signal representation, i.e., determining $U > 1$ different OFDM symbols for a given data frame and transmitting the $\tilde{u}$-th symbol with the lowest peak power. The value of $\tilde{u}$ (Side Information) is required at the receiver side to correctly recover the signal. Since almost $\log_2(U)$ bits are required to represent this information, many practical solutions have been proposed to send the SI to the receiver. Almost all the proposed techniques introduce a small amount of redundancy at the transmitter and send the SI through this redundancy, at the cost of decreasing the effective bit-rate.

In this article, a new algorithm for joint decoding of trellis code modulation (TCM) and detection of PTS SI is presented. Our approach is very general and can also be applied to SLM. The proposed algorithm exploits the redundancy introduced by the TCM code in order to estimate the correct PTS, and represents a practical solution to avoid the explicit insertion of SI into the OFDM signal. Note that the proposed solution is different from the one proposed in [15]: in particular, [15] exploits the fact that the set of uncoded $U$ symbols may have large Hamming distances and uses this inherent diversity, along with hard decision, to derive a practical decoder. On the other hand, our approach exploits the redundancy introduced by a very commonly used channel code, such as TCM, along with soft-decision processing, to jointly detect the transmitted information sequence and the correct multiple signal representation used to reduce the PAR. Moreover, note that the proposed algorithm has a very general formulation: as a consequence, it can be straightforwardly applied to a wide class of convolutional codes. Simulation results have confirmed that the proposed solution presents good performance over a variety of PLC channels and amplifier nonlinearities.

II. PAR DEFINITION FOR OFDM SIGNALS

An OFDM system transmits an information symbol formed by a block of $N$ modulation symbols over $N$ orthogonal subcarrier. The complex envelope of the OFDM signal may be expressed as

$$x(t) = \sum_{i=-\infty}^{+\infty} \sum_{n=0}^{N+N_g-1} b_i[n] p(t - nT - iN_T T)$$

where $\{b_i[n]\}$ defines the sequence of modulation symbols (cyclically extended by $N_g$ samples), i.e.,

$$b_i[n] = \frac{1}{N} \sum_{k=0}^{N-1} c_i[k] e^{j2\pi k(n-N_g)/N} \quad n = 0, 1, \ldots, N_T - 1$$
with \( \{c_i[k]\} \) the code sequence belonging to a QAM constellation, \( p(t) \) a shaping pulse with square-root raised cosine Fourier transform with roll-off \( \alpha \), \( N_T = N + N_g \) the total number of samples transmitted into each OFDM symbol, \( T = 1/(N\Delta f) \) with \( \Delta f = 1/T_s \), the adjacent subcarrier separation and \( T_s \) the OFDM symbol duration without cyclic extension. The PAR of the continuous-time OFDM signal may be defined as [16]

\[
\gamma[x(t)] = \frac{\max \{ |x(t)|^2 \}}{\sigma_x^2}
\]

where \( \sigma_x^2 = \mathbb{E}\{ |x(t)|^2 \} \) denotes the average power of the OFDM signal. In this paper, we consider PAR reduction based on an oversampled discrete-time OFDM signal. In [17] it was shown that a good approximation of the continuous PAR may be achieved using discrete-time PAR with oversampling. In this paper, the discrete-time OFDM signal is evaluated assuming an oversampling factor \( \beta = 8 \). The quality of PAR reduction schemes is often evaluated based on the complementary cumulative distribution function (CCDF) of the PAR, i.e., \( P\{\gamma[x(t)] > \gamma_{th}\} \).

### III. PTS Algorithm

In the PTS algorithm [12], the data vector \( c_i \) is partitioned into \( V \) disjoint subblocks of \( N/V \) code symbols \( c_i^{(v)}, v = 0, \ldots, V - 1 \). Symbols in \( c_i^{(v)} \) which are already represented in another subblock are set to zero, i.e., \( c_i = \sum_{v=0}^{V-1} c_i^{(v)} \). Each block is then rotated by a phase angle \( \varphi_i^{(v)} \), belonging to an alphabet of cardinality \( W \), to obtain a multiple signal representation \( \tilde{c}_i = \sum_{v=0}^{V-1} \varphi_i^{(v)} c_i^{(v)} \). Phase angles of carriers in each subblock are iteratively changed in order to find the optimal phase sequence which minimizes the discrete-time \( T \)-spaced PAR. Denoting with \( b_i^{(v)} \) the so-called partial transmit sequence, i.e., the vector at the output of the inverse discrete Fourier transform (IDFT) \( b_i^{(v)} = \text{IDFT}\{c_i^{(v)}\} \) and exploiting the linearity of the IDFT, the optimum phase vector \( \varphi_i \) for the \( i \)-th OFDM symbol can be found according to [12]

\[
\varphi_i = \arg\min_{\varphi_i} \left( \max_{0 \leq n < N} \left| \sum_{v=0}^{V-1} \varphi_i^{(v)} \cdot b_i^{(v)}[n] \right| \right)
\]

from which the sequence of modulation symbols with the minimum discrete-time PAR can be obtained

\[
\tilde{b}_i = \sum_{v=0}^{V-1} \varphi_i^{(v)} \cdot b_i^{(v)}.
\]

For coherent detection, the optimized phase sequence \( \{\varphi_i^{(v)}\}_{i=0}^{V-1} \) is required at the receiver. Finally, note that without any performance loss, the first rotation factor may be fixed, i.e., \( \varphi_i^{(0)} = 1 \), leaving the first subblock unrotated and only \( V - 1 \) rotation angles to be optimized.

### IV. Joint Decoding of TCM and Detection of PTS Side Information

In this section, a new algorithm for joint decoding of trellis code modulation (TCM) and detection of PTS SI is presented. From now on, we assume a perfect synchronization between the transmitter and the receiver. The \( N/V \) different discrete-time observables at the output of the demodulation block, performed at the receiver via a direct discrete Fourier transform (DFT), belonging to the \( v \)-th subblock and \( m \)-th OFDM received symbol, may be written as

\[
R_m[k] = H_v[k]c_m[k]\varphi_m^{(v)} + N_m[k]
\]

for \( vN/V \leq k \leq (v+1)N/V \) and \( v = 0, 1, \ldots, V - 1 \). In (1), \( H_v[k] \) represents the channel frequency response evaluated at the carrier frequency \( k\Delta f \) whereas \( N_m[k] \) denotes a thermal noise realization over the \( k \)-th carrier and \( m \)-th OFDM symbol with variance \( \sigma_n^2 \). Note that every group of \( N/V \) code symbols \( c_m[k] \) have been rotated by the same coefficient \( \varphi_m^{(v)} \) introduced by the PTS algorithm.

Collecting the \( N \) observables associated with the \( m \)-th OFDM symbol into a suitable vector \( R_m \) and defining \( R = (R_0, \ldots, R_{K-1})^T \) and \( \varphi = (\varphi_0, \ldots, \varphi_{K-1})^T \), as, respectively, the vector collecting all the observables associated with a transmission of \( K \) OFDM symbols and the vector of phase sequences introduced by the PAR-minimization algorithm, the “generalized likelihood” strategy [18] can be formalized as

\[
\hat{a} = \arg\max_a \left( \max_{\varphi} p(R; \varphi|a)P(a) \right)
\]

where \( p(R; \varphi|a) \) denotes the probability density function (PDF) of the observable, indexed by the deterministic but unknown vector \( \varphi \), and \( a \) denotes the information sequence with a-priori probability \( P(a) \). Since \( a \) and \( \varphi \) are discrete variables, it is possible to exchange the order of maximization in (2). As a consequence, one can express (2) as

\[
\hat{a} = \max_a \hat{a}(\varphi)
\]

where \( \hat{a}(\varphi) \) denotes the maximum a-posteriori probability (MAP) information sequence under the hypothesis of a given vector \( \varphi \) chosen among all the possible vectors \( \varphi \), i.e.,

\[
\hat{a}(\varphi) = \arg\max_a p(R; \varphi|a)P(a).
\]

Applying the chain factorization rule to the conditional PDF, assuming that the information symbols are independent and uniformly distributed and discarding irrelevant terms, Eq. (4) may be written as

\[
\hat{a}(\varphi) = \arg\max_a \prod_{m=0}^{K-1} p(R_m; \varphi_m|a_m^N) \prod_{m=0}^{K-1} p(R_m; \varphi_m|a_m^N)
\]

(5)

\[
= \arg\max_a \prod_{m=0}^{K-1} p(R_m; \varphi_m|a_m^N)
\]

(6)

\[
= \arg\max_a \prod_{m=0}^{K-1} \prod_{v=0}^{V-1} p(R_m; \varphi_m^{(v)}|a_m^N)
\]

(7)
where in (5) the absence of intersymbol interference (ISI) allows one to conclude that, given the information sequence, the samples belonging to the \( m \)-th OFDM symbol \( R_m \) are independent from the past realization \( R_{m-1}^m \); in (6) a factorization with respect to the \( V \) subblocks of carriers with the same phase coefficient has been carried out and the vectors
\[
R_{m,v} = (R_m[v/V/N], \ldots , R_m[(v+1)V/N-1])^T \\
R_{m,0} = (R_m[0], \ldots , R_m[vV/N-1])^T
\]
have been introduced; and in (7) the independence of the observable samples within the \( m \)-th OFDM symbols has been used together with the fact that the \( v \)-th subblock of \( N/V \) carriers is rotated by the same phase coefficient \( \varphi_v^m \).

Finally, applying another factorization inside the \( v \)-th subblock of carriers and defining a deterministic state transition law, as a function of the information symbol \( a_k \), which describes the evolution of the system, i.e., \( \mu_m[i] = f(\mu_m[i-1], a_m[i-1]) \), (7) may be written as
\[
\hat{a}(\varphi) = \operatorname{argmax}_a \prod_{m=0}^{K-1} \prod_{v=0}^{V-1} \prod_{k=0}^{N-1} p(R_m[k+vV/N]; \varphi_v^m) \quad a_m[k+vV/N], \mu_m[k+vV/N].
\]

Assuming that the noise process has Gaussian distribution, the observation \( R_m[k] \) is conditionally Gaussian, given the data. The application of the chain factorization rule allows us to factor the conditional PDF in (2) as a product of \( KN \) complex conditional Gaussian PDFs, completely defined by the conditional means and variances
\[
\hat{R}_m[i] = \mathbb{E} \{R_m[i] \mid a_m[i], \mu_m[i]\} = c_{m}[i] \cdot \varphi_v^m
\]
\[
\sigma_{R_m[i]}^2 = \mathbb{E} \{ |R_m[i] - \hat{R}_m[i]|^2 \mid a_m[i], \mu_m[i]\} = \sigma_N^2.
\]

As a consequence, the detection strategy (4) may be formalized in the logarithmic domain as
\[
\hat{a}(\varphi) = \operatorname{argmin}_a \sum_{m=0}^{K-1} \sum_{v=0}^{V-1} \sum_{k=0}^{N-1} \lambda_m[v,k]
\]
with branch metric expressed as
\[
\lambda_m[v,k] = |R_m[k+vV/N] - \hat{R}_m[k+vV/N]|^2.
\]

Using (3), it is possible to obtain the decoded sequence \( \hat{a} \).

V. OPTIMAL SOLUTION: MULTIPLE TRELILSES (MT)

The proposed detection strategy (8) can be realized by defining a set of \( W^{(V-1)} \) parallel trellises in a one-to-one correspondence with all the possible combinations of the phase vector \( \varphi_m \) during the \( m \)-th OFDM symbol. The evaluation of the branch metric (9) can be performed following these simple steps:

- for the \( m \)-th OFDM received symbol, \( W^{(V-1)} \) parallel trellises have to be considered, each of them associated with a particular hypothesis \( \varphi_m \);
- for each of these \( W^{(V-1)} \) parallel trellises, the branch and path metrics have to be evaluated, assuming the \( W \) phases belonging to each of the \( V \) subblocks in the OFDM signal perfectly known;
- at the end of the process, for the considered \( m \)-th OFDM received symbol, the smallest survivor metrics in each of the \( W^{(V-1)} \) parallel trellises have to be compared, in order to select the smallest one. This selected metric belongs to a trellis with the most likely phase sequence used at the transmitter;
- finally, the decoded symbols have to be extracted from the trellis with the smallest path metric.

Fig. 1 shows an example of the proposed joint detection and decoding algorithm based on MT considering \( V = 4, W = 3 \) and assuming \( N = 16 \) with a four-state bidimensional (2D) TCM code. Note that since the first subblock is not rotated by the PTS algorithm (\( \varphi_m^{(0)} = 1 \)), the construction of MT should be considered starting with the second subblock only.

VI. SUBOPTIMAL SOLUTION: PARTIAL MULTIPLE TRELILSES (PMT)

The proposed optimal detection strategy can be directly implemented considering \( W^{(V-1)} \) multiple trellises, as explained in Sect. V. The major limit of MT may be the required complexity, which increases exponentially with the number of subblocks \( V \). A different suboptimal approach may be based on partial multiple trellises (PMT), i.e., inside the \( v \)-th subblock it is possible to construct \( W \) multiple trellises in a one-to-one correspondence with all the possible values of the phase vector \( \varphi_v^m \). The PMT are defined locally to a single subblock with respect to \( \varphi_m \), whereas in the MT approach
parallel trellises are defined with respect to the entire phase vector \( \varphi_m \). Then, the detection process can be carried out for each \( W \) trellis: at the end of the subblock, the smallest survivor metrics in each of the \( W \) parallel trellises have to be compared, selecting the smallest one. This selected metric belongs to a trellis with the most likely phase \( \varphi_m^{(w)} \) used at the transmitter during the \( v \)-th subblock. If the subblock length is sufficiently long to observe, with high probability, a merge of survivors paths, i.e., if the subblock length is longer than the decoding depth of the Viterbi algorithm, then the decoded symbols may be extracted from the trellis with the smallest partial path metric. Otherwise, if the subblock length of two consecutive subblocks is longer than the decoding depth of the Viterbi algorithm, then the decoding decision process may be based on \( W^2 \) multiple trellises and delayed at the end of the successive \((v+1)\)-th subblock.

Fig. 2 shows an example of the proposed joint detection and decoding algorithm based on PMT considering \( V=4, W=3 \) and assuming \( N=16 \) with a four-state 2D TCM code. In this example, we have supposed that a merge of survivor paths has occurred within the subblock length.

![Diagram of parallel trellises](image)

**TABLE I**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subcarrier separation ( \Delta f )</td>
<td>1 kHz</td>
</tr>
<tr>
<td>FFT size ( N )</td>
<td>128</td>
</tr>
<tr>
<td>Signaling interval ( T )</td>
<td>7.8125 ( \mu s )</td>
</tr>
<tr>
<td>roll-off ( \alpha )</td>
<td>0.2</td>
</tr>
</tbody>
</table>

**VII. Numerical Results**

We consider two different high-voltage PLC channels, i.e., a 64 kV line with noise power of \(-36.0 \) dBm defined over a 4 kHz bandwidth and a 360 kV line with noise power equal to \(-23.0 \) dBm [19]. However, the proposed algorithms can also be used for PLC systems working over low- or medium-voltage power line channels. The OFDM parameters used in our simulations are shown in Tab. I. The considered eight-state four-dimensional (4D) TCM [20] code has a rotational invariance of \( \pi \), i.e., code sequences rotated by \( \pi \) are valid code sequences, despite the fact that the information sequence which originates them is different. This observation implies that, if the used phase sequences \( \{\varphi_m^{(v)}\}_{v=0}^{V-1} \) have, with respect to each other, a difference of \( \pi \), then the receiver will not be able to correctly distinguish the correct code sequence with respect to the rotated one. As a consequence, the cardinality of the phase alphabet \( W \) should be an odd number, in order to exclude phase rotations of \( \pi \) between the correct code sequence and the code sequence rotated by the PTS algorithm. Note that in [12] the alphabet cardinality \( W \) is always chosen as an even number: as a consequence, the PAR reduction capability of the PTS algorithm with an odd phase alphabet cardinality \( W \) has to be investigated. This point is addressed in the next section.

A. Solid-State Power Amplifier Model and CCDF Analysis

The nonlinear AM/AM characteristic of the solid-state power amplifier (SSPA) used in our simulations has been modeled by a memoryless block with input-output relationship given by [21]

\[
x'(t) = g_c \cdot |x(t)| e^{j \phi(x(t))} = g_c \cdot |x(t)| \left(1 + \frac{|x(t)|}{x_{sat}} \right)^{2p} \frac{1}{2p} e^{j \phi(x(t))}
\]

where \( g_c \) is the amplifier gain associated to a given peak-power \( P_p \), the parameter \( p \) controls the smoothness of the transition from the linear region to the saturation region and \( x_{sat} \) is the saturation amplitude parameter, i.e., \( x_{sat} \) indicates at which amplitude the transition between linear and saturation region is located. The amplifier input back-off (IBO) is defined as

\[
IBO = x_{sat}^2 / \sigma_x^2
\]

so that the reference power is determined by the saturation parameter \( x_{sat} \). Following this IBO definition, IBO= 0 dB represents a condition in which the power of the signal at the input of the SSPA is \( \sigma_x^2 = x_{sat}^2 \), i.e., the input signal is...
B. Bit Error Rate Analysis: MT and PMT

In this section, the bit error rate (BER) performance of the proposed OFDM PLC system is investigated.

The signal-to-noise ratio (SNR) is defined as $E_b/N_0$, where $E_b$ is the received energy per information bit and $N_0$ is the monolateral power spectral density. In Fig. 4 the performance in terms of BER as a function of $E_b/N_0$ is presented, considering different values of IBO, a 64 kV PLC channel, and assuming $V = 4, W = 5$. The time-varying and frequency-selective nature of the PLC channel may be accounted for by means of optimal power distribution and bit loading [1]. However, the optimality of the proposed algorithm is independent from the characteristics of the channel. Continuous lines show the BER of the OFDM system without PTS (“No PTS” curves), whereas dotted lines show the BER for the OFDM system with PTS and assuming a perfect knowledge of the phase sequences by the receiver (“SI known” curves). Finally, dashed lines present the BER performance obtained by jointly decoding the TCM code and detecting the PTS SI using MT, whereas dotted-dashed lines show the BER curves obtained with PMT.

From Fig. 4, one can conclude that for IBO values of 0 dB and 2 dB the BER of the proposed solution (based on MT or PMT) is degraded with respect to the results with SI perfectly known by the receiver: this is justified by the fact that the SSPA operates at a nonlinear point such that the OFDM signal is heavily distorted. As a consequence, the BER curves obtained with the proposed algorithm are affected by (i) errors generated by thermal noise, (ii) signal distortion, (iii) and errors associated with an erroneous choice of the MT or PMT. On the other hand, for IBO greater than 4 dB, the BER curves with estimated SI (using MT or PMT) and the BER curves with perfectly known SI overlap, confirming the validity of the proposed solutions. In particular, the BER curves obtained using the detection approach based on PMT are very closed to those obtained by MT solution: the penalty introduced by the suboptimal PMT approach is negligible. The main difference is the reduced complexity of PMT with respect to MT, i.e., for the used PTS parameters $V = 4, W = 5$, MT requires $W^{V-1} = 125$ trellises, whereas PMT requires only 5 trellises for each of the $V - 1 = 3$ subblock. Finally, note that the proposed algorithm correctly works for operational values of IBO, i.e., for IBO less than 4 dB the BER curves are degraded even if the PTS algorithm is used.

Fig. 5, which shows the BER performance also as a function of the line length, confirms the good behavior of the solution based on MT (the PMT curves, not shown, overlap with the MT curves). Moreover, note that the PLC system with IBO of 8 dB and 10 dB allows a quality of service of BER equal to $10^{-6}$ for a reduced distance with respect to the one achievable with the optimal values of IBO of 4 dB and 6 dB. This is perfectly correct, since the PLC systems with IBO of 8 dB and 10 dB do not efficiently use all the available power given by the SSPA.

Fig. 6 shows the BER performance of the OFDM PLC system assessed considering a 380 kV line, assuming a peak-power of $P_p = 10$ W and jointly decoding the TCM code and detecting the PTS SI using PMT. Note that for operational values of IBO the system employing the joint detection and decoding approach shows the same BER performance of a system with a perfect knowledge of the SI. Finally, from Figures 5 and 6, one can conclude that the nonlinear distortion introduced by the considered SSPA has only a minor effect on the BER, in good agreement with [22], [23]: in fact,
Fig. 5. BER as a function of line length for different values of IBO with joint decoding of TCM and detection of PTS SI with MT ($V = 4, W = 5$), 64 kV line, $P_f = 35$ W and $p = 3$.

Fig. 6. BER as a function of $E_b/N_0$ for different values of IBO with joint decoding of TCM and detection of PTS SI based on PMT ($V = 4, W = 5$), 380 kV line, $P_f = 10$ W and $p = 3$.

relatively rare time-domain clippings simply correspond to some additional noise in the receiver, which may be coped effectively with by the channel code. Some higher degradation in the BER curves may be expected using a nonlinear model with also an AM/PM distortion.

VIII. CONCLUSION

In this paper, new algorithms for joint decoding of a widely used convolutional channel code, such as TCM, and detection of PTS SI is presented. The proposed algorithm represents a practical solution to avoid the explicit insertion of SI into the OFDM signal. The proposed detection strategy can be directly implemented considering (i) an optimal realization based on multiple trellises (MT) or (ii) a suboptimal approach based on partial multiple trellises (PMT). Numerical simulations have demonstrated that both solutions may be characterized by good BER performance at operational values of IBO and amplifier nonlinearities, whereas the PMT approach may be preferable in terms of complexity.

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