The Gaussian Noise Model Extended to Polarization Dependent Loss and its Application to Outage Probability Estimation

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Abstract *We extend for the first time the Gaussian-noise model to account for polarization dependent loss (PDL) and validate it both numerically and experimentally. The model can be used to estimate outage probabilities induced by PDL-nonlinearity interaction in fast simulation times.*

Introduction

There is recently a surge of operators' interest in electrical signal to noise ratio (SNR) monitoring in multi-vendor optical networks for quality-of-transmission path selection[1]. To this aim, the Gaussian Noise (GN) model can be adopted, because of its simplicity and reasonable accuracy[2,3]. However, so far the GN model does not include polarization-related impairments such as polarization mode dispersion (PMD) and polarization dependent loss (PDL) of optical fibers and devices. While PMD has been shown to have a minor effect in dispersion-uncompensated coherent links[4], PDL may lead to substantial penalties and to unexpected outages[5,6]. PDL induces random SNR unbalance between the polarization tributaries and loss of orthogonality. While the second effect can be equalized by a multiple input multiple output (MIMO) equalizer, the SNR-unbalance cannot be equalized by a MIMO without boosting some frequencies of noise. The problem is even more challenging in the nonlinear regime where nonlinear effects are modified by PDL[5,6].

The purpose of this paper is to extend the GN model to include polarization dependent loss (PDL) and provide simulative and experimental evidence of its accuracy.

The Model

According to GN-model theory, the SNR of a signal of power P is related to the amplified spontaneous emission (ASE) variance σ^2_{ASE} and the nonlinear interference (NLI) variance σ^2_{NLI} by:

$$\text{SNR} = \frac{P}{\sigma^2_{\text{ASE}} + \sigma^2_{\text{NLI}}}\,.$$

In presence of PDL, both σ^2_{ASE} and σ^2_{NLI} are random variables. We refer to the link of Fig. 1, where PDL of span n is described by a matrix $\mathbf{T}_n$. After linear equalization, the first order perturbative

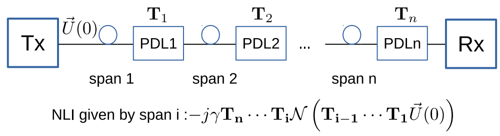

Fig. 1: Notation for the link with PDL. $\mathcal{N}(\vec{U}) \triangleq -j\gamma(\vec{U}^\dagger\vec{U})\vec{U}$.

nonlinear Kerr term given by the electric field $\vec{U}_n$ at frequency f_n and coordinate z changes with PDL as:

$$-j\gamma\vec{U}_k^\dagger\vec{U}_\ell\vec{U}_m \xrightarrow[\text{PDL}]{} -j\gamma\left[\vec{U}_k^\dagger\mathbf{P}_{1i}\vec{U}_\ell\right]\vec{U}_m$$
$$f_n = f_\ell + f_m - f_k$$

where † means transpose-conjugate, while matrix $\mathbf{P}_{1i} = (\mathbf{T}_1\cdots\mathbf{T}_i)^\dagger(\mathbf{T}_1\cdots\mathbf{T}_i)$ is a positive definite matrix accounting for PDL power-unbalance up to span $i = \lfloor z/L \rfloor$, L being the span length.

Such a Kerr effect leads to NLI at the receiver. We refer to it by the signal $\vec{u}$. In order to generalize the GN model to PDL we need the NLI covariance matrix, defined as the following mean:

$$\mathbf{R}_u = \mathbb{E}\left[\vec{u}\vec{u}^\dagger|\text{PDL}\right]$$

We conditioned the covariance matrix to the PDL realization, i.e., to specific realizations of matrices $\mathbf{P}_{1i}$, since PDL is slowly varying in time compared to symbol-induced interference[5]. Elements $(1,1)$ and $(2,2)$ of $\mathbf{R}_u$ are σ^2_{NLI} of x-and y-polarizations, respectively. We evaluated $\mathbf{R}_u$ by generalizing the spatially-resolved theory of the GN model[7], obtaining for a n-span link:

$$\mathbf{R}_u = \sum_{i,k=0}^{n-1}\left(\text{Tr}\left[\mathbf{P}_{1k}\mathbf{P}_{1i}\right]\mathbf{I} + \mathbf{P}_{1k}\mathbf{P}_{1i}\right)R_{ik} \qquad (1)$$

$$R_{ik} = \int_{-\infty}^{\infty}\left|\tilde{H}(\omega)\right|^2\iint_{-\infty}^{\infty}\eta_i\eta_k^*\tilde{S}(\omega+\omega_\nu)\tilde{S}(\omega+\omega_\epsilon) \times\tilde{S}(\omega+\omega_\nu+\omega_\epsilon)\frac{\text{d}\omega_\nu}{2\pi}\frac{\text{d}\omega_\epsilon}{2\pi}\frac{\text{d}\omega}{2\pi} \qquad (2)$$

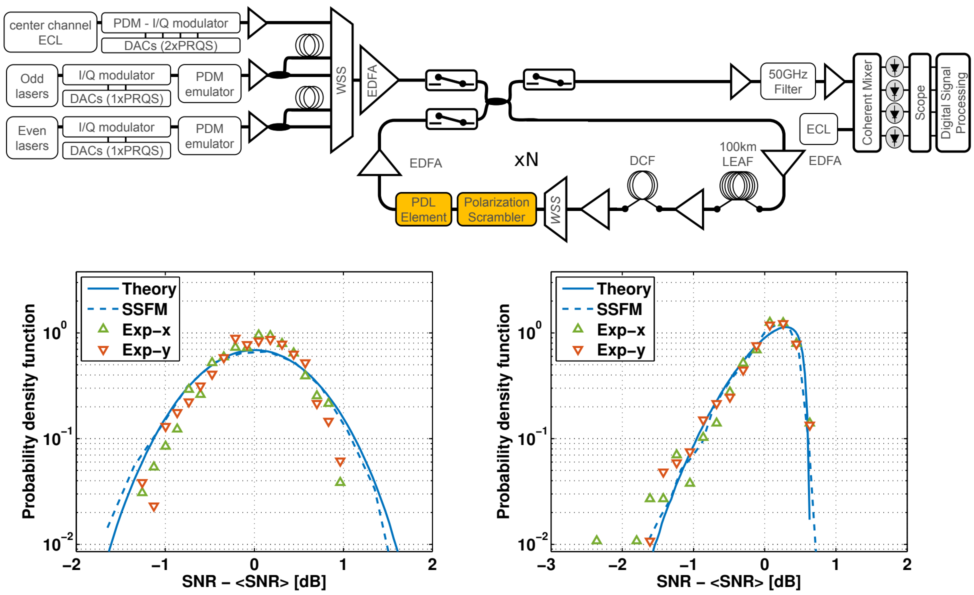

Fig. 2: Top: experimental setup for PDL investigation. Bottom: probability density function of SNR in ASE- (left) and nonlinearity-dominated regime (right) given by i) GN-model extended to PDL, ii) split-step Fourier simulations (SSFM), iii) experimental measurements. $N = 10$ spans.

with $\mathbf{I}$ identity matrix and $\mathrm{Tr}[.]$ matrix-trace. $\tilde{H}(\omega)$ is the transfer function of the detection filter at angular frequency ω, $\tilde{S}(\omega)$ the power spectral density of the transmitted signal while $\eta_i = \frac{8}{9}\gamma P \exp(j\beta_2\omega_\nu\omega_\epsilon L_i)/(\alpha - j\beta_2\omega_\nu\omega_\epsilon)$, with $L_i \gg 1/\alpha$, is the fiber kernel of span i (α attenuation, β_2 dispersion, γ nonlinear coefficient) of the classical GN-model. Although three integrals are present in (2), its value can be computed very efficiently, for instance, through the Monte-Carlo based algorithm[8] with minor modifications. All the randomness of PDL is collected in the double summation of (1), whose numerical simulation is order of magnitudes faster than classical split-step Fourier simulations applied to the entire optical link by iterating random seeds.

Experimental investigation and validation

We double checked the proposed model by an experimental validation of the link reported in Fig.2. We modulated 13 polarization division multiplexing quadrature phase shift keying (PDM-QPSK) channels at symbol rate 32.5 Gbd, spacing 50 GHz. We use the typical approach of modulating all even/odd interfering channels by a unique modulator and then decorrelating the resulting signals by delaying the channels. The link was made of 10×100 km of D=4.3 ps/nm/km fibers, with residual dispersion per span of 40 ps/nm, implemented by a recirculating loop. PDL was emulated by a polarization scrambler followed by a PDL element, calibrated off-line with 1 dB of PDL. The polarization scrambler was synchronized with the loop. We ensure that scrambling did not occur during the acquisition window of the oscilloscope by properly tuning the scrambling window. At the receiver, after recovering linear impairments and carrier phase, we estimated the SNR of each polarization tributary from the variance of the constellation clouds. We worked either 4 dB below or 4.5 dB above the nonlinear threshold, i.e., the power of best SNR, to ensure that either ASE or NLI dominated the total variance.

Fig. 2(left) shows the probability density function (PDF) of SNR due to PDL fluctuations in ASE dominated regime. We observe a good match between theory[9], split-step Fourier method (SSFM) based numerical simulations and experimental data, represented by different symbols for X and Y polarizations, respectively. We used this figure as a first sanity check for our PDL modeling. Fig. 2(right) refers to the case with dominant NLI. The solid line is the new GN-model accounting for PDL. In this case, we estimated PDL-free PSD of the NLI, and then applied random realizations of PDL as described in the previous section. The numerical simulation of (1) is very fast, allowing simulation of thousands of random seeds per minute. The dashed line is the results of split-step Fourier simulations after testing 1500 random seeds. We observe an excellent match among all results.

Numerical extensions

The experimental validation was for a dispersion-managed link. Here we numerically extend the investigations to dispersion uncompensated links

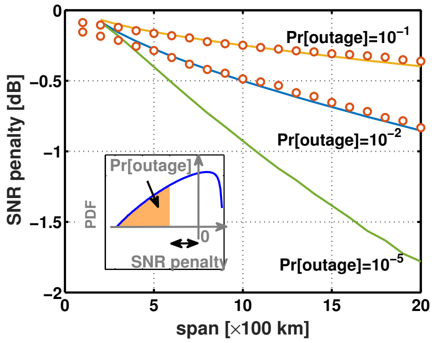

Fig. 3: SNR penalty [dB] at PDL outage probability of 10^{-1}, 10^{-2} or 10^{-5}. PDL per span: 0.5 dB. Solid lines: GN-model extended to PDL. Symbols: Monte Carlo split-step Fourier simulations. Dispersion uncompensated link.

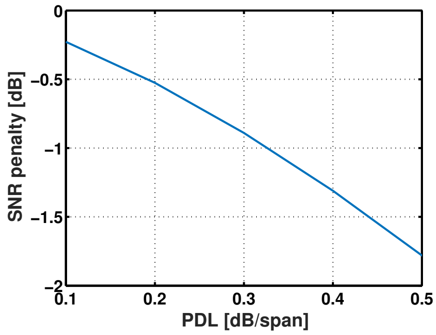

Fig. 4: SNR penalty at outage probability of 10^{-5} (see inset of Fig. 3) after 20 spans versus PDL.

with the following differences with the previous setup: 1) modulation format is 16QAM, 2) 21 channels spaced 37.5 GHz, 3) PDL of 0.5 dB per span, 4) single-mode fibers, 5) theory is double-checked by split-step simulations only.

We ran SSFM simulations by collecting at least 2000 seeds of PDL. For each PDL seed we sent 28800 symbols to get accurate SNR Monte-Carlo estimations. After each span we locally received the central channel by a coherent detector with matched filter recovering all the linear accumulated impairments. From the collected data we estimated the PDF of the SNR by the histogram method, and then evaluated the area under the left tail up to a prescribed SNR, as sketched in the inset in Fig. 3. We interpret the area as the outage probability at the selected SNR penalty with respect to the average SNR. Such SNR penalty can be read as the margin that one should allocate against PDL impairments.

SNR penalty at fixed outage probability is plotted versus link length in Fig. 3. Symbols refer to SSFM simulations, solid lines to the theoretical model. Reliable Monte Carlo estimations of SSFM simulations could be achieved only down to 10^{-2} outage probabilities for practical time limitations. The good match is an indication of the validity of the proposed theory. The theoretical model, however, can even be used to estimate much rare events, such as a practical outage probability of 10^{-5} as reported in Fig. 3 versus link length and in Fig. 4 versus PDL at 20 spans.

From Fig. 4 we observe that even a small value of PDL of 0.1 dB may yield a sizable penalty of $0.2-0.3$ dB.

Conclusions

We extended the GN-model to account for PDL. While according to GN-model both ASE and NLI are Gaussian distributed and generated along the link, we showed that PDL acts on NLI statistics in a different way than on ASE statistics. The model showed a good match both with experimental data and with split-step Fourier simulations. One of the main advantages of the model is its computational speed, where each PDL realization can be tested in a fraction of second.

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