A Stratified Sampling Monte Carlo Algorithm for Efficient BER measurement and its Application to DQPSK Terrestrial Systems

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Abstract We propose an efficient stratified sampling algorithm for BER estimation and apply it to FEC-coded DQPSK systems. We measured computational savings up to 70% compared with standard Monte Carlo.

Introduction

The traffic growth of current optical systems has increased the impact of nonlinear effects, leading to a complex optical propagation that cannot be analyzed exactly through analytical tools. Numerical simulations are commonly used to estimate some performance indicators such as the bit error rate (BER). An efficient BER estimation approach is based on a signal propagation using the split step Fourier method (SSFM), followed by receiver analysis using the Karhunen-Loéve (KL) algorithm [1]. The KL algorithm is based on Gaussian noise statistics, an assumption that may fail in presence of strong nonlinear phase-noise. A correct description of such effect has been so far achieved by brute force Monte Carlo (MC) error counting. Unfortunately, MC simulations are very slow because they require long bit patterns for accurate estimations. Such a problem can be partly alleviated at small BER values ($< 10^{-6}$) by multi-canonical Monte Carlo (MMC) algorithms [2]. MMC is however less efficient than MC at practical BER$\sim 10^{-3}$ in systems employing forward error correction (FEC).

In this work we present an improved Monte Carlo method based on stratified sampling (SS) [3] which is more efficient than MC even at BER=10^{-3}. The proposed algorithm is very simple, stable, with the same MC stopping criterion based on the estimated variance. It also has a significant speed gain compared to standard Monte Carlo. As a case study, we apply it to both non-return to zero (NRZ) and RZ differential quadrature phase shift keying (DQPSK) propagation on a dispersion managed ultralong haul system and compare the performance with standard MC.

The Idea

Any Monte Carlo estimation is based on the observation that the true average value of a random variable I, $\mathrm{E}\{I\}$, can be estimated as [3]:

$$\mathrm{E}\{I\} \approx \langle I\rangle \pm \widehat{\sigma} = \langle I\rangle \pm \sqrt{\frac{\langle I^2\rangle - \langle I\rangle^2}{n}} \qquad (1)$$

where $\langle I\rangle = \frac{1}{n}\sum_k I^{(k)}$ is the sample mean and $I^{(k)}$, $k = 1,\ldots,n$ are independent realizations of random variable I. $\widehat{\sigma}$ in (1) represents the estimated standard deviation of $\langle I\rangle$ and it is a measure of the accuracy. The estimated standard deviation σ of I is related to $\widehat{\sigma}$ by $\widehat{\sigma} = \sigma/\sqrt{n}$. It is thanks to the $\sqrt{n}$ factor that the relative accuracy $\widehat{\sigma}/\langle I\rangle$ decreases for increasing n, making $\langle I\rangle$ a good measure of the exact average value for large n. In our context, I is the error indicator so that $I^{(k)} = 1$ in presence of an error, 0 otherwise. $\mathrm{E}\{I\}$ is the desired BER. Such an approach is inefficient since at small BERs one has to wait many samples from error to error. An improvement to the method uses stratified sampling [3], which runs r different MC samplings on r disjoint subsets (or strata) of the sample space Ω. We applied the method with $r = 2$, as shown in Fig. 1. A point X in Ω is a vector containing all the random variables (modulating bits and added noise samples along the line) of a single run of the simulation. The true transmission system maps X_k to $I^{(k)}$ at each run k, and uniquely determines the true error set $\mathcal{H}: \{I = 1\}$. A faster, approximate model of the transmission system maps X_k to a different $I_F^{(k)}$, hence determines an approximate error set $\mathcal{H}_F$ (stratum 1) which hopefully well overlaps with $\mathcal{H}$. Stratum 2 is the complementary set, i.e. $\Omega\backslash\mathcal{H}_F$. The probability of each stratum p_s is estimated by running n_F times the fast system. Using a subset of the same X_k samples, $k = 1,..,n_F$, we now run separate MC simulations using the true system within each stratum, and estimating the conditional BER $\langle I_s\rangle$ and variance $\widehat{\sigma}_s^2$ in each stratum as per (1), thus generalizing (1) as [3]

$$\mathrm{E}\{I\} \approx \sum_{s=1}^{2} p_s \langle I_s\rangle \pm \sqrt{\sum_{s=1}^{2} p_s^2 \widehat{\sigma}_s^2}. \qquad (2)$$

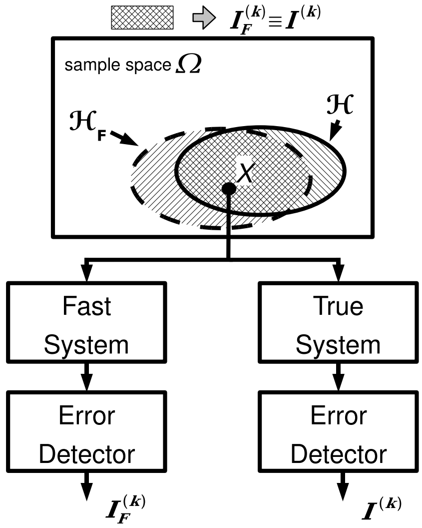

Fig. 1: Stratified sampling Idea.

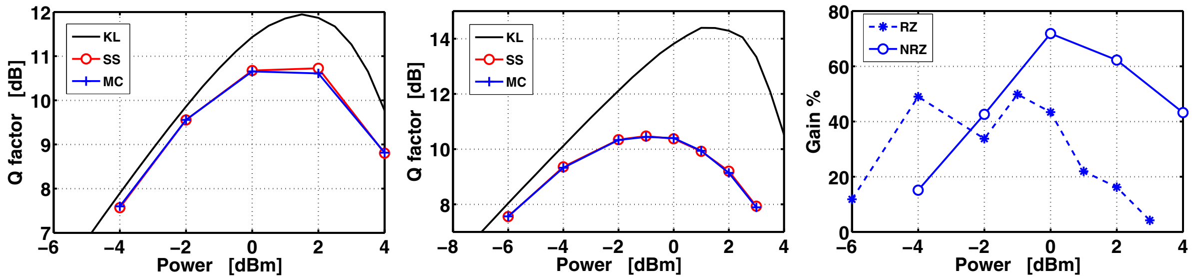

Fig. 2: Left: 10×100 km NRZ-DQPSK. Center: 20×100 km RZ-DQPSK. Right: SS algorithm FFT-reduction factor. KL: Karhunen-Loève. MC: Monte Carlo. SS: proposed algorithm based on stratified sampling.

The number of MC samples within each stratum is adaptively chosen by propagating sample X_k in stratum 1 in the true system with probability $\min(1, \frac{\sigma_1}{\sigma_2})$, and in stratum 2 with probability $\min(1, \frac{\sigma_2}{\sigma_1})$. If $\mathcal{H}_F$ is well chosen, the conditional error probability in stratum 1, $\langle I_1 \rangle$, is close to 1. Hence a gain in efficiency is achieved since very few samples falling in stratum 1 need to be propagated in the true system.

However SS has an overhead due to the runs into the fast system. Efficiency comes from a trade-off between overhead and spared true runs in stratum 1. The simulation ends when the relative accuracy in (2) is less than a given tolerance. Note that in each run we measure the BER over all the N_b bits of the fast Fourier transform (FFT) window, hence n true-SSFM runs yield $n\cdot N_b$ samples $I^{(k)}$.

Results

Clearly the efficiency of the SS algorithm depends on the choice of the fast system. We simulated the true system using a fine SSFM with variable step-size having a maximum nonlinear phase rotation per step equal to $\Delta\Phi = 3\cdot 10^{-3}$ rad [4]. For the fast system we used a coarse SSFM with phase rotation per step 5 times greater, which we separately verified yield more than 1 dB error in the Q-factor over a wide range of powers. The system under investigation had Teralight™ transmission fiber with in-line dispersion compensation and pre- and post-compensating fibers before/after transmission. The pre-fiber had a residual dispersion of -290 ps/nm, while post-fiber dispersion was adjusted to have zero overall dispersion. We used 5 DQPSK 40 Gb/s ($R = 20$ Gbaud) channels, 50 GHz spaced, shaped with both NRZ and RZ pulses, using de Bruijn sequences of 64 bits x channel. The interferometric receiver used a 2nd order supergaussian optical filter of bandwidth $1.5R$ and a post-detection 5th order Bessel electrical filter of bandwidth $0.65R$. In order to test different configurations we analyzed a 20×100 km system with RZ pulses and amplifiers with noise figure $F = 4$ dB, and a 10×100 km system with NRZ pulses and $F = 9$ dB. In the first case we overlooked four wave mixing (FWM), while in the second we accounted for all nonlinear effects in the SSFM. We used 25 ps/nm/span residual dispersion along the line for the NRZ case and 12.5 ps/nm/span for the RZ one. All BER measurements were taken on the central channel at 1550 nm, at a relative accuracy 0.1, both with the MC and the SS algorithm. Fig. 2(left) shows the Q factor vs. the channel power measured using MC (crosses), SS (circles) and the KL algorithm (no symbols) for the NRZ case. First, note the inaccuracy of KL at large powers due to the large nonlinear phase noise induced by the line, which is neglected by the white noise-based KL algorithm. Second, the MC and SS show comparable values, consistently with the same tolerance of 0.1. We repeated the same test for the RZ system obtaining again the same behavior, with a larger impairment due to phase noise, see Fig. 2(center).

Having verified that the BER is consistent, we moved to measure the algorithm efficiency. Since more than 98% of the simulation time was spent in the fibers, we used as a fair cost-criterion the number of FFTs used by standard MC and SS. With SS we have to sum the FFTs of the fast system and the ones of the true system. The FFT-reduction factor of SS w.r.t MC (gain), expressed in %, is shown in Fig. 2(right) vs. power. Note that we averaged the gain among 4 different simulations with different seeds in order to exclude possible best/worst cases. We note that at best SS requires 70% less FFT than MC for NRZ, and 50% for RZ in absence of FWM. As a reference, with $P = 0$ dBm in the NRZ case the MC algorithm used $1.6\cdot 10^7$ FFTs while SS, at the same accuracy, used 3.3×10^6 FFT in the fast system and 1.3×10^6 in the true system. It turns out that the trade-off overhead/variance reduction of SS does give a significant computational time advantage.

Conclusions

We showed a fast and simple Monte Carlo stratified-sampling strategy for BER measurement and applied it to BER estimation of DQPSK systems.

This work was supported by Alcatel-Lucent Bell Labs.

References

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