A parametric gain approach to DPSK performance evaluation in presence of nonlinear phase noise

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Abstract *We show that in dispersion-managed DPSK systems the nonlinear phase noise can be described by a Gaussian stochastic process whose statistics can be simply derived from a linearized solution of the nonlinear Schrödinger equation.*

Introduction

During propagation in an optical fibre link, the amplified spontaneous emission (ASE) noise interacts with the transmitted signal through a four-wave mixing (FWM) process. Such an effect, also known as parametric gain (PG), is the main impairment of phase modulated signals, such as differential phase shift keying (DPSK), and manifests itself as a nonlinear phase noise [1]. For bit error rate (BER) evaluation, the PG statistics are needed.

Recently, an experiment has shown that, in 10 Gb/s DPSK systems, the statistics of the sampled noise after demodulation are not Gaussian, but resemble an exponential distribution [2]. It has also been shown theoretically that, in the absence of group velocity dispersion (GVD), the received optical noise (i.e. nonlinear phase noise) statistics can be described by a linear combination of independent chi-square random variables [3].

In this paper, we show that in actual systems, working at sufficiently large optical signal-to-noise ratios (OSNR) and where a small amount of local GVD is present, the received optical noise can still be described by a Gaussian stochastic process. Hence, for BER evaluation, it is possible to use a standard Karhunen-Loéve expansion of a Gaussian colored noise in quadratic receivers, avoiding the simplified receiver model used in [3]. Instead of evaluating the BER by the time-consuming algorithm proposed in [4], which correctly accounts for the actual non-stationarity of the received Gaussian noise, we provide a novel small-signal model for the ASE propagation, using a linearized solution of the NLSE versus a slow-varying reference signal, which accounts for an “average” impact of the signal on the phase noise. Aided by the periodic behaviour of DPSK formats, our model describes the received PG-noise as a Gaussian cyclostationary process, whose power spectral density (PSD) can be rapidly evaluated in a closed form, from which one can evaluate the BER for instance by using the algorithm proposed in [5] .

Phase noise statistics

In [3] it has been shown that in a single channel DPSK system propagating in absence of GVD the nonlinear phase noise rotation due to the self-phase modulation (SPM) can be described by a linear combination of chi-square random variables. We verified such a conclusion by estimating the probability density function (PDF) of the received optical noise through a Monte Carlo simulation of non-return to zero NRZ-DPSK of a one fully compensated span operating at OSNR=25 dB in a resolution bandwidth of 0.1 nm, with transmission fibre GVD equal to either 0 or 4 ps/nm/km. At zero GVD we obtained the results shown in Fig. 1(left), which clarify that SPM alone yields a PDF far from Gaussian. By including fibre GVD, we obtained the contour plots shown in Fig. 1(right), which have elliptical shape, typical of a Gaussian distribution. Hence, at sufficiently large OSNRs, a small amount of fibre GVD tends to reshape the noise PDF towards a Gaussian distribution. Being the single span a worst case, such a conclusion holds also for multiple spans, providing that the OSNR or the local GVD are not too small, according to [6].

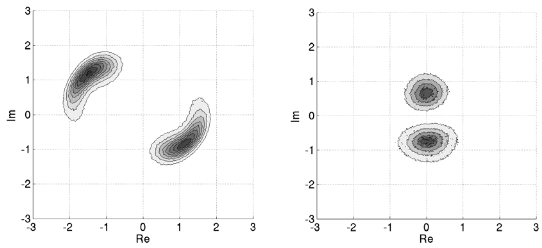

Fig. 1. Contour plot of ASE PDF before reception. Left: D= 0 ps/nm/km. Right: D=4 ps/nm/km. OSNR = 25 dB. Nonlinear cumulated phase 0.3π rad.

The model

The statistics of a Gaussian distributed received noise can be obtained by a linearized solution of the NLSE [4]. We first start by assuming the transmitted signal is not modulated. By adopting a multiple-scale solution of the noisy NLSE [7] in long periodic dispersion-compensated systems, and neglecting quadratic and higher order ASE terms, one can show that the noise field $a(z,\omega)$, being ω the frequency and z the distance, follows the differential equation:

$$\frac{\partial a(z,\omega)}{\partial z} = -j\frac{\omega^2}{2L_D}a - \frac{j}{L_{NL}}\left[a(z,\omega) + \frac{a^*(z,-\omega)}{1-j\frac{L_A}{L_\Delta}\omega^2}\right] \qquad (1)$$

where L_D is the span-averaged dispersion length, L_Δ accounts for the deviation of the transmission fibre dispersion length L_d from the average L_D, i.e. $1/L_d = 1/L_D + 1/L_\Delta$; L_A is the attenuation (effective) length defined as the inverse of the fibre attenuation, L_{NL} is the span-averaged nonlinear length, proportional to the inverse of the signal power *P*. All the dispersive lengths are referred to the mark duration, and frequency ω is normalized to it. Such an equation can be exactly solved in a closed form.

In the signal modulated case, where the nonlinear length L_{NL} is a function of time, eq. (1) is not valid. However, eq. (1) reveals that noise at time *t* will depend on the neighbouring samples into a proper memory window. Hence, in this case we expect that eq. (1) can still be used for evaluating the noise statistics at any time *t* by substituting the signal power *P(t)* with a low-pass filtered version obtained using a windowed Fourier transform. We found that a proper filtering window is :

$$H(\omega) = \frac{1}{1 + \left(\frac{L_A}{4L_\Delta} \omega^2 \right)^2}$$

which well works for values of L_D close to zero, i.e. at small in-line residual dispersions. For instance, at each sampling time t_k of a return-to-zero RZ-DPSK signal with sinusoidal intensity profile, the noise PSD can be evaluated by solving (1) with the reduced power:

$$P(t_k) = \frac{1 + H(\pi)}{2} P_{peak} \qquad (2)$$

while for NRZ-DPSK $P(t_k)$ still coincides with the peak power P_{peak}. We assumed all equal PSDs at the sampling time, from which we evaluated the BER through a standard Karhunen-Loéve method for quadratic receiver in Gaussian random variables.

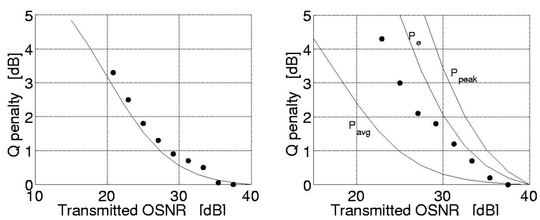

Fig. 2 Q-penalty vs. transmitted OSNR for NRZ-DPSK (left) and RZ-DPSK (right) for the experimental system tested in [2] at P= 7 dBm.

We first tested our model by trying to replicate the results shown in [2]. Fig. 2 shows the experimental Q penalty measured in [2] with circles and the prediction of our model with NRZ-DPSK (left) and RZ-DPSK (right) (duty cycle 33%) for a signal power of 7 dBm. For the system parameters see [2]. For the RZ case we plot the Q penalty as measured by using the average P_{avg}, the peak P_{peak} and the effective P_e power obtained by our approach. With P_e the Q-penalty well approximates the experimental results so that we conclude that the power correction in (2) becomes necessary whenever the signal power is modulated.

We numerically tested the BER obtained by our model, and compared it with the exact BER for a Gaussian noise evaluated with the algorithm proposed in [4]. We used a full in-line compensated RZ-DPSK (duty cycle 50%) amplified system, with 20-span of a 2 ps/nm/km transmission fibre, with optimized pre- and post-compensating fibres. We set L_A/L_Δ=0.35 which corresponds to a bit rate equal to 40 Gb/s while the received average nonlinear phase was 0.3π.

Fig. 3 shows the exact BER (down-triangles), the one obtained with our model (up-triangles), and for comparison the BER as obtained by measuring the PSD at the sampling times with the time-averaged signal power (circles) and peak power (diamonds). The exact BER can be obtained by using an effective signal sampled power as measured with our model, while with the average/peak power one under/over estimates the BER.

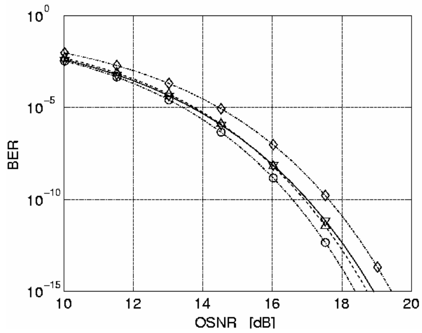

Fig. 3 BER vs. OSNR for a 40Gb/s 20-span full compensated system. Triangles-up: Exact BER. Triangles-down: proposed model. Circles/diamonds: proposed model with the average/peak signal power.

Conclusions

We show that in dispersion-managed systems the nonlinear phase noise can be described through a standard parametric gain approach in Gaussian random variables, for which we provide a model for the statistics.

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