Capacity Maximization of Power-constrained Submarine Systems

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Abstract: We review a novel semi-analytical approach to the achievable information rate maximization of submarine links with gain-shaped EDFAs, and provide the optimal pre-emphasis in presence of both amplified noise and fiber nonlinearity.

1. Introduction

Submarine systems are pump power constrained, and their achievable information rate (AIR) can be increased by both extending the used optical bandwidth beyond that of the erbium-doped fiber amplifiers (EDFA), and/or by using ideally decoupled multiple parallel spatial modes through space division multiplexing (SDM) [1]. We here focus on maximizing AIR on a single mode of an SDM submarine system that uses gain-shaped EDFAs, by accounting for the EDFA physics. Such a maximization was pioneered in [2] by brute-force numerical optimization. In [3] we tackled the maximization by a novel semi-analytical approach, which consists of fixing the inversion x of the first EDFA (the state-variable of the link) and analytically maximizing $AIR(x)$ over the space of x-feasible input wavelength division multiplexed (WDM) signals, called here power spectral distributions (PSD). Then the maximum over all inversions x is numerically obtained. This procedure allows to "look inside" the complex dependence of AIR on the input PSD (a.k.a. preemphasis), and gain useful design insights [3]. This paper reports on advancements of this novel approach in the study of the simplest yet realistic submarine system, called the constant PSD (CPSD) link, where each end-line EDFA is followed by a suitable gain-shaping filter (GSF) that restores the input PSD at every span, so that the PSD is constant along the link. If all spans are identical, with the same pump P_p at every EDFA, then the inversion x and thus the needed input-PSD dependent GSF are the same at all EDFAs. We here extend the analysis in [3] to include fiber nonlinearity.

2. CPSD link

We consider the transmission of a WDM signal along a link of M end-amplified single-mode fiber spans, see Fig. 1a. We discretize the frequency axis in bins of width Δf centered at fequencies $\{f_j\}$. The span attenuation at f_j is $A_j > 1$. Each end-span amplifier consists of an EDFA of gain G_j at f_j followed by a GSF of gain $h_j \le 1$, for a total amplifier gain $\mathscr{G}_j = G_j h_j$. The N_c WDM channels of bandwidth Δf occupy the whole frequency set $\mathscr{B} = \{f_j : G_j(x) > A_j\}$ where gain exceeds attenuation. As shown in Fig. 1b, the GSF gain h_j is chosen such that the shaped-amplifier gain $\mathscr{G}_j$ equals $A_j\chi_{aj}$, with a net span gain (or droop) $\chi_{aj} < 1$, adjusted such that the span-output photon flux Q_j (or equivalently the power P_j) at f_j equals the span-input (i.e., the transmitted (TX)) flux. Such a droop is necessary to keep a constant PSD after addition at each span of the amplified spontaneous emission (ASE) equivalent input noise $\delta Q_{aj} = F_j\Delta f$ generated at the EDFA, where F_j is the noise figure. After

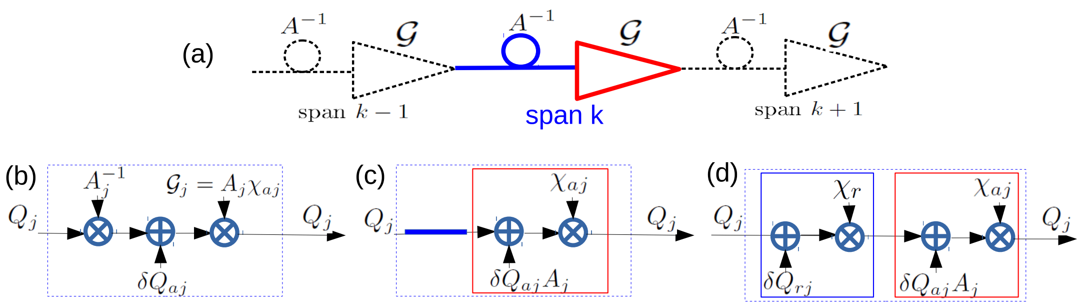

Fig. 1. (a) Span scheme; (b) Span block diagram; (c) equivalent diagram; (d) diagram with NLI

factoring out the attenuation, we obtain the equivalent scheme in Fig. 1c. So far the fiber is an ideal block that just attenuates. It is however possible to include in the model any power-conserving linear or nonlinear crosstalk added as a perturbation to the signal during fiber propagation [4]. To this aim, as shown in Fig. 1d we add at the span input a power-rearrangement sub-block, where the rearrangement perturbation δQ_{rj} is added to the signal Q_j and then multiplied by a *rearrangement droop* χ_r chosen such that total WDM power is conserved at the fiber end: $\sum_{j=1}^{N_c} hf_j Q_j = \left(\sum_{j=1}^{N_c} hf_j(Q_j+\delta Q_{rj})\right)\chi_r$, where h is Planck's constant. In this paper the only rearrangement mechanism we consider is the nonlinear interference (NLI) produced by the Kerr effect within the propagating mode, and calculated by the Gaussian-Noise (GN) first-order regular perturbation model [5], as we detail next. It has been shown that introducing power-renormalization at each span yields a more accurate modeling of NLI that holds over an extended power and distance range [6]. From the augmented model of Fig. 1d, we read off the in/out flux balance $[(Q_j+\delta Q_{rj})\chi_r+\delta Q_{aj}A_j]\chi_{aj}=Q_j$, from which we find the net span gain (i.e., droop): $\chi_j=\chi_{aj}\chi_r\cong\left(1+\frac{\delta Q_{rj}}{Q_j}+\frac{\delta Q_{aj}}{Q_j}A_j\right)^{-1}$ and indeed we choose the GSF as $h_j=A_j\chi_j/G_j$. The received signal at bin j after M spans is thus $Q_j^{SIG,RX}=\chi_j^M Q_j$ and the ASE+NLI noise is its complement to Q_j: $Q_j^{NOISE,RX}=(1-\chi_j^M)Q_j$, since Q_j is the preserved output flux at bin j. Thus the RX SNR at bin j is $SNR_j=({\chi_j}^{-M}-1)^{-1}$ and the AIR (per 2-polarizations mode) of a signal-independent additive Gaussian noise channel is $AIR=\sum_{j=1}^{N_c} 2\Delta f\log_2(1+\Gamma SNR_j)$, where $0\le\Gamma\le 1$ is an implementation penalty.

3. NLI model

If we account only for the dominant cross-phase modulation effects in highly dispersive links we get for the NLI rearrangement noise from the GN model [7]: $\frac{\delta Q_{rj}}{Q_j}=c_1\sum_n \gamma_{n,j}Q_n^2$ with: 1) $c_1=\frac{16}{27}\gamma^2L_{eff}^2$ (γ [W^{-1}km^{-1}] the nonlinear fiber coefficient and L_{eff} the effective length); 2) $\gamma_{n,j}=\frac{h^2f_n^2}{\Delta f^2}(2-\delta_{n,j})\Psi_{n,j}$, where: $\delta_{n,j}=1$ if $n=j$ and 0 else; the equivalent *per-span* [6, Appendix B] contribution for cross-channel interference (XCI) ($n\neq j$) is [7, eq. (124)]: $\Psi_{n,j}=\frac{A_+-A_-}{4\pi|\beta_2|\alpha^{-1}}$ where β_2 is the fiber dispersion coefficient, α is the bandwidth-average (power) fiber loss, and $A_\pm=\operatorname{asinh}(\pi^2|\beta_2|\alpha^{-1}(f_n-f_j\pm\Delta f/2)\Delta f)$; finally the self-channel interference (SCI) is [7, eq. (125)]: $\Psi_{j,j}=M^\varepsilon\frac{\operatorname{asinh}(\frac{\pi^2}{2}|\beta_2|\alpha^{-1}\Delta f^2)}{2\pi|\beta_2|\alpha^{-1}}$, with ε the coherence factor [7, eq. (126)].

4. EDFA model

The EDFA physical model we use is the Saleh extended model (see [3, Appendix]), where the EDFA gain is $G_j(x)=e^{\ell((\alpha_j+g_j^*)x-\alpha_j)}$ where x is the doped-fiber inversion, ℓ is its length, and g_j^*,α_j (m^{-1}) are the erbium emission and absorption coefficients [2, Fig. 7]. ASE flux amplified inside the EDFA (forward and backward) is: $Q_{ASE}^{F+B}\cong 2\sum_l 2n_{sp,l}(x)(G_l(x)-1)\Delta f$ (ph/s) and is calculated over the ASE frequency bins; $n_{sp,l}(x)$ is the spontaneous emission factor at f_l, and the noise figure is $F_l=2n_{sp,l}\frac{G_l-1}{G_l}$. The equivalent-input forward ASE flux at f_l over band Δf is $F_l\Delta f$. If we use input WDM fluxes Q_j^{in}, $j=1,\ldots,N_c$, the steady-state photon flux balance at the EDFA is given by the extended Saleh equation (ESE) [3, Appendix]: $\sum_{j=1}^{N_c} Q_j^{in}(G_j(x)-1)=K(x,Q_p)$, where the parameter $K(x,Q_p)\triangleq Q_p(1-G_p(x))-\frac{r_M}{\tau}x-Q_{ASE}^{F+B}(x)$ is the pump flux that gets converted into the EDFA output signal flux; here Q_p is the pump flux, $G_p<1$ the pump gain, τ the fluorescence time, and r_M the total number of erbium ions in the EDFA. The ESE therefore includes self-gain saturation by ASE.

5. AIR maximization at fixed x

The trick of fixing the inversion x and optimizing over the set of WDM input PSDs that achieve x at the given pump flux Q_p (the "x-feasible set", i.e., the set of TX flux vectors $\underline{Q}=[Q_1,..,Q_{N_c}]$ that satisfy the ESE with $Q_j^{in}=Q_j/A_j$) drastically simplifies the analysis of the CPSD link. The reason is that the gain $G(x)$ is fixed, and so is the set $\mathscr{B}(x)$ and thus the number of channels $N_c(x)$, as well as the $K(x)$ parameter. Thus our problem is that of maximizing AIR subject to the ESE constraint. For this we form the Lagrangian : $L(\underline{Q})=\sum_{j=1}^{N_c(x)}2\Delta f\log_2(1+\Gamma(\chi_j(x,\underline{Q})^{-M}-1)^{-1})-\lambda\sum_{j=1}^{N_c(x)}\frac{Q_j}{A_j}(G_j(x)-1)$, where from Section 3: $\chi_j(x,\underline{Q})\cong(1+c_1(\sum_n\gamma_{n,j}Q_n^2)+\frac{A_jF_j(x)\Delta f}{Q_j})^{-1}$, and $\lambda>0$ is the Lagrange multiplier. By setting $dL/d\underline{Q}=\underline{0}$ we find the optimal "pre-emphasis" $\underline{Q}$ as the solution of the nonlinear system of equations for all $k=1,\ldots,N_c$:

$$Q_k=\frac{A_kK(x,Q_p)}{G_k(x)-1}\frac{g_k(\underline{Q})}{\sum_{l=1}^{N_c}g_l(\underline{Q})} \tag{1}$$

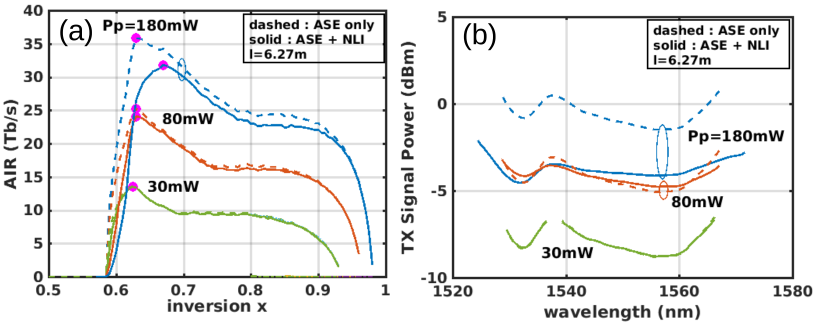

Fig. 2. (a) AIR vs. EDFA inversion x at EDFA length $\ell = 6.27$m at several pump values (dashed: ASE only; solid: ASE+NLI.). (b) TX input PSD at top AIR (circles in (a)) (c) top AIR vs. pump power at optimized EDFA ℓ. Symbols: our CPSD link; lines: values from ([2] Fig. 4a).

where $g_k \triangleq f(\chi_k)\frac{A_k F_k \Delta f}{Q_k} - 2c_1 Q_k^2 \sum_{j=1}^{N_c} f(\chi_j)\gamma_{k,j}$ and $f(\chi) \triangleq \frac{\chi^{M+1}}{(1-\chi^M)(1-\chi^M(1-\Gamma))}$. In absence of NLI, eq. (1) becomes eq. (18) in [3], which is solved with a simple fixed-point algorithm [3, eq. (19)]. The bad news is that with NLI the right-hand side of (1) ceases to be a contraction, and the solution of (1) requires a nonlinear solver.

6. Numerical results

We consider a $M = 287$ span submarine link with single-stage EDFAs as studied in [2, 3]. Spans have 50.9km of pure silica core fiber with attenuation 0.162 dB/km, nonlinear coefficient $n_2 = 2.5 \cdot 10^{-20}$ m^2/W, effective area 130 μm^2 and dispersion 21 ps/nm/km. Including 1.25dB of connectors losses, the total span attenuation is $A = 9.5$dB. The EDFA absorption and emission coefficients are those in ([2] Fig. 7). Fig. 2a shows the AIR versus inversion x at several pump powers and at EDFA length 6.27m, with only ASE (dashed), and also including NLI (solid). Fig. 2b shows the corresponding input TX WDM power (preemphasis) versus wavelength at top AIR (magenta circles in (a)). We note that the AIR decrease due to NLI starts to be visible at pumps $P_p \geq 80$mW, with a marked increase of the optimal inversion at the largest pump. Fig. 2b shows that the inverse gain-shaped optimal preemphasis in absence of NLI [3] requires large powers as the pump increases, while the optimal preemphasis with NLI (solution of eq. (1)) is flatter and tends to converge to a common average power per channel as the pump increases. Finally, Fig. 2c reports with symbols the top AIR versus pump power for our CPSD link with optimized EDFA length ℓ at each point. Fig. 2c also reports for comparison the top AIR values for the "constant-signal" link tackled in ([2], Fig. 4a), and known to be quite close to our CPSD link [3]. We verified that with NLI most of the AIR increase as pump increases seen in Fig. 2c is due to an increase of the supported bandwidth.

7. Conclusions

We reviewed our recent semi-analytical optimization procedure for AIR maximization of power-constrained submarine links, and extended it to include NLI. With only ASE the optimal preemphasis has an inverse gain-shaped wavelength profile, while with NLI it becomes flatter and the optimum inversion shifts to larger values.

References

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