
Transient Gain Dynamics in Saturated Counter-pumped Raman Amplifiers

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Outline

- ✓ Motivation
- ✓ New State-Variable Model
- ✓ Results
- ✓ Conclusions



Motivation

- Saturation-induced gain transients (Cross-gain modulation):

Chen and Wong, OAA 2001, paper OMC2

- Physical reason: pump-signal walkoff
signal front grows and depletes pump
⇒ signal body finds less pump than front

- Need to Quantify:
Power surges, time constants of transients

☞ Need a system model !



Propagation Equations

$$\begin{cases} \left(\frac{\partial}{\partial z} + \frac{1}{v} \frac{\partial}{\partial t} \right) S_j(t, z) = [-\alpha_j + g_j P(t, z)] S_j(t, z) & j = 1, \dots, N \\ \left(\frac{\partial}{\partial z} - \frac{1}{v} \frac{\partial}{\partial t} \right) P(t, z) = \left[\alpha_p + \sum_{j=1}^N \hat{g}_j S_j(t, z) \right] P(t, z) \end{cases}$$

where: S_j signal [W]; v speed [m/s]; g_j Raman gain [1/W/m]; α attenuation [1/m]; P pump [W]; $\hat{g}_j \triangleq g_j \lambda_j / \lambda_p$

Neglect:

- ✗ ASRS noise
- ✗ Rayleigh Backscattering
- ✗ direct signal-signal crosstalk



Implicit Solution

Solution of propagation equations in the signals and pump
retarded time frames $t_s = t - z/v$, $t_p = t + z/v$ is

$$\begin{cases} S_j(t_s, z) &= S_j^{in}(t_s) \exp \left\{ -\alpha_j z + g_j \int_0^z P(t_s + d z', z') dz' \right\} \\ P(t_p, z) &= P_0 e^{-\alpha_p(L-z)} e^{-\Gamma(t_p, z)} \end{cases}$$

where: $S_j^{in}(t_s)$ input signal [W]; $d \triangleq 2/v$ [s/m] walkoff parameter;
 P_0 launched pump [W]; L amplifier length [m]; and

$$\Gamma(t_p, z) \triangleq \sum_{j=1}^N \hat{g}_j \int_z^L S_j(t_p - d z', z') dz'$$

is the *pump depletion in the pump time frame*.



The State Variable

At moderate pump depletion $e^{-\Gamma} \cong 1 - \Gamma$. Signal power at output is thus

$$S_j^{out}(t_s) = S_j^{in}(t_s) \exp \left\{ -\alpha_j L + Q_j(1 - x(t_s)) (1 - e^{-\alpha_p L}) \right\}$$

where $Q_j \triangleq \frac{g_j}{\alpha_p} P_0$ and

$$x(t_s) \triangleq \frac{1}{L_{eff}^{(p)}} \int_0^L e^{-\alpha_p(L-z')} \Gamma(t_s + d z', z') dz'$$

is the *relative change in injected pump power sensed by the signals at retarded time t_s* .

☞ **Once $x(t_s)$ is known, all WDM output signals are known !**



The State Equation

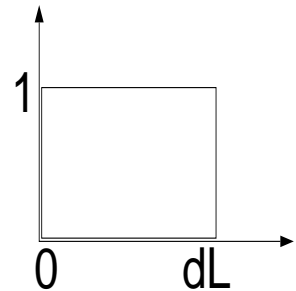
The state variable $x(t_s)$ satisfies the following implicit integral equation

$$x(t_s) = \sum_{j=1}^N S_j^{out}(t_s, x(t_s)) \otimes h_j(t_s)$$

where \otimes denotes convolution, and h_j is the impulse response of a linear filter:

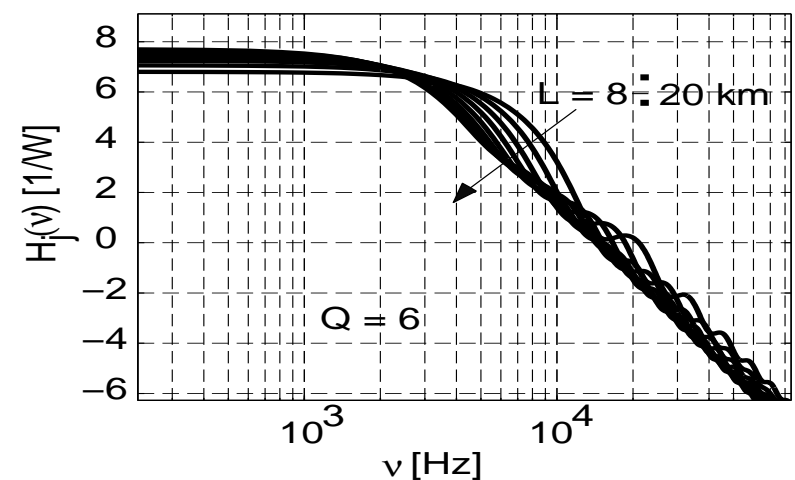
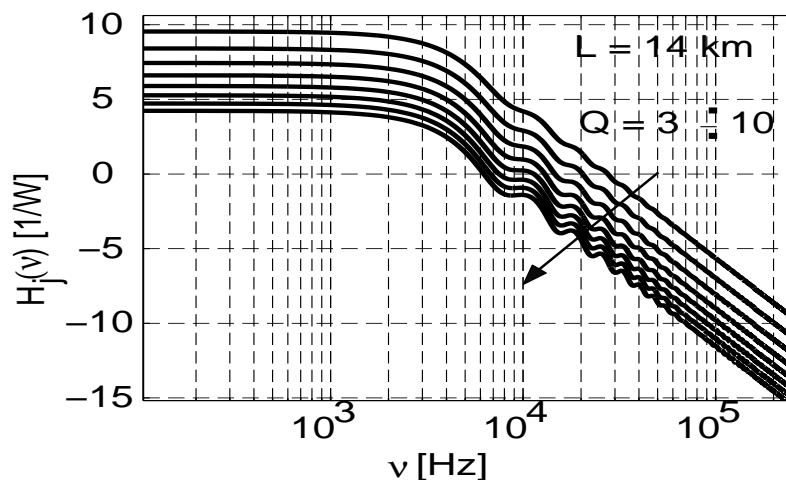
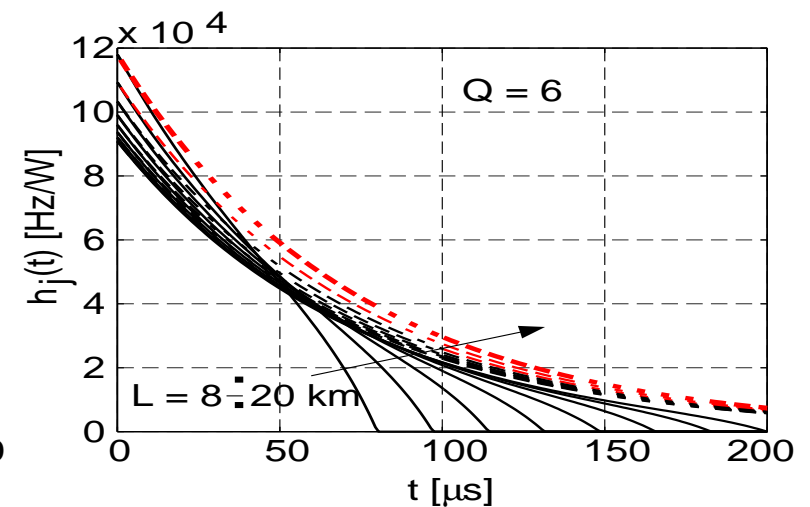
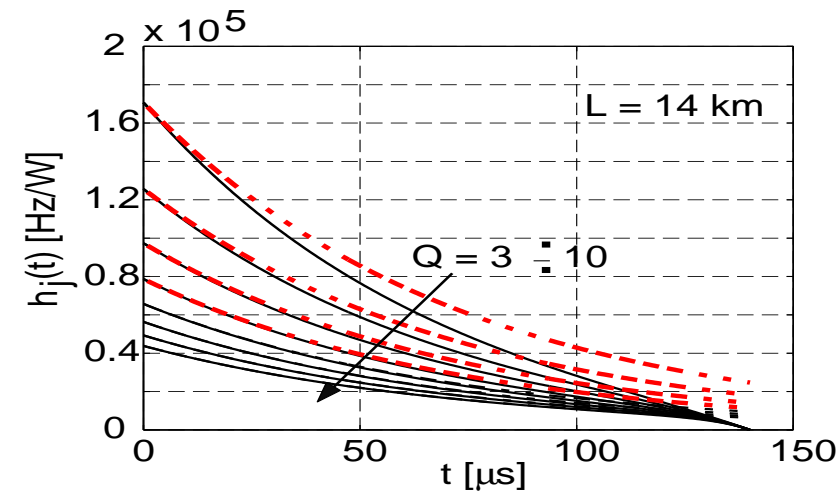
$$h_j(t) = \frac{\hat{g}_j}{d L_{eff}^{(p)} G_j(L)} \left[\int_0^{L - \frac{t}{d}} e^{-\alpha_p(L-z')} G_j\left(z' + \frac{t}{d}\right) dz' \right] \cdot p(t)$$

where $G_j(z)$ is the *unsaturated* gain-versus- z profile, and $p(t) =$



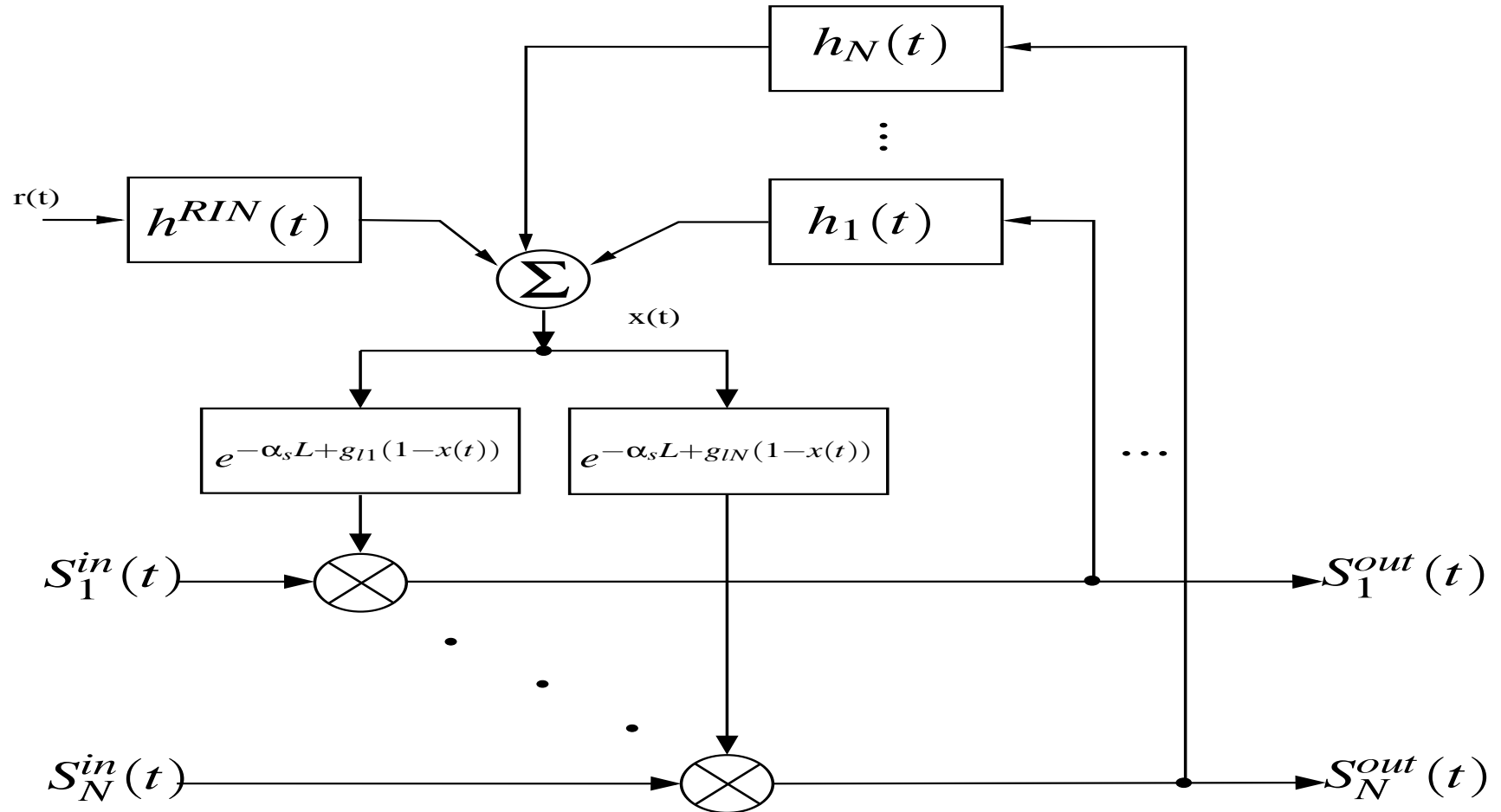
Filter

A **closed-form** of such filter exists. For a DCF with $\alpha_j = 0.46$ dB/km, $\alpha_p = 0.6$ dB/km, $g_j = 2$ [W⁻¹km⁻¹] get: (- - exponential approx.)



Amplifier Block Diagram

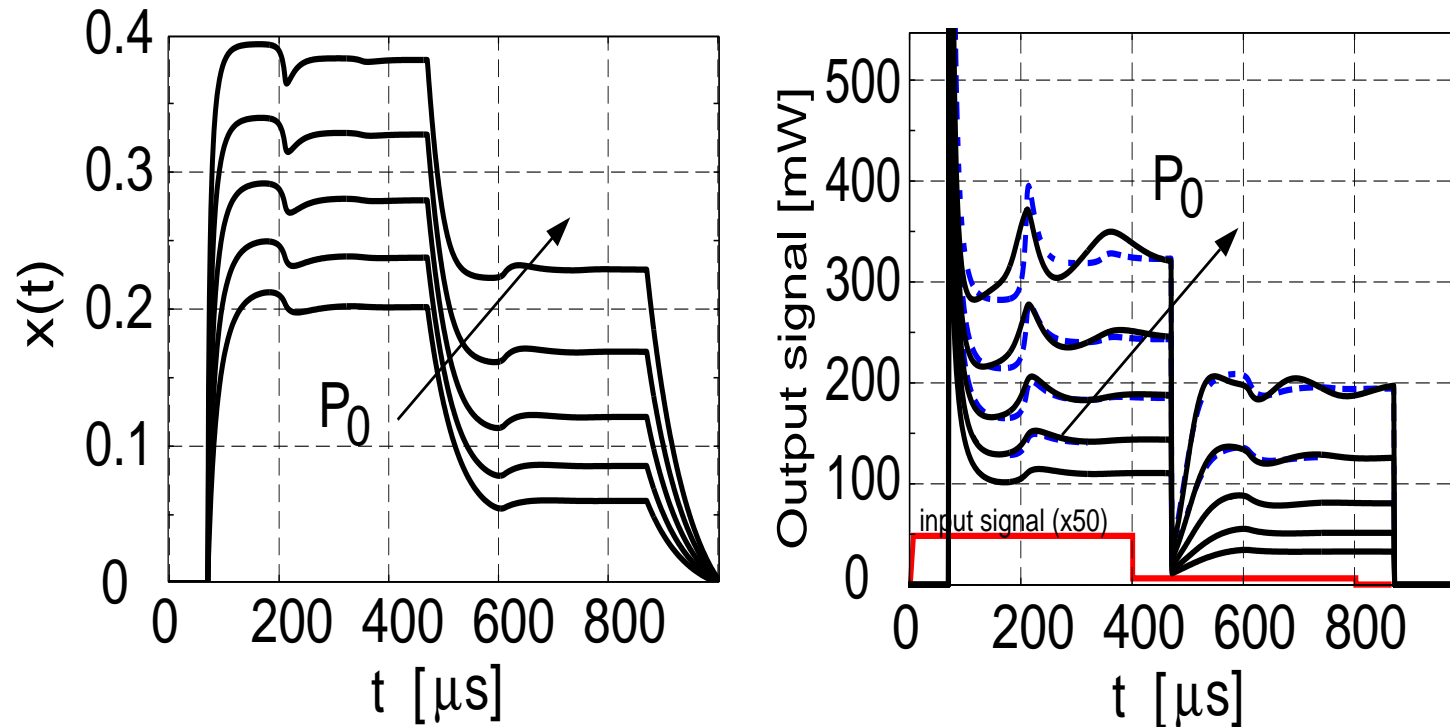
RIN filter: Cfr [Fludger et al, JLT Aug '01](#).



Can be implemented in any Block-Diagram Simulator!

Results: Single Channel

Two Pulses of 1 and 0.1 mW, 400 μ s each. Ampli: L=14 km, DCF.

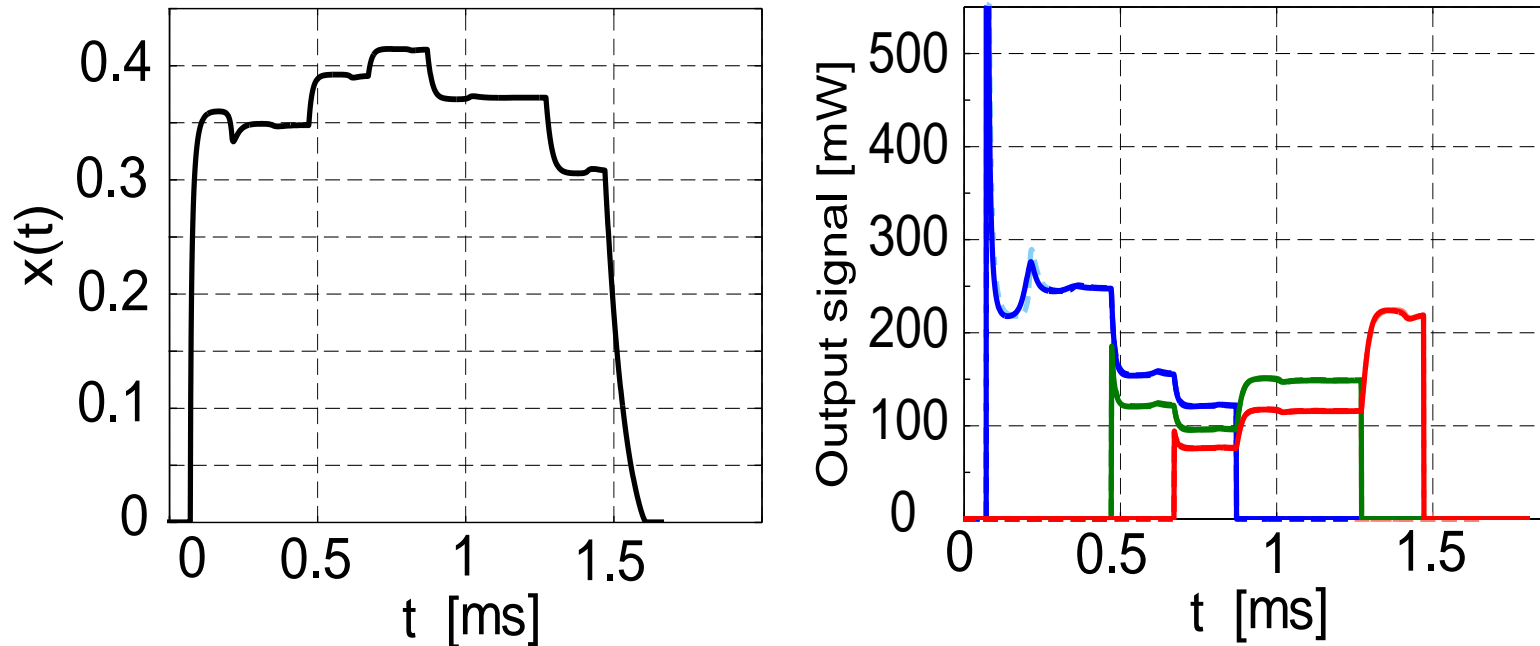


(Left) state variable, and (Right) output signal power, for increasing levels of pump power $P_0 = 0.64, 0.70, 0.77, 0.86, 0.97$ W. Solid lines: exact solution (+DRB); Dashed lines: model.



Results: WDM

Three staggered pulses (1 mW, 800 μ s each) on 3 channels, $\Delta\lambda = 0.4$ nm.
L=14 km, DCF.

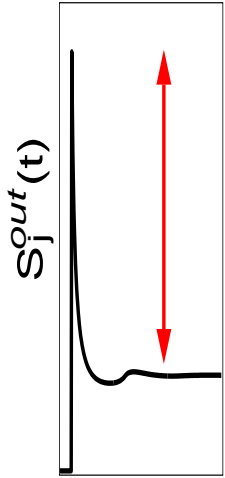


(Left) state variable, and (Right) output signal powers, for pump power $P_0 = 0.86$ W. Solid lines: exact solution (+SS xtalk, +DRB); Dashed lines: model.

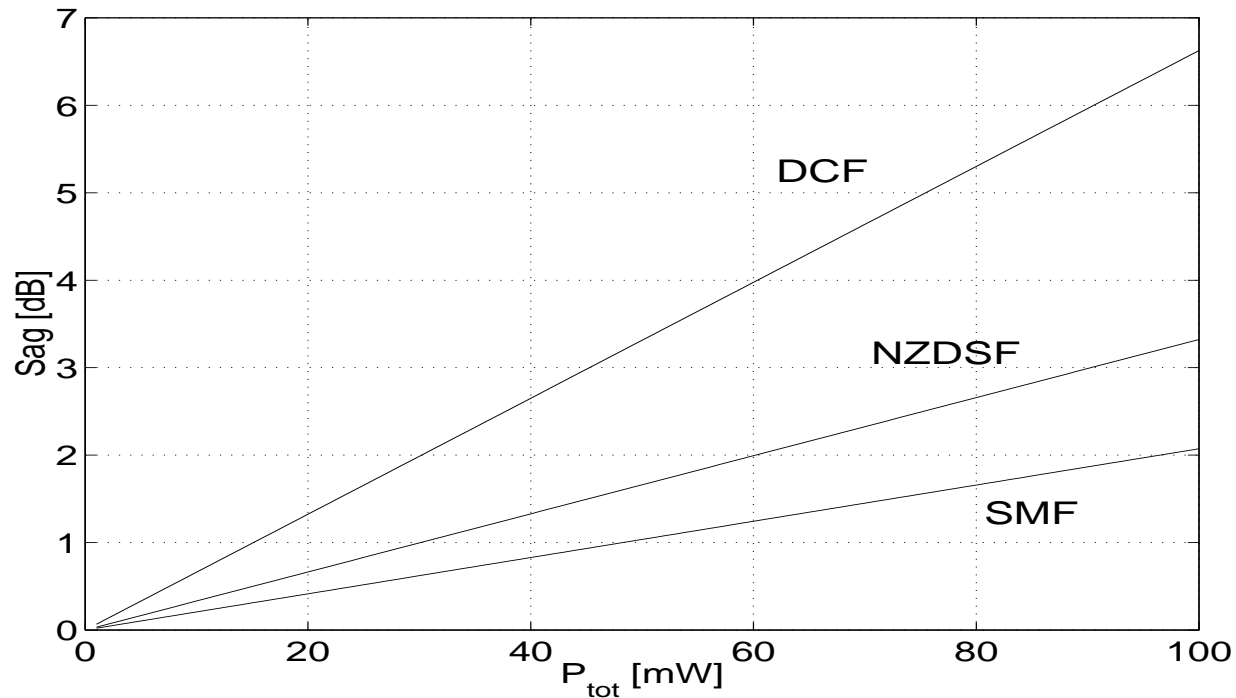


Power surges

The power **Sag** across a pulse is:



$$\left. \frac{S_j^{out}(0)}{S_j^{out}(x^{ss})} \right|_{dB} \cong (10 \log_{10} e) \left[\frac{\hat{g}_s}{\alpha_p} \underbrace{\left(\sum_{i=1}^N S_i^{out}(x^{ss}) \right)}_{P_{tot}} \right]$$



Time constants

Using exponential approximation $h_j(t) \cong h_{j0}e^{-\frac{\alpha_p}{d}t}$, state equation becomes an ordinary differential equation (ODE), similar to that for EDFAs:

$$\dot{x}(t) = -\frac{x(t)}{\tau} + \sum_{j=1}^N h_{j0} S_j^{out}(t, x(t))$$

with $\tau \triangleq d/\alpha_p$ playing the role of the fluorescence time.

Using standard linearization technique ([Y. Sun et al., JLT, July 97](#)) we find:

$$\tau_{eff} \cong \frac{d/\alpha_p}{1 + \left[\frac{\hat{g}}{\alpha_p} \left(\sum_{i=1}^N S_i^{out}(x^{ss}) \right) \right]}$$



Conclusions

- ✓ XGM present also in Raman amplifiers
- ✓ New state-variable model of counter-pumped saturated Raman amplifiers
- ✓ Closed-form expressions for power surges and time constants of transients

