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Analytical Evaluation of Improved Access Techniques in Deflection Routing Networks

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Abstract

This paper presents improvements of existing analytical techniques to evaluate the performance of non-priority deflection routing networks in uniform traffic. Pure hot-potato in a ShuffleNet topology will be presented for three polite-access techniques: Transmit-No-Hold, Transmit-Hold and No-Head-of-Line-Blocking.

1. Introduction

Deflection routing [1],[2] has been extensively studied in the framework of optical mesh networks [3],[4] for the drastic hardware savings and control simplification it involves. Simple analytical models have been developed to evaluate the throughput and delay of slotted networks under uniform traffic for deflection routing without buffers (hot-potato) [5],[6],[3], with a limited number of buffers [7],[8],[4], and also with priority rules [9],[10]. Analytical techniques have also been proposed in the case of non uniform traffic [11]. Most of these works analyze two-connected network topologies, such as ShuffleNet (SN) [12] and Manhattan Street Network (MSN) [13]. They are all based on the fundamental assumption of independence of the arrival processes from the input links at each node. Common assumptions are also that 1) each node is capable of receiving packets from both input links in the same time slot (i.e. the node has two receivers), and that 2) a node will inject a packet whenever it has one ready at its transmitter (TX) and at least one of the input slots is free after reception (RX) of packets destined to the node itself. Transmission occurs even if it will cause a deflection. This polite-access technique will be referred to as Transmit-No-Hold (TXNH).

This paper will show how the standard analysis models have to be modified whenever improved polite-access techniques are adopted. Specifically, the transmitter could hold up its packet whenever injecting it would cause a deflection (Transmit-Hold (TXH)). Alternatively, we could imagine having a shared local input queue at the TX; if the head-of-line packet is in conflict with the packets from the input links and its injection would cause a deflection, another packet is chosen from the input buffer whose injection does not cause a deflection. We refer to this TX technique as No-Head-Of-Line-Blocking (NHOLB), as opposed to what happens in First-In-First-Out (FIFO) input queues.

2. Network model

In this section we will analyze the behavior of regular networks, such as SN and MSN, in uniform traffic. Regularity means that each node has the same spanning tree, so that all nodes are topologically equivalent. In uniform traffic, the destinations of packets ready at each node's TX are chosen uniformly among all nodes (except the source) in the network, and independently slot by slot. Regularity and uniform traffic pattern guarantee that the traffic flowing through a node is statistically the same for every node. Hence network performance evaluation is accomplished by focusing on a single node. To make the theory as simple and clear as possible, we choose the simplest node structure for a two-connected network, the one shown in Fig. 1a. The node consists of two local crossbar switches for injection/absorption of traffic destined to the node (Add/Drop block), followed by a crossbar routing switch. The logical flow of node operations is absorption, injection and routing, as depicted in Fig. 1b.

Flow-through slots arrive aligned at the node's inputs i_1 and i_2. They can be empty (E), can have a packet for the node (FN), or a packet that cares to exit on output 1 (C1), on output 2 (C2), or a don't care (DC) packet whenever both node outputs provide equivalent shortest-paths to its destination. When two

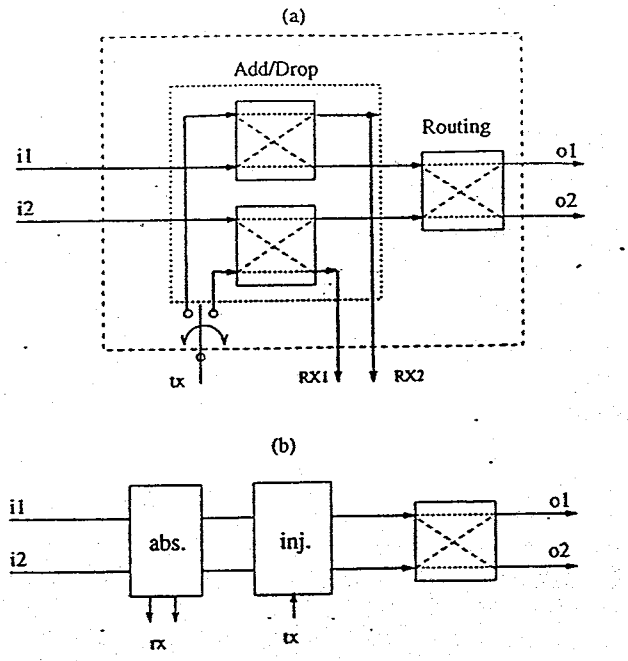

Figure 1: *(a) Physical and (b) logical node structure.*

care competing packets are present at the input links, the non-priority hot-potato routing algorithm will assign one at random to the wished output, and will deflect the second to the remaining output.

Define u as the input slot utilization, i.e. the probability that an input slot carries a packet. Define P_{dc} and r as the probability that an incoming packet is DC and FN respectively. We make here the fundamental assumption that the input arrivals $i_1(k), i_2(k)$ are independent, white, discrete-time wide-sense stationary (WSS) processes with the same first order distribution $f_i(s) = Pr[i_j(k) = s]$, $s \in \{E, DC, C2, C1, FN\}$, $j = 1, 2$, for every time slot k. From the above definitions we have:

$$\begin{aligned} f_i(E) &= 1 - u \\ f_i(DC) &= u\,P_{dc} \\ f_i(C2) &= u(1 - P_{dc} - r)/2 \\ f_i(C1) &= u(1 - P_{dc} - r)/2 \\ f_i(FN) &= u\,r \end{aligned} \qquad (1)$$

where it was assumed that C1 and C2 packets have equal probability. C1 and C2 can be combined in a care set C by defining $f_i(C) \triangleq f_i(C2) + f_i(C1)$.

The TX has no local input queue. TX packets arrive in each slot with probability g, the offered load. If both input links contain a flow-through packet not destined to the node, local blocking occurs and the local packet is discarded. The uniform traffic pattern assures that all destinations except the source are equally likely for TX packets. Let P_{dc0} be the fraction of DC destinations, i.e. those that can be reached from the source from either output link in the same minimal number of hops. Regularity of the network ensures that half of the remaining destinations will be C1 and half C2. With these definitions, the local arrival process $tx(k)$ is a WSS white discrete time process, independent of $i_1(k), i_2(k)$, with first order distribution

$$\begin{aligned} f_{tx}(E) &= 1 - g \\ f_{tx}(DC) &= g\,P_{dc0} \\ f_{tx}(C2) &= g(1 - P_{dc0})/2 \\ f_{tx}(C1) &= g(1 - P_{dc0})/2\,. \end{aligned} \qquad (2)$$

and we define $f_{tx}(C) \triangleq f_{tx}(C2) + f_{tx}(C1)$.

Note that $tx(k)$ is the known network forcing, while the assumptions on $i_1(k), i_2(k)$ are just simplifying ones. Hopefully,

1) the routing will keep a balanced load so that the input distribution f_i will be the same for every link. Simulations show that actually the two input links of each node may have slightly different distributions.

2) i_1 and i_2 will be independent. One can verify that correlations may indeed exist even in uniform traffic, depending on network topology and on the deflection probability [7]. This will in general cause the model to overestimate the throughput.

If conditions 1) and 2) are satisfied, then the model accurately predicts the network behavior.

3. Throughput and Delay evaluation

Given the model, we will now detail the throughput and delay analysis for three polite-access techniques: 1) TXNH, where a TX packet is injected whenever at least one input slot is empty, that is when injection is possible; 2) TXH, where a TX packet is injected when injection is possible *and* there is no conflict with a (possibly present) flow-through packet; and 3) NHOLB, where, if injection is possible and there is a conflict, the present packet is discarded and another packet is drawn uniformly among all destinations non conflicting with the flow-through packet. NHOLB will be evaluated only at full load, $g = 1$, in which case it is equivalent to the case of a saturated infinite shared input queue at the TX.

Intuitively, TXH has lower link utilization u than TXNH, but also lower deflection probability and hence lower average number of hops H [1]. By Little's law [14]

[1] H is the average lifetime of packets before reaching their destination, i.e. the average number of times packets hop before being absorbed.

the throughput per node T, i.e. the average number of packets injected/absorbed per slot by a node, in two-connected networks is given by

$$T = \frac{2u}{H}. \quad (3)$$

Therefore TXH should give higher throughput than TXNH at high loads g, being the lower utilization compensated by the lower H. Even higher throughput should be obtained by NHOLB, since the utilization is the same as for TXNH, while the deflection probability is comparable to that of TXH.

Finally, in hot-potato, the average packet delay is a multiple W of H, where W is the number of slots in flight on each link at any time.

3.1. Slot utilization

Expressions for the steady-state slot utilization u will now be derived. Refer to Fig. 1b. After the absorption block, packets FN are removed and replaced by empty slots. Hence, after absorption, the distribution of the two inputs changes so that $f_i'(E) = (1-u+ur)$ and $f_i'(FN) = 0$. This is the new input distribution. We can now evaluate after absorption the following quantities:

$$\begin{aligned} P_{bce} &= (1-u+ur)^2 \\ P_{ofoe} &= 2u(1-r)(1-u+ur) \\ P_{bcf} &= u^2(1-r)^2 \\ P_{ocoe} &= 2u(1-P_{dc}-r)(1-u+ur) \end{aligned} \quad (4)$$

indicating, respectively, the probability of having both input channels empty (bce), one full and one empty (ofoe), both full (bcf), and one care and one empty (ocoe).

At steady state, at each node, the average number of absorbed packets T_{abs} must equal the average number of injected packets per slot T_{inj} (and both equal the throughput T). Having two independent receivers,

$$T_{abs} = 2ru. \quad (5)$$

From this and (3) one immediately gets

$$r = \frac{1}{H}. \quad (6)$$

The injected throughput is

$$T_{inj} = \sum_{s \in \mathcal{S}} Pr[tx \text{ is inj.}/tx = s]\, Pr[tx = s] \quad (7)$$

where $\mathcal{S} = \{DC, C2, C1\}$. For TXNH, $Pr[tx \text{ is inj.}/tx = s]$ is independent of s and thus

$$T_{inj} = g(1-P_{bcf}), \quad (8)$$

being $(1-P_{bcf})$ the probability that an injection is possible.

In TXH the injection probability is somewhat higher for DCs:

$$T_{inj} = f_{tx}(DC)(1-P_{bcf}) + f_{tx}(C)((1-P_{bcf}) - \frac{P_{ocoe}}{2})$$

since DCs are always transmitted when injections are possible, while Cs are, except when a flow-through care is present and in conflict (with probability 1/2) with tx. This equation can be rewritten as

$$T_{inj} = (1-P_{bcf})\,[f_{tx}(DC) + f_{tx}(C)(1-P_{b0})] \quad (9)$$

where

$$P_{b0} = \left[\frac{P_{ocoe}/2}{(1-P_{bcf})}\right] \quad (10)$$

represents the block probability for TX care packets, i.e. the probability of having a conflicting care flow-through packet conditioned to the event "injection is possible".

Also in NHOLB the injection probability is different for care and don't care packets. When a conflict between, say, a C_1 TX packet and a C_1 flow-through packet occurs, another TX packet is drawn uniformly in the non-conflicting set $\{DC, C_2\}$. A care packet is thus re-drawn with probability

$$\alpha_R = \frac{f_{tx}(C_2)}{f_{tx}(DC) + f_{tx}(C_2)} = \frac{1-P_{dc0}}{1+P_{dc0}}, \quad (11)$$

while a don't care packet is redrawn with probability $(1-\alpha_R)$. Therefore, reasoning as before,

$$\begin{aligned} T_{inj} = {} & (1-P_{bcf})[\{f_{tx}(DC) + f_{tx}(C)P_{b0}(1-\alpha_R)\} \\ & + f_{tx}(C)[(1-P_{b0}) + P_{b0}\alpha_R]], \end{aligned} \quad (12)$$

where the inner term in curly brackets indicates the contribution of don't care packets. As intuition suggests, this reduces to equation (8) as in TXNH, since at $g = 1$ the probability that a TX packet is injected is just the probability that there is room for injection. However we can rewrite (12) as

$$\begin{aligned} T_{inj} = {} & (1-P_{bcf})[f_{tx}(DC)\{1 + 2\alpha_R P_{b0}\} \\ & + f_{tx}(C)\{1 - P_{b0} + P_{b0}\alpha_R\}] \end{aligned} \quad (13)$$

to get an expression similar to (9). The curly brackets indicate the injection probabilities of DCs and Cs given that a packet is ready at the TX and injection is possible.

The equation $T_{abs} = T_{inj}$ can be solved, obtaining $u = u(g, r, P_{dc}, P_{dc0})$.

An equivalent procedure to get u is that of balancing the average number of input and output empty slots. The average number of empty slots arriving from both input links is $2(1-u)$, by the assumed independence of i_1 and i_2. The average number of empty slots on output links for TXH is:

$$(2\,P_{bce}+P_{ofoe})f_{tx}(E)+P_{bce}\,g+\frac{1}{2}P_{ocoe}\,f_{tx}(C)$$

since input empty slots remain empty when TX is empty, only one is filled when both are empty and TX full, and the third term accounts for empties caused by hold-ups. It is absent in the expression for TXNH and NHOLB.

3.2. Deflection probability

In uniform traffic, and with the independence assumption made in the previous sections, the global network traffic is a merging of independent traffic streams directed to each destination. Each stream can be analyzed independently of other streams. Regularity will force all streams to be statistically equivalent. Any packet will be a "typical" packet, whose trajectory towards destination can be seen as a random walk in a homogeneous "gas" of interfering packets. We now evaluate the deflection probability d of a test packet entering a care node (with respect to its destination) from an input link, and the deflection probability d_0 of a test care packet at its injection node.

Refer again to Fig. 1b. The flow-through care test packet is at one of the two inputs and bypasses the absorption and injection blocks, reaching the routing block. The probability P_b that another care conflicting packet reaches the routing block on the second channel is, for TXNH:

$$P_b=\frac{1}{2}\{f_i(C)+f_i'(E)\,f_{tx}(C)\} \qquad (14)$$

where 1/2 is the probability that the care test and the other care collide.

In TXH and NHOLB this reduces to

$$P_b=\frac{1}{2}\,f_i(C) \qquad (15)$$

since a conflicting packet is never injected at the TX.

P_b is the probability of a conflict for the test packet. Thus, in no-priority hot-potato, the flow-through deflection probability for the care test is:

$$d=P_b/2\,. \qquad (16)$$

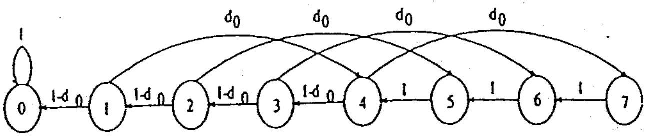

Figure 2: *Markov chain describing the random walk of the test packet in a SN(2,4) topology.*

As for the initial deflection probability of a care test, d_0, this is trivially zero for TXH and NHOLB, while for TXNH it is

$$d_0=P_{b0}/2\,, \qquad (17)$$

where P_{b0} was derived in (10).

3.3. Average number of hops and don't care probability

This section will derive expressions for the average number of hops H and the don't care probability P_{dc}.

The random walk of the test packet towards its destination can be modeled as an absorbing markov chain whose states coincide with the network nodes [3], [4].

For some topologies, like SN, it is possible to speed up the computation by drastically reducing the number of states in the chain. This is done by combining together in a single state all nodes with same distance to destination. The test packet thus performs a random walk on the integers $0,1,..d_{max}$, where d_{max} is the maximum distance to destination [15].

The solution procedure presented next can be applied to any regular topology, whether or not a reduced state-space can be obtained. However, for illustration purposes, a SN topology will be used.

A specific example of the absorbing markov chain describing the random walk of the test packet toward its destination is given in Fig. 2 for a 64-node SN(2,4) topology.

A SN(q,k) topology has $N=k\,q^k$ nodes arranged in k columns of q^k nodes each, and there is a perfect shuffle connection among nodes in adjacent columns [12]. The maximum distance between nodes is $d_{max}=2k-1$. Fix a destination node. All nodes reachable in less than $k+1$ hops proceeding backwards are care with respect to that destination. All the remaining nodes, at distance $k+1,..,2k-1$ are don't care. A deflection of the test packet flowing towards that destination at a node at distance i brings the packet back to the set of nodes at distance $i+k-1$. Finally, the number of nodes $n(i)$ at distance i is

$$n(i)=\begin{cases} q^i & 1\le i\le k-1 \\ q^k-q^{i-k} & k\le i\le 2k-1 \end{cases} \qquad (18)$$

Fig. 2 refers to the initial step of the walk, where the packet is at its injection node. The labels indicate the transition probabilities. For every step after the first hop, in which the packet is at the TX port of the node, label d_0 gets changed into d. The states represent the distance in hops of the test packet to its destination. State 0 is the absorbing state of the chain.

This reduced-state chain is not directly applicable to topologies like MSN in which a set of nodes at the same distance can be partly care and partly don't care.

For all steps $t = 1, 2, ...,$ the transition probabilities $\pi(l, m)$ from state m to state l, $l, m = 0, 1, .., 7$, can be organized in a transition matrix $\Pi = \{\pi(l, m)\}$. Analogously, a matrix Π_0 can be written for the injection step $t = 0$.

Since 0 is the only absorbing state, matrix Π is in canonical form. Taking off the first row and the first column, a matrix Q^T is obtained. From this, the fundamental matrix of the absorbing chain $\mathcal{N} = (I - Q)^{-1}$ is obtained, where I is the 7x7 identity matrix. The entries of $\mathcal{N} = \{n(l, m)\}$, $l, m = 1, .., 7$, give the expected number of times in each nonabsorbing state m for each possible nonabsorbing starting [2] state l [16].

Let $\mathbf{p}_0$ be the probability state (column) vector at the injection step. The state after the first hop is $\mathbf{p}_1 = \Pi_0 * \mathbf{p}_0$. Let $\hat{\mathbf{p}}_1$ and $\hat{\mathbf{p}}_0$ indicate respectively $\mathbf{p}_1$ and $\mathbf{p}_0$ with the first component removed. The i^{th} entry of vector $\mathcal{N}^T \hat{\mathbf{p}}_1$ represents the average number of visits before absorption of state i after the first hop. The sum of all entries [3] is thus the average number of hops excluding the first hop

$$\hat{H} \triangleq ||\mathcal{N}^T \hat{\mathbf{p}}_1||_1 .$$

Since $\hat{\mathbf{p}}_0$ represents the visits at the first hop, the average number of hops before absorption is

$$H = ||\hat{\mathbf{p}}_0 + \mathcal{N}^T \hat{\mathbf{p}}_1||_1 = \hat{H} + 1 . \qquad (19)$$

The sum of the entries of $\mathcal{N}^T \hat{\mathbf{p}}_1$ relative to don't care states $i = 5, 6, 7$ represents the expected number of visits EN_{dc} before absorption at don't care nodes at which the test packet is flow-through. The don't care probability P_{dc} is estimated as the fraction of time the test packet is don't care flow-through

$$P_{dc} = \frac{EN_{dc}}{H} . \qquad (20)$$

[2] Where *starting* here means after the first hop.

[3] $|| \bullet ||_1$ indicates vector norm 1.

This procedure, making use of the fundamental matrix of the absorbing chain, can be faster than previously reported iterative methods [3], [4] when efficient matrix-inversion algorithms are available.

A very delicate point is to properly establish the value of $\mathbf{p}_0$. $\mathbf{p}_0$ must sum to one, as we are following the random walk of a single injected test packet destined to 0. Each entry $\mathbf{p}_0(i)$ represents the probability that a packet destined to 0 has been *injected at distance* i, conditioned on the event "one packet destined to 0 has been injected in the network". Now, from (18) and the uniform traffic assumption, the conditional probability that a packet for 0 has been *generated* at distance i is $\frac{n(i)}{N-1}$. This also represents $\mathbf{p}_0(i)$ when TXNH is adopted, since care and don't care TX packets have the same injection probability. However, in TXH, the injection probabilities for TX don't care packets and TX care packets are in ratio 1 to $(1 - P_{b0})$, as can be inferred from equation (9). Therefore $\mathbf{p}_0$ must first be changed in

$$\mathbf{p}_0(i) = \frac{n(i)}{N-1}(1 - P_{b0}) \qquad (21)$$

for all distances i corresponding to care nodes, namely $i = 1, 2, 3, 4$ in our example, and then renormalized to one.

Finally, in NHOLB, an analogous rescaling is done by looking at equation (13). $\mathbf{p}_0$ must first be modified in

$$\mathbf{p}_0(i) = \frac{n(i)}{N-1}\{1 - P_{b0} + P_{b0}\,\alpha_R\}$$

for the care states $i = 1, 2, 3, 4$, and

$$\mathbf{p}_0(i) = \frac{n(i)}{N-1}\{1 + 2\alpha_R P_{b0}\} \qquad (22)$$

for the don't care states $i = 5, 6, 7$, and then renormalized to one.

4. Results and Conclusions

It will now be shown how the results obtained so far must be used to get the desired expressions of the throughput T as a function of the free parameter g, the offered load. The procedure is essentially iterative. We start with an initial guess of the quantities $[d, d_0]$. Using the results of section 3.3, the average number of hops H and the don't care probability P_{dc} are expressed as functions of d, d_0, P_{dc0}. Then $r = 1/H$ is obtained. Next $u = u(g, P_{dc0}, r, P_{dc})$ is evaluated as outlined in section 3.1. Finally new values for $[d, d_0]$ are obtained from equations (10)–(17). The process is repeated up to convergence of $[d, d_0]$. Note that P_{dc0}

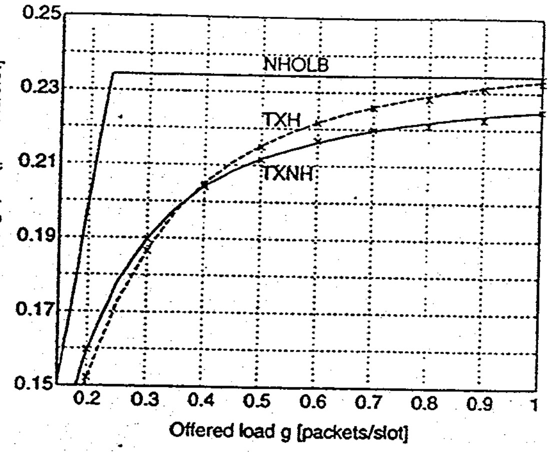

Figure 3: *Throughput in a 64-node SN for Transmit-No-Hold (TXNH), Transmit-Hold (TXH) and infinite shared input buffers (NHOLB).*

is a known quantity when the topology is chosen. In SN(q,k), using (18), one gets

$$P_{dc0} = \frac{k\,q^k - \frac{q\,(q^k-1)}{q-1}}{k\,q^k - 1}.$$

Fig. 3 shows the analytical results for a 64-node SN. Simulations results are marked with crosses. There is excellent agreement between the analytical model and the simulations. It is confirmed that TXH gives improved throughput at high offered loads g. The highest achievable throughput for pure hot-potato is that of NHOLB, for which only the saturation throughput is meaningful.

Aside from the numerical values of this simple case, it is important to note the precision of the analytical model.

The key point in the derivation, where the present model extends the previously published models, lies in equations (21)–(22). If the initial state probability p_0 is erroneously chosen as the *generation* instead of the *injection* probability for TXH and NHOLB, then extremely optimistic throughput values are obtained.

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