

Optical Communications

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Optical Communications is one of the key segment in nowadays global communication networks. Main features of an optical communication system are the large bandwidth, the low losses and the the ultra-fast bit rates that only photonics can support. Not surprisingly, the growing capacity demand of internet can be satisfied thanks to the optical fibers that allow digital transmissions over long distances, thus connecting the entire earth.

This course covers basic and advanced topics in optical communications. We begin by a brief summary of the ray optics, summarizing the main properties of optical fibers. Then we move to describe the fiber optic channel including linear group velocity dispersion (GVD). Several investigations of GVD will be proposed to clarify its impact on the electric field. Next we examine the amplification process within an erbium doped fiber amplifier (EDFA), both from a physical and a state model point of view. We also analyze the noise figure of an EDFA and its impact on system performance.

The next topic is the detection of an off-keying (OOK) modulated signal, for which we provide an exhaustive analysis of the bit error rate (BER) evaluation in presence of different kind of noise sources (shot noise, amplified spontaneous emission noise, thermal noise). Basic models of direct-detection schemes will be proposed.

In the second part of the course we analyze the nonlinear Kerr effect in optical fibers. We first provide a model for the Kerr effect deriving the nonlinear Schrödinger equation, then we start a detailed analysis of each nonlinear effect separately: self phase modulation (SPM), cross phase modulation (XPM), four wave mixing (FWM), modulation instability (MI), Raman effect. We analyze the interaction of GVD with the Kerr effect, for instance in the special case of the soliton transmission where the two effects counteract perfectly. We will also discuss a numerical algorithm to simulate the nonlinear propagation within optical fibers, showing advanced methods to speed up computation. Such an algorithm is one of the main algorithms of an open source software that will be used in this course by the students.

The next part of the course concerns polarization effects, both in the linear and non-linear regime. In the first case we analyze polarization mode dispersion (PMD), in the second cross polarization modulation (XPolM).

Finally, we analyze modern optical communication systems based on advanced modulation formats and coherent detection. We also enter in the details of the digital signal processing (DSP) unit and of its main building algorithms.

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Lecture 1 (03-Mar-2014)

Introduction, presentation of the course, motivations. Brief history of optical communications.

Notes: The slides of the presentation can be found at LeA.

Supplementary reading: An interesting analysis of the future of optical communications can be found in [Des06]. An introduction to the history of optical communications can be found in [Agrbase]. A very good analysis of the main effects in optical communications with emphasis to channel capacity can be found in [Ess10]. A list of the main companies in optical communications is in [Ecocex]. A list of the main Italian companies is in [ListITA].

Lecture 2 (04-Mar-2014)

Ray optics. Fermat's principle. Snell's law. Total reflection. Numerical aperture of an optical fiber. Multi-mode fibers. Problems of multi-mode fibers. Single-mode fibers (overview). V-number (overview). Systems theory approach to the optical fiber. Phase delay and group delay.

Notes: The principles of ray optics can be found in [Alb05] and [Saleh, Agrbase]. A rigorous proof of GVD can be found in [AgrNL, Agrbase].

Supplementary reading: The carrier and envelope delay theory can be found in "Carrier and envelope delay" in [Carlson].

Lecture 3 (05-Mar-2014)

Group velocity dispersion (GVD). GVD: examples. Rigorous proof of GVD using Maxwell's equations.

Notes: For the rigorous proof of GVD see [AgrNL].

Supplementary reading: For a general description of Maxwell's equation see [Saleh]. A more rigorous proof of GVD (and nonlinear Kerr effect) using the multiple scales approach can be found in [Men99, Men06]. A list of the main optical fibers can be found in [Cisco].

Lecture 4 (10-Mar-2014)

Attenuation. Group delay. Impact of GVD over a Gaussian pulse. Dispersion length.

Notes: GVD is described in [Alb05] and [Agrbase, AgrNL].

Supplementary reading: Other details can be found in [Saleh].

Lecture 5 (11-Mar-2014)

Anomalous and normal dispersion. GVD in presence of signal's chirp. Instantaneous frequency. Dispersion Management (DM). GVD in presence of signal chirp. Matched filter interpretation of GVD with chirp. Third order dispersion. Eye closure penalty in presence of GVD.

Notes: GVD and GVD-chirp interaction are described in [Alb05] and [Agrbase, AgrNL].

Supplementary reading: Other details about GVD can be found in [Saleh]. For the matched filter interpretation in the frequency domain see [Proakis]. For the definition of the variance of a signal and the Heisenberg's principle see the appendix of [Saleh].

Lecture 6 (12-Mar-2014)

Eye closure penalty in presence of GVD. Chen's formula for the GVD induced eye closure penalty. Fourier transform induced by strong GVD.

Notes: details can be found in [Alb05].

Supplementary reading: Chen's formula was introduced in [Chen99] regarding polarization mode dispersion (PMD), but the idea still works for GVD. The GVD induced Fourier transform is similar to the Fresnel diffraction effect [Saleh].

Lecture 7 (17-Mar-2014)

Memory of GVD.

Erbium doped fiber amplifier (EDFA). Cross sections. Propagation equation for the photon flux over distance. Reservoir.

Notes: details can be found in [Alb05]. A detailed description of the reservoir is in [Alb98].

Supplementary reading: A discussion about the cross section is in [Saleh]. The models of the EDFA are in [Sal90, Sun96].

Lecture 8 (19-Mar-2014)

Reservoir. State model interpretation of reservoir. Small signal gain. Gain saturation. Propagation equation with gain saturation. Fixed output power of an EDFA in saturation. Amplified spontaneous emission (ASE) noise. Noise figure of an EDFA: definition.

Notes: details can be found in [Alb05]. For the noise figure definition see [Hau98].

Supplementary reading: The models of the EDFA are in [Sal90, Sun96].

Lecture 9 (20-Mar-2014)

Friis's formula. Excess noise figure. Dual stage amplification: evaluation of noise figure.

Photo-detectors: photo-diode. Quantum efficiency. Responsivity. Reasons for photocurrent: electron-holes contributions to current.

Notes: details can be found in [Alb05]. For the noise figure definition see [Hau98]. The property that one photon contributes to one charge to the net current is described in [Saleh] in section 17.1 "Properties of semiconductor photodetectors".

Supplementary reading: a more general discussion about photo-diodes can be found in [Alexander] and [Agrbase].

Lecture 10 (24-Mar-2014)

P-i-n junction. Junction capacity. Photo-diode bandwidth. Avalanche photo-diode (APD).

Poisson statistics. Poisson counting process. Shot noise. Campbell's theorem with proof. Power spectral density (PSD) of shot noise.

Notes: details can be found in [Alb05].

Supplementary reading: An alternative proof of Campbell's theorem can be found in [Saleh].

Lecture 11 (25-Mar-2014)

PSD with APD.

Optical receivers. Matched filter. Amplifiers for the photo-current: low impedance, high impedance, trans-impedance. Bit error rate (BER) for on-off keying (OOK) transmission. Quantum limit. Sensitivity power. Thermal noise. Gaussian approximation.

Notes: details can be found in [Alb05]. Quantum limit is also discussed in [Agrbase].

Supplementary reading: other details can be found in [Saleh].

Lecture 12 (26-Mar-2014)

Gaussian approximation and Personick's formula. Gaussian approximation with APD. Optimal multiplication factor with APD.

Notes: details can be found in [Alb05].

Supplementary reading: other details can be found in [Saleh].

Lecture 13 (31-Mar-2014)

Relation between Sensitivity penalty and Eye closure penalty for PIN and APD. Exercise regarding the amount of chirp yielding a given sensitivity penalty. Pre-amplified receivers. Signal to spontaneous and spontaneous to spontaneous noise beat.

Notes: details can be found in [Alb05].

Supplementary reading: general discussions can be found in [Agrbase]. For the Rice representation of a bandpass stochastic process see [Papoulis, Carlson].

Lecture 14 (01-Apr-2014)

BER with ASE noise: Gaussian approximation. Isserlis's theorem. Average and variance of signal/spontaneous, spontaneous/spontaneous, shot, thermal noise. Optical signal to noise ratio (OSNR). Comparison signal/spontaneous, spontaneous/spontaneous.

Notes: details can be found in [Alb05].

Supplementary reading: general discussions can be found in [Agrbase]. Isserlis's theorem is well described in wikipedia.

Lecture 15 (02-Apr-2014)

Marcuse's formula. Pre-amplified receivers: comparison with quantum limit. Exercises.

Bergano's method to estimate BER. Threshold error using the Gaussian approximation.

Notes: details can be found in [Alb05].

Supplementary reading: Better methods to evaluate the BER without the Gaussian approximation can be found in [Hum91, For00].

Lecture 16 (07-Apr-2014)

Nonlinear Schroedinger equation (NLSE). Reasons for the cubic nonlinear effect. Dimensional units of the electric field.

Notes: for the NLSE see [AgrNL]. For the reasons of the cubic nonlinearity see [Iizuka] at p. 523.

Supplementary reading: A rigorous proof of NLSE can be found in [Men06, Men99]. A tutorial about NLSE is in [Agr11].

Lecture 17 (08-Apr-2014)

Self phase modulation (SPM). Comparison between temporal interpretation of SPM and frequency interpretation of GVD. SPM in frequency domain. SPM with sinusoidal power. Bandwidth enlargement induced by SPM. Impact of chirp induced by SPM and GVD over a Gaussian pulse.

Notes: SPM is well described in [AgrNL]. The physical interpretation of the chirp is described in [And92].

Supplementary reading: Further information on the superposition of chirps can be found in [And93].

Lecture 18 (09-Apr-2014)

Wave breaking (WB). Impact of chirp induced by SPM and GVD over a Gaussian pulse.

Noise figure of optical amplifiers measured in the electrical domain. OSNR budget. Amplifier chains: limitations of ASE noise and nonlinear Kerr effect. Inhomogeneous amplifier chains.

Notes: Wave breaking is described in [And92]. Details about the noise figure measurement can be found in [Alb05].

Supplementary reading: Additional notes on WB are described in [AgrNL] and [And93]. Further details about the noise figure measurement can be found in [Agrbase].

Lecture 19 (14-Apr-2014)

Inhomogeneous amplifier chains. Lagrange multipliers method. Best amplifiers gain in inhomogeneous chains.

Solitons. Proof of fundamental soliton.

Notes: For the inhomogeneous amplifier chains see [Mec98]. Details about solitons can be found in [AgrNL].

Supplementary reading: Further details on solitons can be found in [Agr11]. A good book on solitons in the Unipr library is [Hasegawa].

Lecture 20 (15-Apr-2014)

Proof of fundamental soliton. Higher order solitons. Solitons: from dimensionless to standard units. Collision length and symbol rate of solitons. Scaling laws of solitons. Perturbation of solitons: solitons of non-integer order, impact of chirp. Solitons in amplified systems: impact of losses.

Notes: details can be found in [AgrNL].

Supplementary reading: Further details on solitons can be found in [Agr11]. A good book on solitons in the Unipr library is [Hasegawa].

Lecture 21 (16-Apr-2014)

Numerical examples of soliton propagation: 3rd order soliton, dark soliton, soliton of non-integer order, interaction of solitons. Notes on the impact of ASE noise on solitons: sliding filters.

Wavelength division multiplexing (WDM) systems. Unique and separate fields in linear regime.

Notes: For cross-channel nonlinear effects see [AgrNL].

Supplementary reading: The numerical examples about solitons are taken from [AgrNL].

Lecture 22 (28-Apr-2014)

NLSE with separate fields. Cross-phase modulation (XPM) and four wave mixing (FWM). Intra- and inter-channel GVD. XPM with inter-channel GVD: probe/pump case. XPM filter for single fiber. Walk-off coefficient. Bandwidth of XPM filter. XPM filter for multi-span systems in absence of intra-channel GVD.

Notes: For cross-channel effects see [AgrNL]. XPM without intra-channel GVD is discussed in [Kaz96].

Supplementary reading: XPM impact on ASE noise without intra-channel GVD is discussed in [Ho04].

Lecture 23 (29-Apr-2014)

Numerical results. Example: hybrid OOK/DQPSK system.

Split-step Fourier method (SSFM). Formal solution using operators. Non commutative operators. SSFM with symmetrized and asymmetric step: accuracy.

Notes: for the XPM filter see [Bel98, BelVar98]. The basic idea of SSFM is discussed in [AgrNL].

Supplementary reading: An alternative derivation of the XPM filter can be found in [Car99]. XPM in hybrid systems is discussed in [Alb09].

Lecture 24 (30-Apr-2014)

Choice of the SSFM step: constant step, step based on the nonlinear phase criterion, step based on the local error. Richardson extrapolation. Block diagrams of SSFM.

The Matlab programming language.

Notes: The basic idea of SSFM is discussed in [AgrNL]. The step size is analyzed in [Sin03]. The method based on the local error is described in [Feldman]. The slides about Matlab can be found at LeA. A Matlab primer is [Sig93].

Supplementary reading: An extension of the method of the nonlinear phase including GVD is described in [Zha08]. An analysis of the spurious resonances induced by a constant step is shown in [Bos00].

Free versions of Matlab are Octave and SciLab. With Octave a graphical user interface may be useful, e.g., gnuplot (in wikipedia at the voice Octave there is an exhaustive list of alternatives). The basic toolboxes of Matlab are available in Octave at Octave-Forge.

Lecture 25 (05-May-2014)

The Matlab programming language.

Notes: The slides about Matlab can be found at LeA. A Matlab primer is [Sig93].

Supplementary reading: An advanced tutorial of Matlab is [Ack03]. A collection of general purpose Matlab functions can be found at Matlab-File-Exchange.

Some useful code lines regarding saving the body of a Matlab file into a variable can be found in the forum of the Optical Communications course at LeA, date 9 May 2012.

Lecture 26 (06-May-2014)

Software Optilux. Examples. Discretization of a signal in the time and frequency domain.

Notes: Optilux can be downloaded at OptiluX.

Supplementary reading: I suggest to keep update Optilux with Subversion, both for Windows and Linux. In Linux, the command is the following:

`<svn checkout svn://svn.code.sf.net/p/optilux/code/trunk optilux-code>`. This commands creates a copy of the global Optilux repository, such that updating the files to the latest version can be done (e.g., in the bash of Linux) by typing `<svn up>` in the directory of the repository. In any case, the Optilux repository can be found at Sourceforge.

I suggest to compile the `mex` files inside Optilux (e.g., run the function `comp_mex.m` in the `Optilux_files` directory). In this case, under Linux it may be useful the installation of the package `build-essential`.

Lecture 27 (07-May-2014)

Software Optilux. Examples. Unique and separate fields: numerical cost comparison.

Notes: Optilux can be downloaded at OptiluX.

Supplementary reading: I suggest to keep update Optilux with Subversion.

Lecture 28 (12-May-2014)

Four wave mixing (FWM). Regular perturbation (RP) method to approximate the solution of the NLSE. FWM with CW signals. FWM efficiency. Phase matching coefficient.

Notes: FWM can be found in [AgrNL].

Supplementary reading: The RP method is discussed in [Van02].

Lecture 29 (13-May-2014)

Gaussian Nonlinear (GN) model. Best power using the GN model. Application of the GN model: best SNR, scaling of SNR.

Notes: The nonlinear Gaussian model for the signal to noise ratio can be found in [Alb11].

Supplementary reading: Further details on the GN model can be found in [Pog11] and [Car12].

Lecture 30 (14-May-2014)

Matlab exercises.

Notes: The exercises are available at LeA..

Supplementary reading: see the Matlab documentation for basic exercises.

Lecture 31 (19-May-2014)

Exercise: getting the entire SNR curve by two measurements. Constrained performance: scaling of nonlinear asymptote with the number of spans.

Modulation instability (MI): linearized NLSE.

Notes: The nonlinear Gaussian model for the signal to noise ratio can be found in [Alb11]. The scaling laws of the SNR can be found in [Alb12]. Modulation instability is discussed in [AgrNL].

Supplementary reading: Further details on the GN model can be found in [Pog11] and [Car12].

Lecture 32 (20-May-2014)

Modulation instability: solution in absence of attenuation. Eigenvalues of MI.

Optical parametric amplifier (OPA). Two pumps OPA.

Raman amplification. Motivations (distributed amplification). Memory induced by Raman effect. SPM, XPM and FWM in presence of Raman. Raman impact on XPM.

Notes: Modulation instability is discussed in [AgrNL]. A good tutorial about OPA is [Han02]. The pump/signal model of Raman is discussed in [AgrNL].

Supplementary reading: Two pumps OPA is described in [MKi02]. A book on Raman amplification is [AgrRaman]. Basics about Raman are in [Blo89].

Lecture 33 (21-May-2014)

Raman amplification: pump-signal case. Brief notes about the amplified spontaneous Raman scattering and Rayleigh back scattering. Forward and backward Raman pumping.

Polarization of light. Birefringence. Jones formalism. Ellipse of polarization. Polarimeter. Stokes space. Poincaré sphere. Degree of polarization (DOP).

Notes: A good book on polarization of light is [Damask]. A good description of linear algebra is in [Gor00]. A software to examine the state of polarization of light is in LeA.

Supplementary reading: Advanced details on linear algebra are in [Gor00].

Lecture 34 (26-May-2014)

Unitary matrices. Local behavior of birefringence. Hermitian matrices. Eigenvalues and eigenvectors of Hermitian matrices. Polarization mode dispersion (PMD). Motion in omega. Differential group delay (DGD). First order PMD.

Manakov equation. Cross polarization modulation (XPolM). Memoryless XPolM.

Notes: A good book on polarization of light is [Damask]. A good description of linear algebra is in [Gor00].

Supplementary reading: An introduction on XPolM is in [Kar06]. A rigorous proof of the Manakov equation is in [Wai96].

Lecture 35 (27-May-2014)

Advanced modulation formats: motivations. Phase modulator and Mach Zehnder (MZ) modulator. Return to zero(RZ) pulses and its variants (carrier-suppressed (CS-RZ), chirped-RZ (CRZ), alternate phase-RZ (APRZ)). Duobinary transmission. Differential phase shift keying (DPSK). Generation and detection of DPSK. Nonlinear phase noise. Differential quadrature phase shift keying (DQPSK). Generation of M-ary PSK.

Coherent Detection: motivations. Historical background. Optical hybrid. Detection of in-phase and quadrature components.

Notes: The slides are available at LeA. A good tutorial of DPSK is [Win06].

Supplementary reading: A detailed bibliography can be found into the slides.

Lecture 36 (28-May-2014)

Polarization division multiplexing (PDM). Polarization diversity receiver. Digital signal processing (DSP). Analog to digital conversion (ADC): choice of the number of samples per symbols. Electronic dispersion compensation of GVD. Electronic dispersion compensation of PMD: constant modulus algorithm (CMA). Phase estimation: Viterbi & Viterbi algorithm. Numerical and experimental results. Interaction of PMD and nonlinear Kerr effect. Cross polarization modulation (XpolM): impact of channel walk-off. Nonlinear threshold (NLT) of optical links. Digital back-propagation (DBP) algorithm. Polarization switched quadrature phase shift keying (PS-QPSK).

Notes: The slides are available at LeA. Good tutorials on coherent detection are [Cha08, Kik10, Sav10].

Supplementary reading: A detailed bibliography can be found into the slides.

References

- [Abd08] M. O. Abdelgawad, "Writing and Publishing Good Journal Papers," tech. report, University of Toronto, 2008.
- [Ack03] P. J. Acklam, "MATLAB array manipulation tips and tricks," tech. report, 2003. [Online] Available: <http://home.online.no/~pjacklam/matlab/doc/mtt/index.html>.
- [Agrbase] G. P. Agrawal, "Fiber-Optic Communication Systems," Wiley, 3rd ed., 2002.
- [AgrNL] G. P. Agrawal, "Nonlinear Fiber Optics," San Diego, CA: Academic Press, 2001.
- [AgrRaman] C. Headley and G. P. Agrawal, "Raman Amplification in Fiber Optical Communication Systems," San Diego, CA: Academic Press, 2005.
- [Agr11] G. P. Agrawal, "Nonlinear fiber optics: its history and recent progress," J. Opt. Soc. Am. B, v. 28, n. 12, pp. A1–A10, Dec. 2011.
- [Alb98] A. Bononi and L. A. Rusch, "Doped-Fiber Amplifier Dynamics: A System Perspective," J. Lightw. Technol., v. 16, n. 5, pp. 945–956, May 1998.

- [Alb05] A. Bononi, "Appunti di Comunicazioni Ottiche A," Università degli Studi di Parma, 2005.
- [Alb11] E. Grellier and A. Bononi, "Quality parameter for coherent transmissions with Gaussian-distributed nonlinear noise," *Opt. Express*, v. 19, n. 13, pp. 12781–12788, June 2011.
- [Alb09] A. Bononi, M. Bertolini, P. Serena, and G. Bellotti, "Cross-Phase Modulation Induced by OOK Channels on Higher-Rate DQPSK and Coherent QPSK Channels," *J. Lightw. Technol.*, v. 27, n. 18, pp. 3974–3983, Sept. 2009.
- [Alb12] A. Bononi, N. Rossi and P. Serena, "On the Nonlinear Threshold versus distance in long-haul highly-dispersive coherent systems," *Opt. Express*, v. 20, n. 26, pp. B204–B216, Nov. 2012.
- [Blo89] K. J. Blow and D. Wood, "Theoretical Description of Transient Stimulated Raman Scattering in Optical Fibers," *J. Quantum. Electron.*, v. 25, n. 12, Dec. 1989.
- [Alexander] S. B. Alexander, "Optical Communication Receiver Design," SPIE press, IEE, 1997.
- [And92] D. Anderson, M. Desaix, M. Karlsson, M. Lisak and M. L. Quiroga-Teixeiro, "Wave Breaking-free pulses in Nonlinear Optical Fibers," *J. Opt. Soc. Am. B*, v. 9, n. 8, pp. 1358–1361, Aug. 1992.
- [And93] D. Anderson, M. Desaix, M. Lisak and M. L. Quiroga-Teixeiro, "Wave Breaking in Nonlinear Optical Fibers," *J. Opt. Soc. Am. B*, v. 10, n. 7, pp. 1185–1190, July 1993.
- [Bel98] A. Bononi, C. Francia, and G. Bellotti, "Impulse response of crossphase modulation filters in multi-span transmission systems with dispersion compensation," *Opt. Fiber Technol.*, v. 4, n. 4, pp. 371–383, 1998.
- [BelVar98] G. Bellotti, M. Varani, C. Francia and A. Bononi, "Intensity Distortion Induced by Cross-Phase Modulation and Chromatic Dispersion in Optical-Fiber Transmissions with Dispersion Compensation," *IEEE Photon. Technol. Letters*, v. 10, n. 12, pp. 1745–1747, Dec. 1998.
- [Bos00] G. Bosco, A. Carena, V. Curri, R. Gaudino, Member, P. Poggiolini, and S. Benedetto, "Suppression of Spurious Tones Induced by the Split-Step Method in Fiber Systems Simulation," *IEEE Photon. Technol. Lett.*, v. 12, n. 5, pp. 489–491, May 2000.
- [Car12] A. Carena, V. Curri, G. Bosco, P. Poggiolini, and F. Forghieri, "Modeling of the Impact of Non-Linear Propagation Effects in Uncompensated Optical Coherent Transmission Links," *J. Lightw. Technol.*, v. 30, n. 10, pp. 1524–1539, May 2012.
- [Carlson] A. B. Carlson, "Communication Systems," Mc Graw Hill, 2002.

- [Car99] "Cross-Phase Modulation in Intensity Modulation– Direct Detection WDM Systems with Multiple Optical Amplifiers and Dispersion Compensators," J. Lightw. Technol., v. 17, n. 2, pp. 178–190, Feb. 1999.
- [Cha08] G. Charlet, "Coherent detection associated with digital signal processing for fiber optics communication," Comptes Rendus Physique, v. 9, n. 9-10, pp. 1012–1030, 2008.
- [Chen99] C.-J. Chen, "System Impairments due to Polarization mode dispersion," in Proc. OFC99, paper We2-1, p. 77–79, 1999.
- [Cisco] A. Barbieri, "A Guide to Select Single-Mode Fibers for Optical Communications Applications," Tech. report, Cisco Technology Marketing, pp. 1–37, 2002.
- [Damask] J. N. Damask, "Polarization optics in telecommunications, " New York: Springer, 2004.
- [Des06] E. Desurvire, "Capacity Demand and Technology Challenges for Lightwave Systems in the Next Two Decades," J. Lightw. Technol., v. 24, n. 12, p. 4697–4710, Dec. 2006.
- [Ecocex] European Conference of Optical Communications (ECOC), list of exhibitors, [Online] Available: <http://www.ecocexhibition.com/edirectory/list>.
- [Ess10] R.-J. Essiambre, G. Kramer, P. J. Winzer, G. J. Foschini, B. Goebel, "Capacity Limits of Optical Fiber Networks," J. Lightw. Technol., v. 18, n. 4, p. 662–701, Feb. 2010.
- [Feldman] J. Feldman, "Variable step size methods," Tech. report, Dept. of Mathematics, University of British Columbia Vancouver, BC Canada, [Online] Available: <http://www.math.ubc.ca/~feldman/math/vble.pdf>.
- [For00] E. Forestieri, "Evaluating the error probability in lightwave systems with chromatic dispersion, arbitrary pulse shape and pre- and postdetection filtering," J. Lightw. Technol., v. 18, n. 11, pp. 1493–1503, Nov. 2000.
- [Fri86] N. J. Frigo, "A Generalized Geometrical Representation of Coupled Mode Theory," IEEE J. Quantum. Electron., v. QE-22, n. 11, pp. 2131–2140, Nov. 1986.
- [Hasegawa] A. Hasegawa and M. Matsumoto, "Optical Solitons in Fibers," Springer, 3rd ed., 2002.
- [Han02] J. Hansryd, P. A. Andrekson, M. Westlund, J. Li and P.-O. Hedekvist, "Fiber-based Optical Parametric Amplifiers and Their Applications," J. Sel. Top. Quantum Elec., v. 8, n. 3, pp. 506–520, May/June 2002.
- [Hau98] H. A. Haus, "The Noise Figure of Optical amplifiers," IEEE Photon. Technol. Letters, v. 10, n. 11, p. 1602–1604, Nov. 1998.

- [Ho04] K. Po Ho, “Error Probability of DPSK Signals with Cross-Phase Modulation Induced Nonlinear Phase Noise,” *J. Sel. Top. Quantum. Elec.*, v. 10, n. 2, pp. 421–427, March/Apr. 2004.
- [Hum91] P. A. Humblet, “On the Bit-error rate of Lightwave Systems with Optical Amplifiers,” *J. Lightw. Technol.*, v. 9, n. 11, pp. 1576–1582, Nov. 1991.
- [Iizuka] Keigo Iizuka, “Elements of Photonics, Volume I: In Free Space and Special Media”, Wiley, 2002.
- [Ip08] E. Ip, A. P. T. Lau, D. J. F. Barros, and J. M. Kahn, “Coherent detection in optical fiber systems,” *Opt. Express*, v. 16, n. 2, pp. 753–791, 2008.
- [Kar06] M. Karlsson and H. Sunnerud, “Effects of Nonlinearities on PMD-Induced System Impairments,” *J. Lightw. Technol.*, vol. 24, no. 11, pp. 4127–4137, Nov. 2006.
- [Kaz96] T.-K. Chiang, N. Kagi, M. E. Marhic, and L. G. Kazovsky, “Cross-Phase Modulation in Fiber Links with Multiple Optical Amplifiers and Dispersion Compensators,” *J. Lightw. Technol.*, v. 14, n. 3, pp. 249–260, March 1996.
- [Kik10] K. Kikuchi, “Coherent optical communications: Historical perspectives and future directions,” *High Spectral Density Optical Communication Technologies*, pp. 11–49, 2010.
- [Kyl04] P. Kylemark, P. O. Hedekvist, H. Sunnerud, M. Karlsson and P. A. Andrekson, “Noise characteristics of fiber optical parametric amplifiers,” *J. Lightw. Technol.*, v. 22, n. 2, pp. 409–416, Feb 2004.
- [Gor00] J. P. Gordon and H. Kogelnik, “PMD fundamentals: Polarization mode dispersion in optical fibers,” *Proc. Nat. Acad. Sci.*, v. 97, n. 9, pp. 4541–4550, Apr. 2000.
- [ListITA] S. Pelli, G. C. Righini, “Optics in Italy: directory 2010,” Tech. report, Siof, 2010. [Online] Available: <http://www.siof-ottica.it/documenti/annuario10.pdf>.
- [MKi02] C. J. McKinstrie, S. Radic and A. R. Chraplyvy, “Parametric amplifiers driven by two pump waves,” *J. Sel. Top. Quantum Elec.*, v. 8, n. 3, pp. 538–547, May/June 2002.
- [Mec98] A. Mecozzi, “On the Optimization of the Gain Distribution of Transmission Lines with Unequal Amplifier Spacing,” *IEEE Photon. Technol. Lett.*, v. 10, n. 7, pp. 1033–1035, July 1998.
- [Men99] C. R. Menyuk, “Application of multiple-length-scale methods to the study of optical fiber transmission,” *J. of Engineering Mathematics*, v. 36, p. 113–136, 1999.
- [Men06] C. R. Menyuk and B. S. Marks, “Interaction of Polarization Mode Dispersion and Nonlinearity in Optical Fiber Transmission Systems,” *J. Lightw. Technol.*, v. 24, n. 7, p. 2806–2826, 2006.

- [Papoulis] A. Papoulis, "Probability, random variables, and stochastic processes," McGraw-Hill, 2002.
- [Pet92] J. Wang and K. Petermann, "Small-signal Analysis for Dispersive Optical Fiber Communication Systems," *J. Lightw. Technol.*, v. 10, n. 1, pp. 99–100, Jan. 1992.
- [Pog11] P. Poggiolini, A. Carena, V. Curri, G. Bosco and F. Forghieri, "Analytical Modeling of Nonlinear Propagation in Uncompensated Optical Transmission Links," *IEEE Photon. Technol. Lett.*, v. 23, n. 11, pp. 742–744, June 2011.
- [Proakis] J. G. Proakis, "Digital Communications," Mc Graw Hill.
- [Saleh] B. E. A. Saleh and M. C. Teich, "Fundamentals of Photonics," Wiley, 2007.
- [Sal90] A. A. M. Saleh, R. M. Jopson, J. D. Evankow and J. Aspell, "Modeling of Gain in Erbium-Doped Fiber Amplifiers," *IEEE Photon. Technol. Letters*, v. 2, n. 10, pp. 714–716, Oct. 1990.
- [Sav10] S. Savory, "Digital coherent optical receivers: Algorithms and subsystems," *IEEE J. Sel. Topics Quantum Electron.*, v. 16, n. 5, pp. 1164–1179, 2010.
- [Seimetz] M. Seimetz, "High-Order Modulation for Optical Fiber Transmission," Springer: Berlin, 2009.
- [Sig93] K. Sigmon, "Matlab Primer", tech. report, University of Florida, 1993. [On-line] Available: <http://www.math.toronto.edu/mpugh/primer.pdf>.
- [Sin03] O. V. Sinkin, Member, R. Holzlohner, J. Zweck, and Curtis R. Menyuk, "Optimization of the Split-Step Fourier Method in Modeling Optical-Fiber Communications Systems," *J. Lightw. Technol.*, v. 21, n. 1, pp. 61–68, Jan. 2003.
- [Sun96] Y. Sun, G. Luo, J. L. Zyskind, A. A. M. Saleh, A. K. Srivastava and J. W. Sulhoff, "Model for gain dynamics in erbium-doped fibre amplifiers," *Electron. Letters*, v. 32, n. 16, pp. 1490–1491, Aug 1996.
- [Zha08] Q. Zhang, and M. I. Hayee, "Symmetrized Split-Step Fourier Scheme to Control Global Simulation Accuracy in Fiber-Optic Communication Systems," *J. Lightw. Technol.*, v. 26, n.2, pp. 302–316, Jan. 2008.
- [Van02] A. Vannucci, P. Serena and A. Bononi, "The RP method: a new tool for the iterative solution of the nonlinear Schroedinger equation," *J. Lightw. Technol.*, vol. 20, no. 7, pp. 1102–1112, 2002.
- [Wai96] P. K. A. Wai and C. R. Menyuk, "Polarization mode dispersion, decorrelation, and diffusion in optical fibers with randomly varying birefringence," *J. Lightw. Technol.*, vol. 14, no. 2, pp. 148–157, 1996.
- [Whi04] G. M. Whitesides, "Whitesides' Group: Writing a Paper," *Advanced Materials*, v. 16, n. 15, pp. 1375–1377, 2004.
- [Win06] P. J. Winzer and R. J. Essiambre, "Advanced modulation formats," *Proceedings of the IEEE*, v. 94, n. 5, pp. 952–986, May 2006.