

# Modeling of Signal-Noise Interactions in Nonlinear Fiber Transmission with Different Modulation Formats

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Let me first acknowledge my co-authors, Paolo Serena and Nicola Rossi of my Dept at the Unversity of Parma



## Outline

- Motivation and objectives
- Models of nonlinear signal-noise interaction (NSNI):
  - > single channel
  - > WDM
- When is NSNI the dominant nonlinearity?
  - simulations of Nonlinear threshold (NLT)
- Conclusions

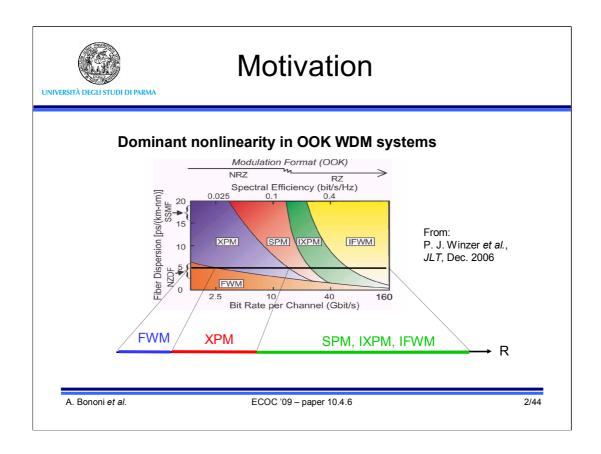
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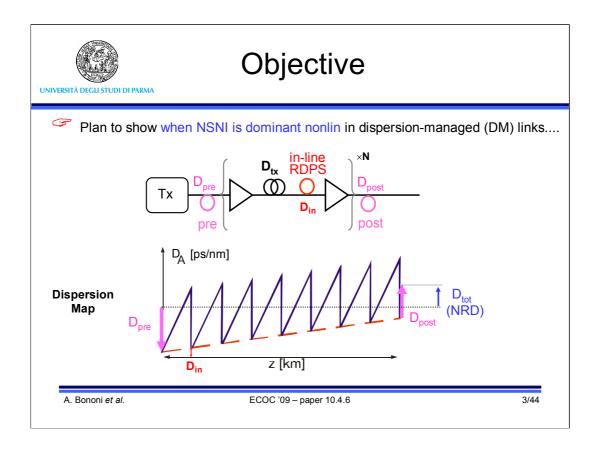
Here is the outline of the talk.

- I'll first provide motivation and objectives of this presentation
- then talk about existing models of NSNI both for single channel and for WDM
- I'll next show when NSNI is the dominant nonlinearity setting the nonlinear threshold in periodic dispersion-managed systems.
- Finally, I'll draw my conclusions.

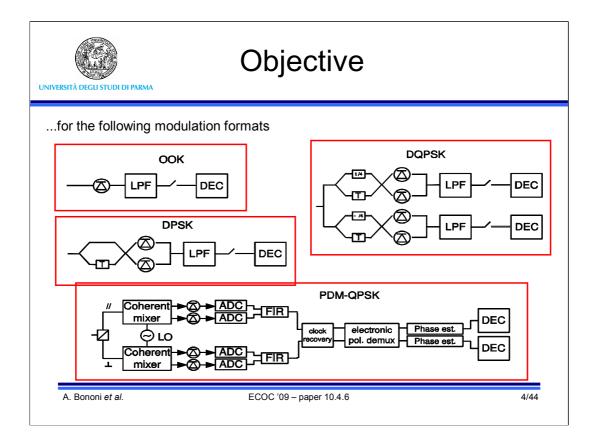


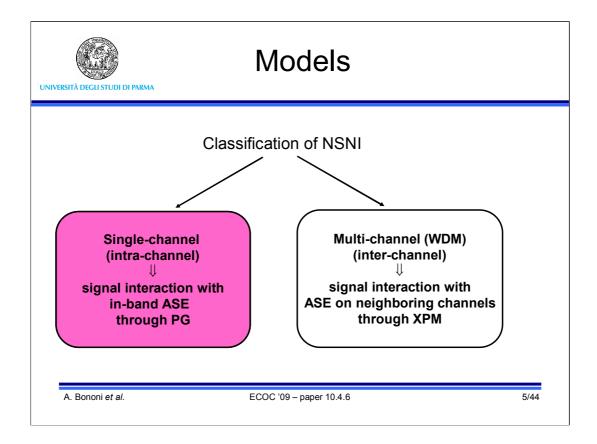
It is well known that the dominant nonlinearity in OOK WDM systems, as the baudrate is increased, is first FWM, then XPM, and then single channel effects, ie SPM, with its variants of single-pulse distortion, interpulse XPM and I-FWM.

We would like to know where in this picture NSNI comes into play.



More precisely, we plan to show when NSNI is the dominant nonlinearity in DM links composed of N identical spans with in-line compensation and a given residual dispersion per span Din, with pre and post compensating fibers at the beginning and end of the link for a total or net residual dispersion Dtot.....

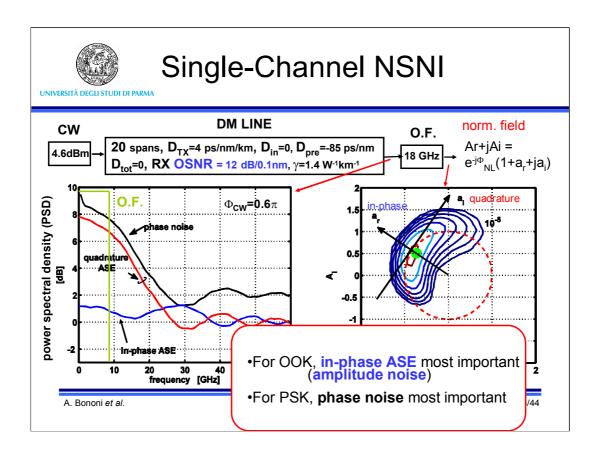




#### NSNI can be classified as

- single-channel, where we have nonlinear parametric interaction of the reference signal with in-band amplified spontaneous emission (ASE) noise.....or
- 2) multi-channel, where the interaction with the reference signal comes from ASE on neighboring channels through XPM.

Let's start with models for single-channel NSNI



To fix the ideas, consider a DM line composed of 20 spans, with nonzero dispersion shifted fiber with dispersion 4 ps/nm/km, full in-line compensation, pre comp of –85 ps/nm and zero net residual dispersion. The transmitted signal is CW with 4.6 launched dBm for a cumulated nonlinear phase of 0.6 pi and a received OSNR of 12 dB/0.1nm.

What you see on the right is the probability density function (PDF) resolved down to 10<sup>-5</sup> of the normalized RX optical field at the output of an optical filter of 18 GHz. The average field is marked as a green dot. You note that the RX ASE has a non ellyptical, bean-like PDF which is the signature of single-channel NSNI, so that RX ASE is not Gaussian.

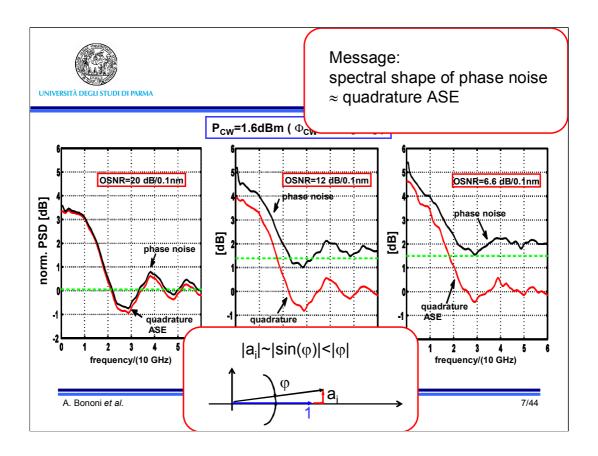
If we factor out the phase rotation of the average field, we can define

a radial or in-phase ASE component, and a tangent or quadrature ASE component.....

whose PSD is shown on the left graph, taken upstream of the optical filter.

We alo show the PDF of the phase noise, i.e. of the phase of the field with respect to that of the average field.

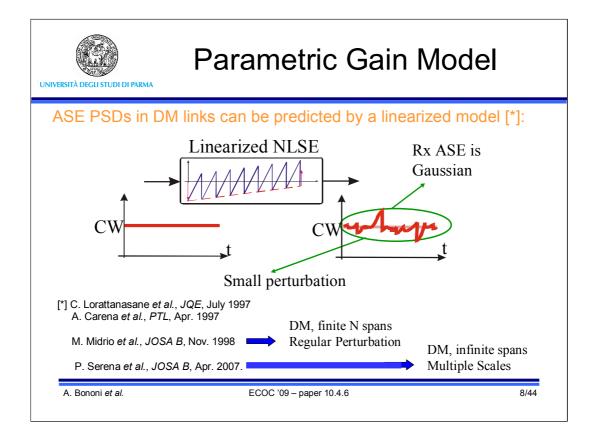
For OOK, in-phase ASE (ie amplitude noise) is most important, while for phase modulated formats phase noise is most important.



- here, for the same DM line but with a reduced CW, we see the difference in PSD between quadrature ASE and phase noise for various OSNR levels from 20 to 6.6 dB/0.1nm.
- PSDs are normalized to the white ASE level in absence of nonlinearity. The green line represents the PSD of phase noise in absence of nonlinearity, which is larger than that of the quadrature ASE since

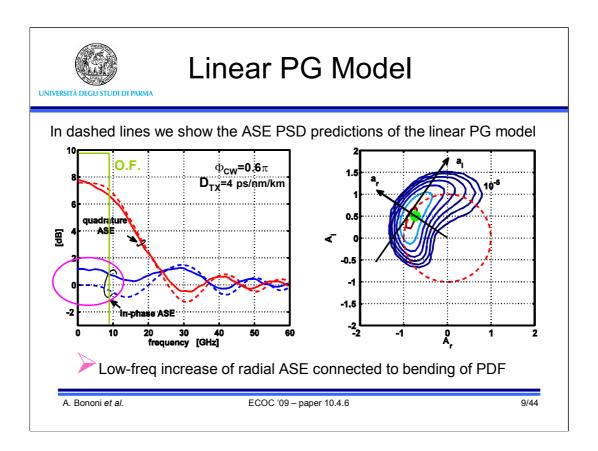
the phase is larger than the sin of the phase which is approx the quadrature ASE.

The message we get from these curves is that, except for this high-frequency offset, the shape of the phase noise PSD can always be inferred from that of the quadrature ASE.



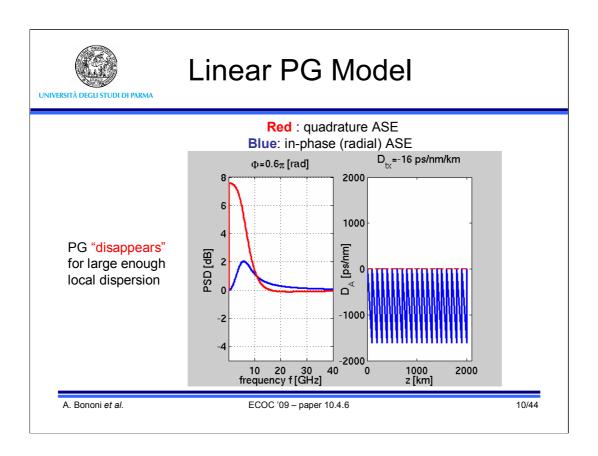
The ASE PSD can be well predicted by a linearized model developed more than 10 years ago, and adapted to DM lines (see references)

The model comes from a linearization of the NLSE around a CW solution, and thus provides the output field as a small perturbation of the CW. The output ASE is thus a linear filtering of the white generated ASE, and keeps the Gaussan statistcs, i.e. the linear PG model does not predict the bending of the PDFs. However it provides fairly accurate PSD predictions......

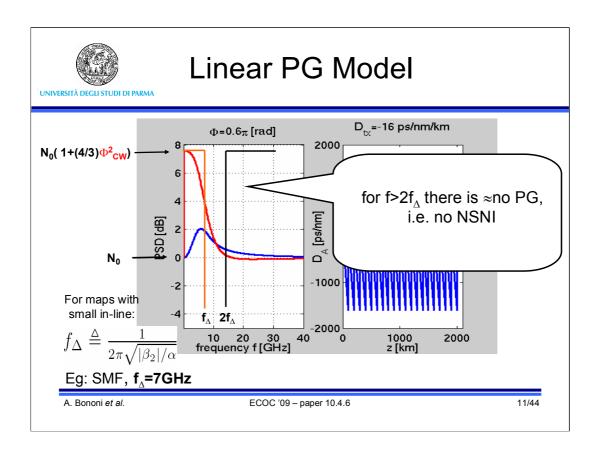


We show in dashed lines the PSD predictions of the linear PG model: they are fairly accurate, except in the radial (in-phase) ASE at low frequency:

the true PSD level is higher because the PDF bending produces an increase of the variance of the radial ASE (along with a negative average value, i.e. a sinking of the green dot average field inside the unit circle) which is not captured by the linear PG model.

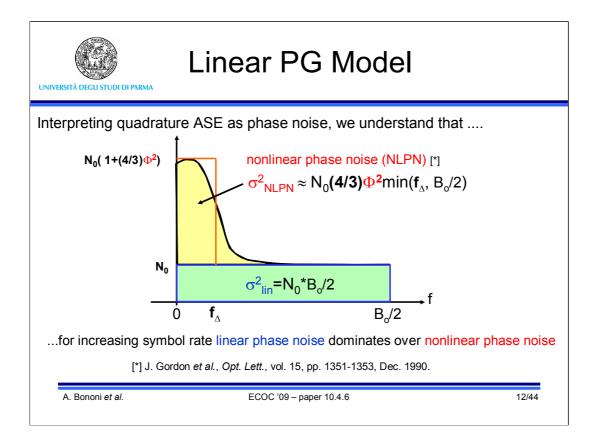


- The linear PG model helps us appreciate the role of DM parameters.
- in this animation we show the ASE PSD on the left and the map on the right, as we vary the dispersion of the transmission fiber [start animation and let it go]
- - when local dispersion is close to zero, signal-noise interaction extends over a huge frequency range and the PSDs are almost flat;
  - when local dispersion is large, the range of significant signal-noise interaction shrinks, so that PG "disappears" for large-enough dispersion.



- we also learn from the PG model that
- 1) PSD level of quadrature ASE scales as the square of the CW nonlinear phase (ie power) and
- 2) for maps with small in-line RDPS a particular frequency  $f_{\Delta}$  exists, within which most of the PG interaction takes place. It scales as the inverse of the square root of the dispersion, and for SMF links its value is 7 GHz.

At twice  $\ f_{_{\! \Delta}}$  the signal-noise interaction has vanished.



Hence, if we interpret the quadrature ASE as phase noise,

we see that the total phase noise variance is the sum of a linear component due to the white PSD at level No, and of a nonlinear phase noise (NLPN) component, whose variance scales approximately as  $\Phi^{2*}f_{\Delta}$ , i.e. with the squared power of the CW and inversely with the square root of dispersion.

...and it is clear that for increasing baudrate and thus optical filter bandwidth, the linear component increases and eventually dominates over NLPN.

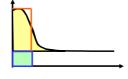


## Models of NLPN

■ K.-P. Ho [\*] computed the probability density function (PDF) of nonlinear phase noise on a CW signal.

Exact result if:

- zero GVD (i.e. no DM)
- ASE exists only over signal's bandwidth



In DM systems, Ho's analysis well matches simulations when  $B_o/2{\approx}f_{_{\! \varLambda}}$  (e.g. SMF systems at 10Gbaud)

[\*] K.-P. Ho, JOSAB, pp. 1875-1879, Sept. 2003.

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Regarding NLPN, I'd like to mention a well known model due to

- K. Po Ho, which gives the exact PDF of NLPN on a CW signal when
- 1) no chromatic dispersion is present and
- 2) when ASE exists only over the signal's bandwidth ....
- i.e. when the PG-induced increase of the phase noise PSD is present only over the optical filter bandwidth, such as for instance in SMF systems at 10 Gbaud.

However, at different baudrates on the same SMF line the results of Ho's model are not accurate.

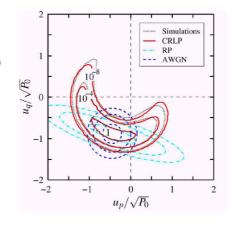


## Models of NLPN

- Secondini *et al* [\*] developed an analytical model yielding the probability density function (PDF) of ASE plus a CW:
- based on ansatz:

Ar+jAi = 
$$(1+a_r+j(a_i+\delta\phi)) \exp(-j(\Phi_{NL}+\delta\phi))$$

- captures bending of PDF
- model works for DM systems



[\*] M. Secondini at al., JLT, Aug. 2009.

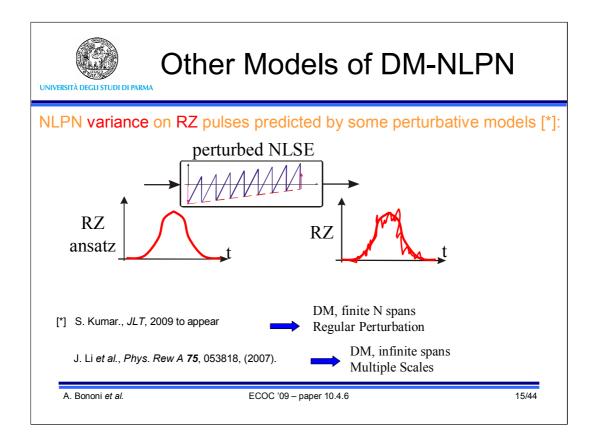
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More recently, Secondini and co-workers have developed a novel analytical model yielding the PDF of the received optical field on a CW sgnal.

The model is an extension of the previously described linear PG model, with a different ansatz:

[read out]



It is only fair to mention that several works exist on NSNI when the supporting pulse is RZ, such as in dispersion-managed solitons. Here the technique is a perturbation analysis of the NLSE based on an RZ ansatz.

I'd like to mention the recent work of Kumar based on a regular perturbation approach and yielding the NLPN variance in DM systems, and the elegant work of the group of Biondini based on a perturbation analysis of the dispersion-managed nonlinear Schroedinger equation.



# single-channel BER methods

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	Method	suitable for DM analysis
based on RX ASE statistics	PG approach, Karhunen Loeve method, ASE: Gaussian  [*] P. Serena et al., JLT, Aug. 2005 ⇒ OOK, CW  [**] P. Serena et al., JLT, May 2006 ⇒ D(Q)PSK, CW  [***] R. Holzlohner et al., JLT, Mar. 2003 ⇒ OOK, non-CW (NCM)	YES
	extended PG approach, Karhunen Loeve method [*] M. Secondini at al., JLT, Aug. 2009 ⇒ OOK, CW	YES
based on NLPN	"exact" NLPN PDF, Blachman's method [*] KP. Ho, JOSAB, Sept. 2003 ⇒ D(Q)PSK, coh. (Q)PSK, ideal phase est. [**] A. P.T. Lau et al., JLT, Oct. 2007 ⇒ coh. M-PSK, M-QAM, ideal ph. est.	NO

■ A model including NSNI is still missing for realistic coherent reception with M-power feedforward phase estimation.

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# Models

## Classification of NSNI

Single-channel (intra-channel)

signal interaction with in-band ASE through PG

Multi-channel (WDM) (inter-channel)

signal interaction with ASE on neighboring channels through XPM

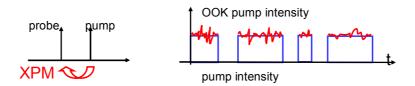
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# Multi-Channel NSNI



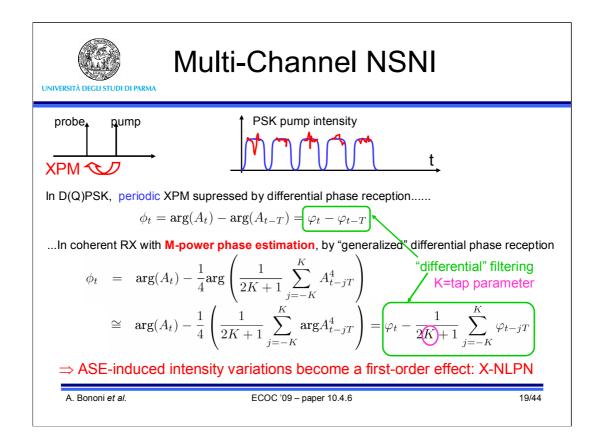
In OOK, XPM mostly due to modulation-induced intensity variations.

ASE-induced intensity variations are a second-order effect

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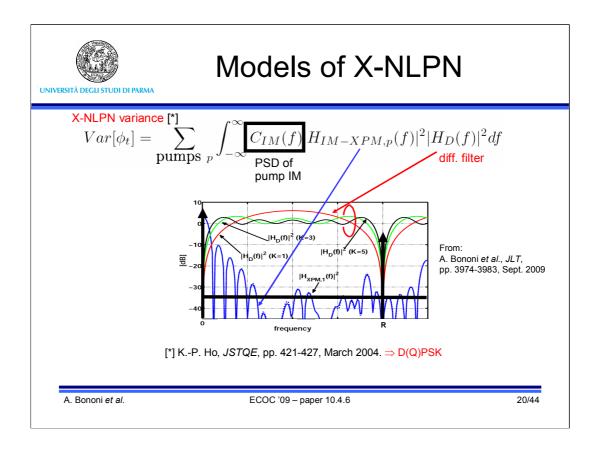


however, in phase-modulated channels the XPM induced by the periodic intensity is completely suppressed by the differential phase reception in DQPSK, and ...

in coherent reception with M-power phase estimation (or Viterbi and Viterbi, V&V) by the generalized differential filter....

in green you see the differential operation in both cases. K will be called the tap (or smoothing) parameter in V&V (number of taps is 2K+1)

Since the large periodic IM-induced XPM is suppressed, then ASE induced non-periodic intensity variations become a first-order effect: we call it cross-nonlinear phase noise (X-NLPN).



The variance of X-NLPN can be evaluated with techniques similar to those employed for classical XPM for OOK systems, as first suggested by Ho in 2004.

In the variance formula,  $H_D$  is the differential filter shown here for several values of K, then the blue curve is the IM-XPM filter which depends on the details of the DM map, and finally  $C_{\rm IM}$  is the PSD of the pump intensity modulation, which has periodic components (the deltas) that are completely suppressed by  $H_D$ , and a white level due to the pump ASE.



# Outline

- Models of nonlinear signal-noise interaction (NSNI):
  - > Single channel
  - > WDM
- When is NSNI the dominant nonlinearity?
  - ➤ simulations of Nonlinear threshold (NLT)
- Conclusions

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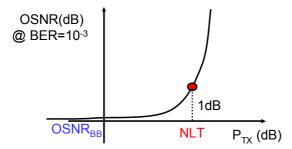
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With these models in mind, we can now better interpret the quantitative performance results on NLT I am going to show you in this second part of the talk.



A key parameter for any long-haul DM line and modulation format is the nonlinear threshold (NLT) at 1dB of OSNR penalty at BER=10<sup>-3</sup>

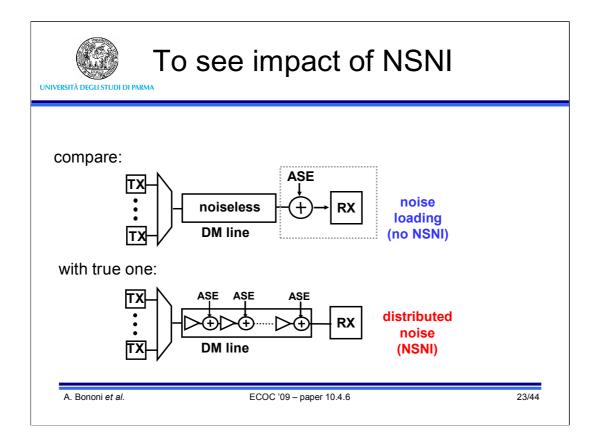


We will show estimates by simulation of NLT vs. Symbol Rate R for various modulation formats in optimized DM lines, both with noise loading and with distributed noise.

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We define here the NLT as the transmitted power level at which we have 1 dB of OSNR penalty at a reference BER of 10<sup>-3</sup>, typical of FEC coded systems.

We will show estimates ....[read out]



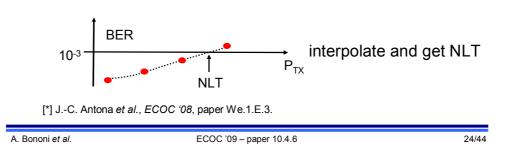
....in other words, we will compare NLT results in these two cases:

- 1) the classical strategy adopted in simulations, where we have a noiseless SSFM propagation of the signals, and all ASE is loaded at the receiver, so that no signal-noise interaction is present, and
- 2) the more realistic but longer simulations where ASE is distributed along the line.



## **NLT** procedure

- 1. Set OSNR=OSNR<sub>BB</sub>+1 (dB)
- 2. fix  $P_{TX}$
- 3. get ampli noise figure NF from ASE power  $N_A$ =OSNR/ $P_{TX}$
- measure BER by Monte-Carlo error counting, i.e: in DM line with that NF, propagate blocks of 256 random symbols per channel. Stop simulation when 100 errors counted [\*]
- 5. if BER<10<sup>-3</sup> increase  $P_{TX}$  and goto 2.



The procedure to get the NLT is the following:

- 1) we set the OSNR to 1 dB of penalty above the back-to-back OSNR
- 2) we fix a tentative power
- 3) and thus get the noise figure of the amplifiers in the DM line
- 4) we simulate the split-step Fourier propagation along that line of blocks of 256 purely-random symbols, and count errors after the RX. We keep propagating independent blocks until 100 errors have been collected, then BER is estimated. This method, presented by Antona et al at ECOC 08, is equivalent to the use of pseudo-random sequences (PRS) at small strengths, but has the advantage of working even at large strengths, when the use of PRS is problematic.
- 5) if we measure a BER less than  $10^{-3}$  we increase  $P_{tx}$  and repeat, till we hit the  $10^{-3}$  level.

Why 256 symbols? Actually, it is sufficient that the FFT window be larger than the memory of the system.



## In the next 5 slides I will provide the details of the simulations, and next I will show the NLT results

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# Optimized DM line

The simulated DM line had N=20 spans of SMFiber ( $D_{tx}$ =17 ps/nm/km), and  $D_{in}$ =30 ps/nm/span. TX fibers had zero slope, inline DCF were linear.

#### Optimized DM parameters:

### For OOK:

$$D_{pre} = -\frac{D_{tx}}{\alpha} - \frac{N-1}{2}D_{in}$$

[\*] J. Frignac *et al.*, Proc. OFC' 02, paper ThFF5. [\*\*] A. Bononi *et al.*, *JLT*, pp. 3617-3631, Nov. 2008.

$$\begin{split} D_{tot} &= D_{pre} + ND_{in} + D_{post} \\ &= \frac{\sqrt{2}}{8} \frac{\Phi}{1 + \frac{4}{3\sqrt{3}}\Phi^2} \text{ nonlin phase} \end{split}$$

 $D_{tot} = 0$ 

#### For PSK:

$$D_{pre} = -\ln(2)\frac{D_{tx}}{\alpha} - \frac{N-1}{2}D_{in}$$

[\*] R. I. Killey *et al.*, *PTL*, pp. 1624-1626, Dec. 2000. [\*\*] P. Serena *et al.*, *JLT*, pp. 2026-2037, May 2006.

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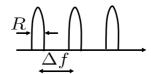
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## More Simulation Data

Supporting pulse: NRZ

• 
$$\eta \triangleq \frac{R}{\Delta f}$$
 = 0.4

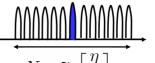


Eg: R=40 Gbaud  $\Rightarrow \Delta f=100 \text{ GHz}$ 

No filtering at TX ⇒ linear Xtalk

Number of WDM channels [\*]:

Eg: SM Fiber at 10 Gbaud ⇒ Nch=5+1+5



 $S=rac{|eta_2|}{lpha}R^2$ Map Strength

[\*] P. Serena et al., ECOC '06, paper We3.P.129.

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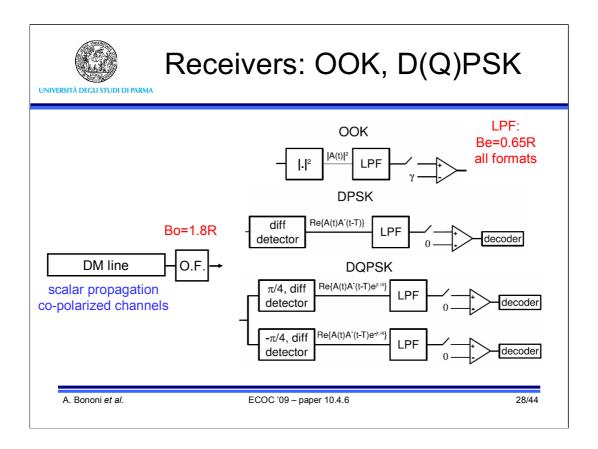
#### More simulation data:

- the supporting pulses are NRZ;
- the bandwidth efficiency  $\eta$  [symbols/sec/Hz] defined as the ratio of baudrate R to channel spacing was 0.4

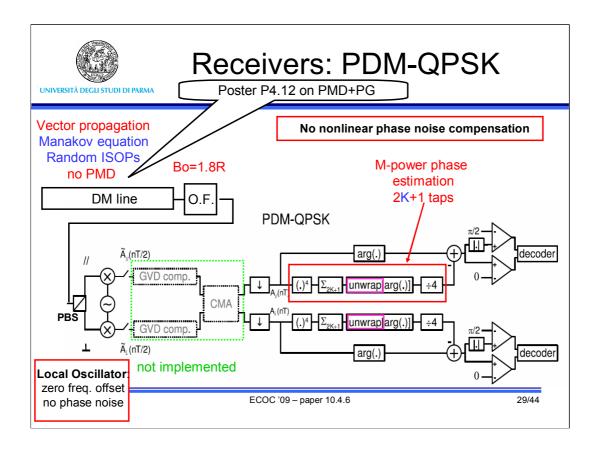
(Note: bandwidth efficiency is related to spectral efficiency (SE) as:

SE=  $\eta$  \*(bits/symbol) [bits/sec/Hz] );

- -no filtering was applied at the TX, so that some spectral overlap is present and thus some linear cross-talk;
- the number of WDM channels was scaled as the ratio  $\,\eta\,$  over strength, as shown by our group at ECOC 06, to correctly reproduce XPM effects.
- -All WDM channels have the same modulation format.



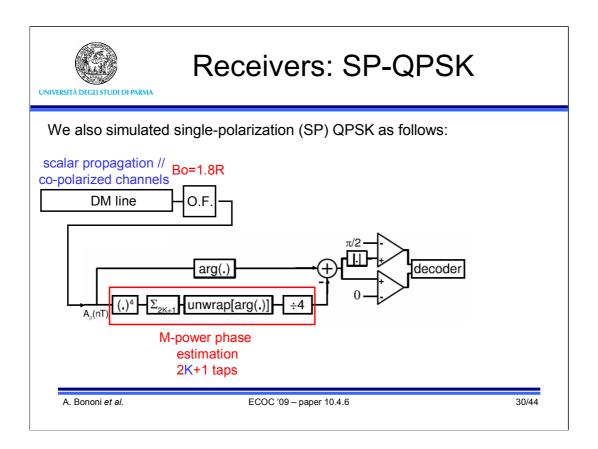
The receivers for OOK and D(Q)PSK are standard ones, with optical filter bandwidth 1.8 times the baudrate, electrical filter 0.65 the baudrate, and scalar propagation along the DM line (i.e. in the worst-case of all co-polarized WDM channels).



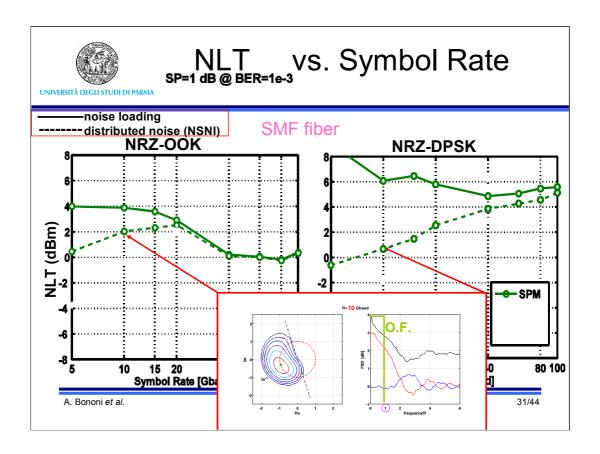
In the coherent PDM-QPSK case, the propagation was vectorial, based on the Manakov equation (with random WDM input SOPs), and PMD was set to zero (if you are interested in the combined action of PMD and PG, please see our ECOC 09 poster P4.12). Optical filter scaled again as 1.8 times the baud rate. Electronic GVD compensation was not implemented since a post-compensation optical fiber was considered. Also CMA was not implemented, since there was no residual linear birefringence. This allowed us to concentrate on the action of the feedforward M-power phase estimation, equipped with the unwrapping function to eliminate equivocation (which appears at lower OSNRs).

### More assumptions:

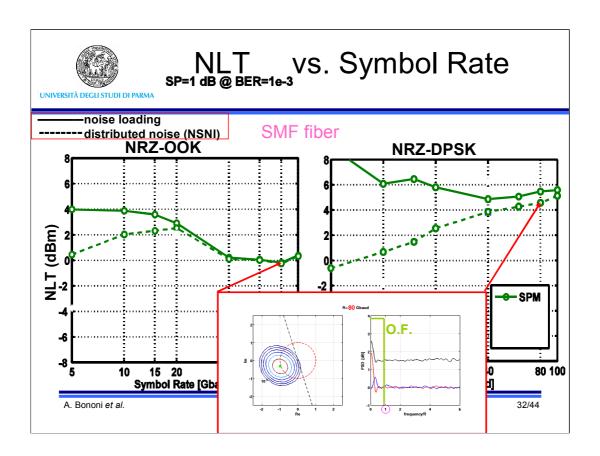
- -- the local oscillator had zero frequency offset for the reference central channel of the WDM comb
- -- the coherent receiver had no electronic nonlinear phase noise compensation.



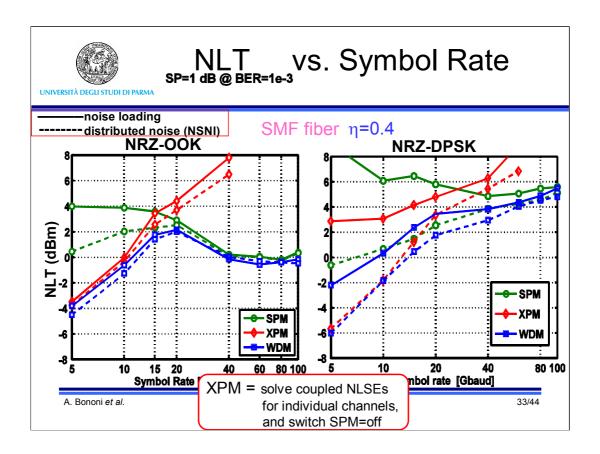
We also simulated single-polarization coherent QPSK, where propagation was scalar along the X axis (i.e. all WDM channels co-polarized). The receiver is just one branch of the PDM receiver.



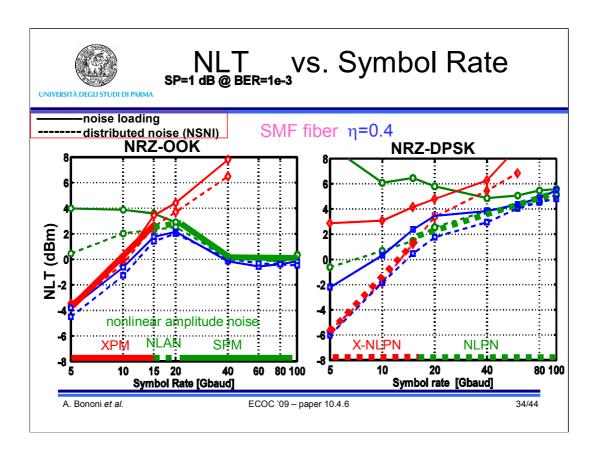
- elet's move to the results. We start with binary formats, OOK on the left, DPSK on the right. In green I am showing single-channel results ("SPM" label) both with noise loading (solid) and with distributed ASE (dashed).
- The ASE PDFs have significant bending (important for OOK) and NLPN is dominant over the optical filter bandwidth (important for DPSK).



but if we move to higher baudrates, eg 80 Gbaud, then the PG interaction band is a very small portion of the optical filter bandwidth, and thus NSNI vanshes for OOK (no PDF bending) and is reduced for DPSK, although the NLPN is seen to be significant even beyond 100 Gbaud.

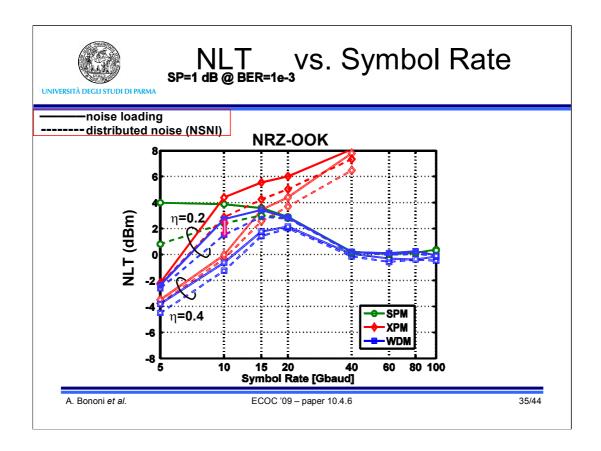


- The work with the WDM case at a bandwidth efficiency 0.4, blue lines.
- We note that here the NSNI is marginal in OOK, while it is fundamental for DPSK.
- To investigate the dominant nonlinear effect, we also performed simulations in which XPM is the only nonlinearity, see red curves: we solved the coupled NLSEs for all WDM channels (which automatically excludes FWM and spectral overlap during propagation) and we switched SPM "OFF".
- Two mote that in OOK the solid red curve (noiseless XPM) is essentially matching the dotted red curve (XPM+distributed ASE), thus confirming the secondary role of ASE on pumps
- ©On the contrary, the red dotted line is dominant in DPSK: it is thus X-NLPN the dominant nonlinearity at lower baudrates.
- The offset between the solid red and the solid blue lines in DPSK at lower baudrates cannot be explained by FWM (it would be too small in this SMF line with 30 ps/nm RDPS): more likely explanations of the offset are i) interaction of SPM and XPM and ii) the linear Xtalk due to spectral overlap, which acts as an intensity noise during propagation and thus causes nonlinear phase noise exactly as ASE would do.



If I were to draw simple conclusions, I would thus state that:

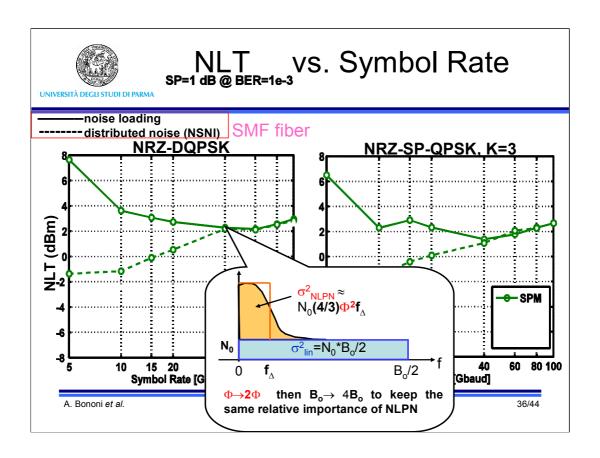
- For OOK, noiseless XPM is the dominant nonlinearity at lower baudrates, up to 15 Gbaud, then single-channel NSNI dominates (in the form of "nonlinear amplitude noise" (NLAN), mostly due to PDF bending) between 15 and 20 Gbaud, next noiseless SPM is the dominat effect beyond 20 Gbaud.
- For DPSK, the low baudrates are dominated by multi-channel NSNI (ie X-NLPN) up to around 15 Gbaud, then single-channel NSNI dominates (in the form of NLPN) well beyond 100 Gbaud, after that noiseless SPM would dominate.
- At lower spectral efficiency, the red XPM curves would shift upards, and so would the low baudrate portion of the blue curves: this would extend the dominance of single-channel NSNI to lower baudrates....



For instance, I am showing here the OOK case, where we see that at  $\eta$ =0.2 the single channel NSNI is not negligible anymore, and causes an extra 1.5 dB of decrease of the NLT at 10 Gbaud (worst case).

We also note in passing that in this case the ASE on pumps ceases to be a negligible effect (see difference between re-solid and red-dotted) as it was at  $\eta$ =0.4.

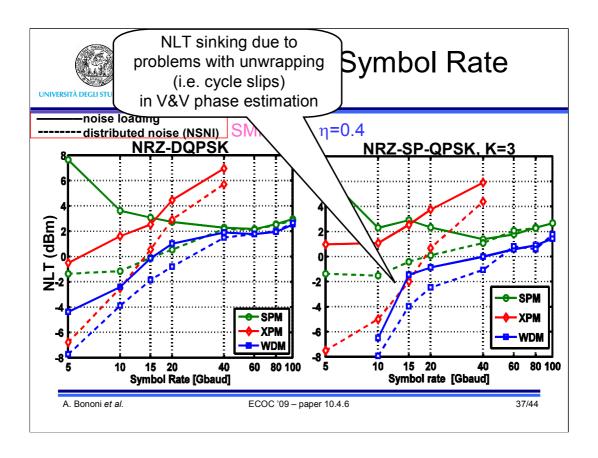
Note finally that at  $\eta$ =0.4 the WDM curves do not converge to the SPM curve, but remain 0.5 dBs below: this is due to the linear Xtalk, ie the spectral overlap at this large spectral efficiency (there was no TX filtering in place). Such an offset disappears completely at  $\eta$ =0.2.



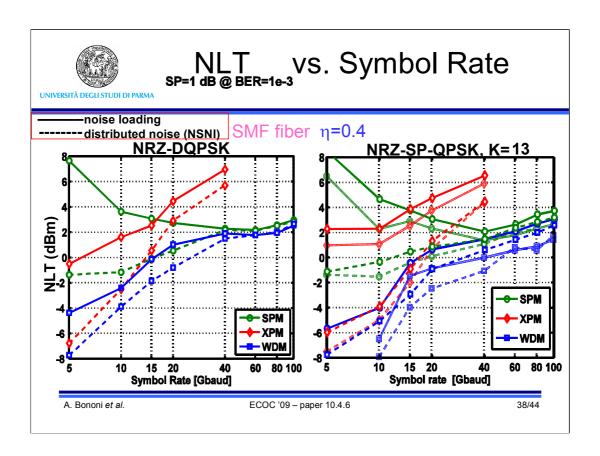
Flet's move to quaternary formats, DQPSK on the left, and SP coherent QPSK on the right, where the tap parameter was set to 3 (ie 3+1+3 taps). We note that the single channel performance is quite similar, with a vanishing of NLPN at 40 Gbaud.

From the previous slide we've seen that in DPSK the vanishing occurs at much higher baudrates and the NLT is 3 dB higher:

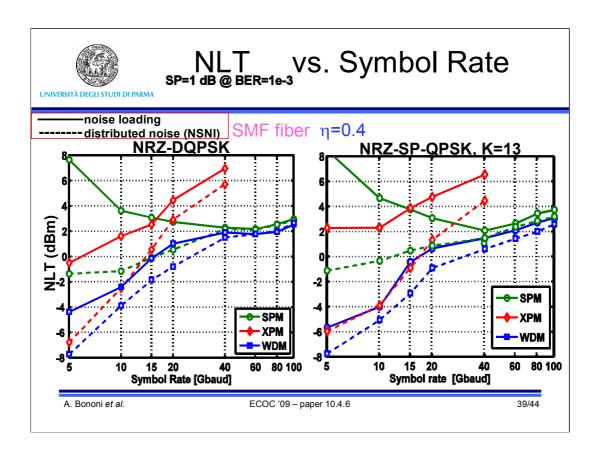
this is explained by the linear PG model, since if we double the nonlinear phase, it is necessary to quadruple the baudrate to have the same relatve importance of NLPN.



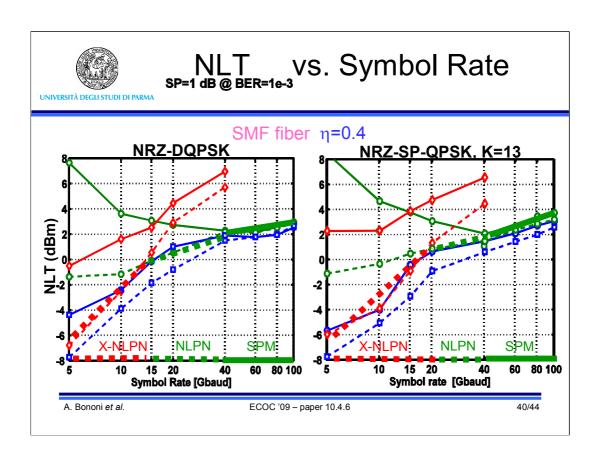
- Let's now look at the blue curves [click] that provide the WDM performance. We see that in both formats NSNI is fundamental. We see here several differences between DQPSK and SP-QPSK, especially at lower baudrates.....
- We also performed simulations with only XPM, but in the SP-QPSK case we see that X-NLPN (red dotted curve) does not completely explain the decrease of the WDM NLT (blue dotted) as happens instead in the DQPSK case.
- we verified that the NLT sinking in the coherent RX is due to problems with the unwrapping (i.e. to cycle slips) in the M-power phase estimator, most likely due to the linear xtalk which induces extra nonlinear phase noise at lower baudrates.
- The gap between WDM (blue) and SPM (green) at high baudrates (the two curves should ideally merge when single-channel effects dominate) is due to the linear Xtalk which adds to the ASE and calls for a larger tap parameter K.



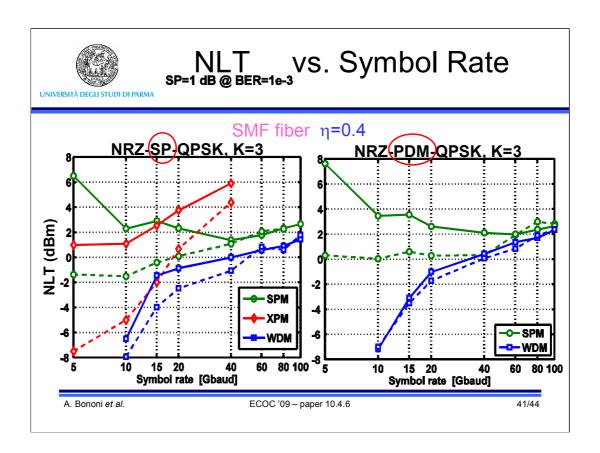
hence we increased the tap parameter K: let me change for convenience the color of the K=3 curves (more transparent)....and then show the NLT curves with K=13: we see an NLT improvement in all cases....



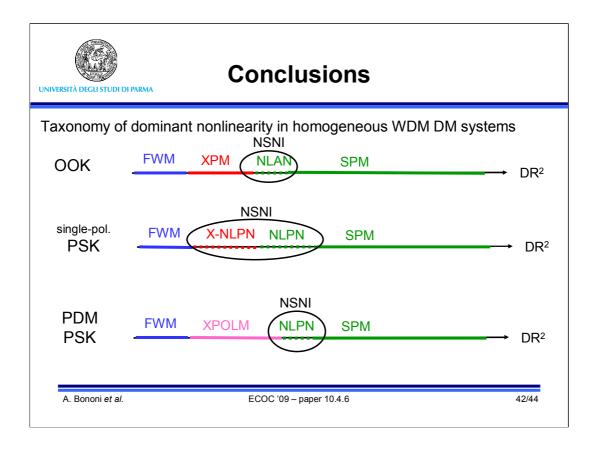
so now let's remove the K=3 curves and leave only the K=13 curves: we see that when the cycle-slip problem is fixed, then also the WDM performance of coherent SP QPSK is similar to that of DQPSK......(increasing K reduces the effect of linear Xtalk at all baudrates).



 $<sup>\</sup>ensuremath{\mathscr{G}}$  and we can summarize that the dominant nonlinearity in both formats is X-NLPN at lower baudrates, up to 15 – 20 Gbaud, then single-channel NLPN, and then noiseless SPM beyond 40 Gbaud.



Finally, we compare SP and PDM coherent reception for the only case in which I have PDM simulations, namely the K=3 case. In the PDM case I do not have the red XPM curves, as it was unclear how to keep XPM while switching XPoIM off in the vector propagation. By looking at the blue WDM curves we see that in the PDM case there is little dependence on NSNI, less than 1 dB from 20 to 60 Gbaud: we believe the cause is the dominant role of phase-modulation-induced XPoIM at lower baudrates, a phenomenon which is important for PDM systems and is basically independent of ASE.



To conclude and graphcally summarize the findings of this work, here is a taxonomy of the dominant nonlinearities in WDM systems with equal-format channels (known as homogeneous systems):

- 1) For OOK, as we increase the baudrate, or better yet the product dispersion times baudrate squared, the dominant nonlinearity is at first FWM (we did not see it here), then noiseless XPM, then nonlinear amplitude noise and finally noiseless self-phase modulation;
- for single-polarization phase modulated formats, both with incoherent and with coherent reception, FWM dominates first, then X-NLPN, then NLPN, and finally noiseless SPM;
- 3) for PDM coherent PSK we have FWM first, then XPOLM, then when XPOLM fades away NLPN emerges, and finally noiseless SPM.

and thus circled in black we have the answer to the NSNI dominance which was the objective of the talk.



# Thank you. Questions?

#### For more details on this work:

[\*] A Bononi, P. Serena, N. Rossi, "Nonlinear signal-noise interactions in dispersion-managed links with various modulation formats," invited paper, Opt. Fiber Technol., submitted July 2009.

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