Estimation and Compensation of Linear Amplitude Distortions

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Abstract

The estimation and the compensation of particular amplitude distortions, affecting the consumer-grade receivers in broadband satellite communications, are considered. We present a novel estimator, analytically derived by exploiting low-order polynomial approximations of the equivalent distortion filter, which is modeled as an exponential function of the frequency. In the bandwidth of practical interest, the proposed solution results to be unbiased and does not exhibit false locks. Since the ideal compensator requires a complex filtering, we also describe an alternative low-complexity compensation filter, almost ensuring the same performance as the ideal one. Some computer simulation results, showing the effectiveness of the proposed solutions, are finally reported.

I. Introduction

Despite the large amount of literature dealing with the estimation and the compensation of unknown parameters, the future broadband satellite digital transmissions, as those based on the DVB-S2 standard [1], raise new issues on the synchronization stage. One of most critical problem is the presence of an amplitude distortion on the received signal, mainly due to the low noise block (LNB) and the coaxial cable at the consumer side. After measurements on these devices, a model based on a linear filter was proposed for this distortion (often referred to as *slope* distortion), whose amplitude response, measured in dB, is linear. Hence, the complex baseband equivalent of the slope filter can be modeled as an exponential function of the frequency. In conditions of large baudrates, namely from 20 Mbaud ahead, the effects of the slope filter on the performance of the system are dramatic, so that the distortion must be properly estimated and compensated. In this paper, a novel all-digital slope equalizer is presented.

We describe the equalizer in both open-loop and closed-loop configurations, but we mainly focus on the latter, since the former results impractical to be implemented. The proposed solution, which exploits the properties of the power spectral density and the autocorrelation function of the received signal, results computationally very simple. Moreover, the S-curve of the estimator [2], whose expression is derived in closed form, proves that neither estimation biases nor false locks can occur in the bandwidths of interest. A further interesting property of the proposed estimator is that it can cope not only with exponential filters, but even with any slope filter whose dependence on the frequency is linear (or approximately linear).

In order to perfectly compensate the slope distortion, the compensation filter should be an exponential function of the frequency. Since this solution is impractical from a complexity viewpoint, we describe a low-complexity compensator based on the first-order Taylor expansion of the true frequency response. We also considered higher-order approximations, but no significant improvement was found. The resulting compensator consists in the sum of a replica of the incoming signal plus, neglecting any constant multiplicative factor, its derivative. We consider a

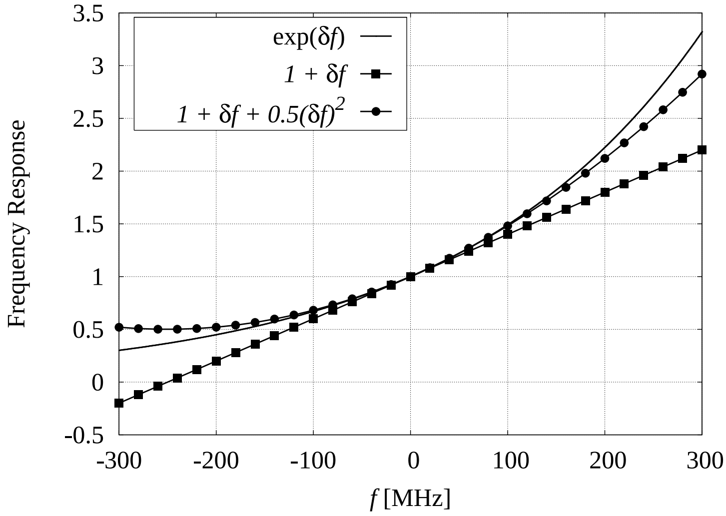

Fig. 1. Frequency responses of the exponential slope filter and of its first-order and second-order polynomial approximations, for $\delta = 4$ ns.

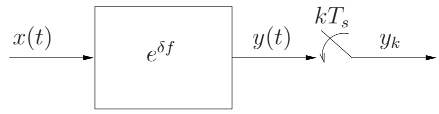

Fig. 2. Basic block diagram.

derivative filter implemented by means of a discrete-time finite impulse response (FIR) filter, whose order is very critical, since the higher its value, the larger the bandwidth over which the filter practically equals the behavior of an ideal differentiator, but the higher its complexity. Extensive computer simulations show that the FIR filter ensures an almost-perfect compensation when an order of at least 10 is adopted.

The paper is organized as follows. In Section II, we describe the basic system model. In Section III, the proposed estimator is described, in both open-loop and closed-loop configurations. In Section IV, after designing a low-complexity slope compensator, we report the results of computer simulations showing the behavior of the proposed equalizer. In Section V, it is shown how to modify the equalizer in order to extend its effectiveness to more general cases with respect to the simple initial model. Finally, Section VI gives some concluding remarks.

II. System Model

Let us consider a complex signal $x(t)$ with limited single-sided bandwidth B_x, and let us suppose that it is distorted to $y(t)$ by the following slope filter

$$H_S(f) = e^{\delta f} \tag{1}$$

where δ is an unknown parameter ranging in the interval $[-4, 4]$ ns. The frequency response of the filter (1) is plotted in Fig. 1 for the largest possible value of δ, together with its first-order and second-order Taylor expansions. In the following, we will exploit the fact that the parabolic approximation perfectly matches the exponential filter up to about 120 MHz, that is the range of interest for B_x. The aim of this work is to design a fully-digital slope equalizer working on the samples $y_k = y(kT_s)$, assuming that the sampling interval T_s satisfies the Nyquist condition $B_x \leq \frac{1}{2T_s}$ [3]. In order to derive the basic equalizer, we also assume that the input signal $x(t)$ has a real-valued autocorrelation function $R_x(\tau)$—we will explain how to cope with more general models in Section V.

Referring to the block diagram reported in Fig. 2, the functions $S_x(f)$ and $S_y(f)$, respectively the power spectral density (PSD) of $x(t)$ and $y(t)$, satisfy the following relation

$$S_y(f) = S_x(f)e^{2\delta f} \approx S_x(f)\Big[1 + 2\delta f + 2\delta^2 f^2\Big] = S_x(f) - j\frac{\delta}{\pi}\Big[(j2\pi f)S_x(f)\Big] - \frac{\delta^2}{2\pi^2}\Big[(j2\pi f)^2 S_x(f)\Big] \tag{2}$$

where the exponential function has been approximated by its second-order Taylor expansion. As stated before, this approximation is practically ideal in the ranges of interest for δ and f. By applying the inverse Fourier transform to (2), the relation among the corresponding autocorrelation functions results

$$R_y(\tau) = R_x(\tau) - j\frac{\delta}{\pi}R'_x(\tau) - \frac{\delta^2}{2\pi^2}R''_x(\tau) \tag{3}$$

where $R'_x(\tau)$ and $R''_x(\tau)$ are the first and the second derivative of $R_x(\tau)$ with respect to τ. Thanks to the hypothesis of real-valued $R_x(\tau)$, by taking the imaginary part of (3) and performing simple algebraic manipulations, we can write

$$\delta = -\frac{\pi}{R'_x(\tau)}\mathrm{Im}\Big\{R_y(\tau)\Big\} \tag{4}$$

provided that $R'_x(\tau) \neq 0$. Since we are interested in fully-digital equalizers working on the samples y_k, let us consider $\tau = nT_s$ (with n positive integer to be chosen such that $R'_x(nT_s) \neq 0$), so that the following equality results

$$\delta = -\frac{\pi}{R'_x(nT_s)}\mathrm{Im}\Big\{R_y(nT_s)\Big\} = -\frac{\pi}{R'_x(nT_s)}\mathrm{Im}\Big\{E\{y_k y^*_{k-n}\}\Big\}\,. \tag{5}$$

In Section III, we will exploit (5) in order to derive effective estimators of the parameter δ. It is easy to prove that (5) is valid not only in the case of exponential slope filters, but in the more general case of slope filters whose dependence on the frequency is linear (or approximately linear). Hence, the estimators derived from (5) can cope with different distortion models.

III. Slope Estimators

A. Open-Loop Estimator

The equality (5) allows to easily derive an open-loop estimator of δ. Indeed, if the statistical mean $E\{y_k y^*_{k-n}\}$ is replaced by a temporal mean over a set of L realizations, the following estimator results

$$\hat{\delta} = -\frac{\pi}{R'_x(nT_s)}\mathrm{Im}\left\{\frac{\sum_{k=1}^{L} y_k y^*_{k-n}}{L}\right\} = -\frac{\pi}{LR'_x(nT_s)}\sum_{k=1}^{L}\mathrm{Im}\{y_k y^*_{k-n}\}\,. \tag{6}$$

It can be proved that the estimator (6) is unbiased, that is $E\{\hat{\delta}\} = \delta$, but unfortunately its relevance is mainly theoretical, since it requires to perfectly know the value of $R'_x(nT_s)$. It is indeed practically impossible to get a statistical characterization of $x(t)$ as detailed as required to achieve a good accuracy in estimating $R'_x(nT_s)$. Because of this and of the fact that any open-loop estimator cannot track possible variations of δ, we will focus on closed-loop slope equalizers.

B. Closed-Loop Estimator

Let us consider the classical block diagram of a closed-loop estimator, reported in Fig. 3. The frequency response $e^{-\hat{\delta}_k f}$ of the digital slope compensator is denoted as if we were working in the continuous-time domain, in order to keep the notation simpler. The same choice is adopted in all the block diagrams shown in this paper.

Let us update $\hat{\delta}_k$, the estimate of δ at the sampling epoch k, according to the following recursion

$$\hat{\delta}_{k+1} = \hat{\delta}_k + \gamma e_k \tag{7}$$

where $\hat{\delta}_0 = 0$, γ is the step-size of the loop, and e_k is a properly defined error signal. The crucial point of the algorithm is the design of the slope error detector (SED), i. e. the generation of e_k in (7), such that the residual error $\Delta_k = \delta - \hat{\delta}_k$ vanishes on average for increasing values of k. Our aim is to derive a SED such that

$$E\{e_k|\Delta_k\} \propto \Delta_k \tag{8}$$

since it is well known [2] that, in the range over which it is satisfied, this condition ensures the effectiveness of a closed-loop estimator. Referring to the block diagram in Fig. 3, it is worth noticing that the notation $e^{-\hat{\delta}_k f}$ denoting the frequency response of the slope compensator is not rigorous, since it involves the time-varying coefficient $\hat{\delta}_k$. In

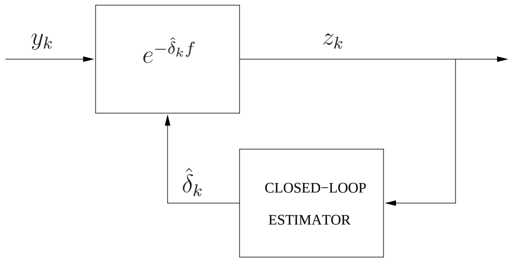

Fig. 3. Block diagram of the closed-loop equalizer.

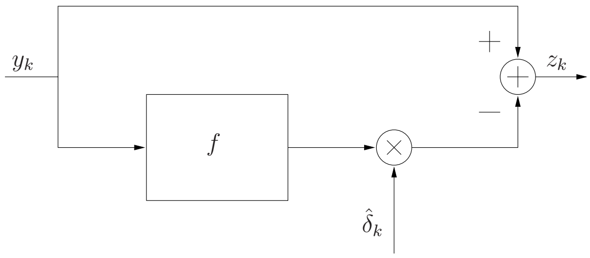

Fig. 4. Block diagram of the proposed low-complexity slope compensator.

Section IV we will replace this filter by a well-defined one, but as a first step let us suppose that the variation of $\hat{\delta}_k$ is slow enough, such that the notation $e^{-\hat{\delta}_k f}$ can be considered well-defined.

By applying considerations similar to those leading to (5), it is easy to prove that the residual error Δ_k results

$$\Delta_k = -\frac{\pi}{R_x'(nT_s)}\mathrm{Im}\Big\{R_z(nT_s)\Big\} = -\frac{\pi}{R_x'(nT_s)}\mathrm{Im}\Big\{E\{z_k z_{k-n}^*\}\Big\} \tag{9}$$

where the samples z_k are the outputs of the compensation filter. Hence, approximating the statistical mean $E\{z_k z_{k-n}^*\}$ in (9) by the actual value $z_k z_{k-n}^*$ and dropping any irrelevant positive factor, we can define

$$e_k = A_n \mathrm{Im}\Big\{z_k z_{k-n}^*\Big\} \tag{10}$$

where $A_n = -\mathrm{sign}\{R_x'(nT_s)\}$. Let us remark that this closed-loop estimator only requires to know the sign of $R_x'(nT_s)$, instead of its exact value as required by the open-loop estimator (6), thus resulting much easier to be implemented even in the case of incomplete knowledge on the spectral properties of $x(t)$.

If we neglect the terms of order higher than two in the polynomial expansion of the slope filter $H_S(f)$, the S-curve corresponding to the proposed SED turns out to be

$$E\Big\{e_k\big|\Delta_k\Big\} = E\Big\{A_n\mathrm{Im}\{z_k z_{k-n}^*\}\big|\Delta_k\Big\} = A_n\mathrm{Im}\Big\{R_z\big(nT_s\big|\Delta_k\big)\Big\} = -A_n\frac{R_x'(nT_s)}{\pi}\Delta_k = \frac{|R_x'(nT_s)|}{\pi}\Delta_k \propto \Delta_k \tag{11}$$

as in the target relation (8). This proves that the described equalizer is unbiased and that no false locks can occur in the region where the adopted expansion can be considered ideal, that is the region of practical interest.

IV. Slope Compensators

In the closed-loop equalizer, the slope compensator is ideally required to exhibit the following transfer function

$$e^{-\hat{\delta}_k f} = \sum_{i=0}^{\infty}\frac{(-\hat{\delta}_k f)^i}{i!} \ . \tag{12}$$

In order to obtain a low-complexity compensation filter, the summation in (12) must be truncated. In particular, by truncating the summation to the first-order term, the block diagram shown in Fig. 4 results. The complexity of this compensator is very low, since it is only composed by one adder, one multiplier, and one derivative filter (scaled by a factor $j2\pi$). As shown in Fig. 1, the quality of the linear approximation is not as good as the parabolic one, but extensive computer simulations prove that it is a good tradeoff—with respect to higher-order approximations, it requires a much simpler filtering and practically ensures the same performance when B_x is lower than 80 MHz. The design of numerical differentiators is a widely-studied topic and it is well-known that a good solution consists in a FIR filter [4]. We consider a proper discrete-time 10^{th}-order FIR filter (implying a 5-samples group delay), which provides a good accuracy in approximating an ideal derivative filter over a bandwidth of about 37% of the sampling rate, as shown in Fig. 5. Hence, the sampling rate must be designed by taking into account that the bandwidth over which the FIR filter approximates the ideal derivative filter must be larger than B_x, that is (in the case of the considered 10^{th}-order filter)

$$B_x < 0.37\frac{1}{T_s} \ . \tag{13}$$

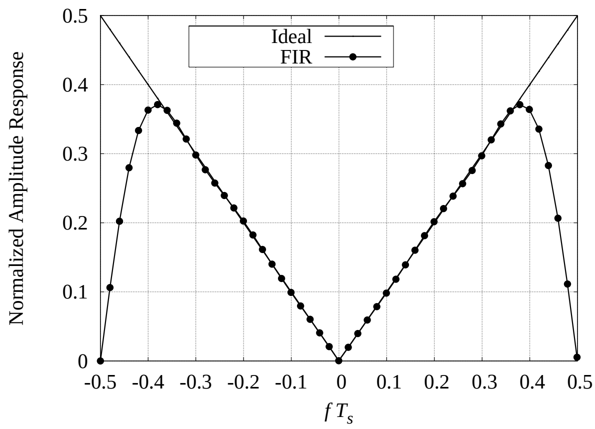

Fig. 5. Normalized amplitude responses of the ideal differentiator and of the considered FIR filter.

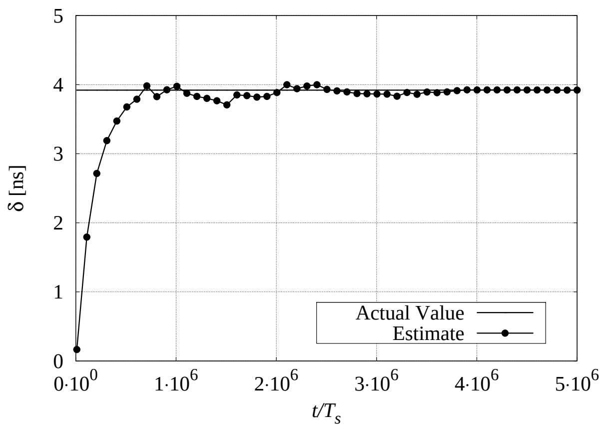

Fig. 6. Acquisition and tracking behavior of the proposed closed-loop equalizer.

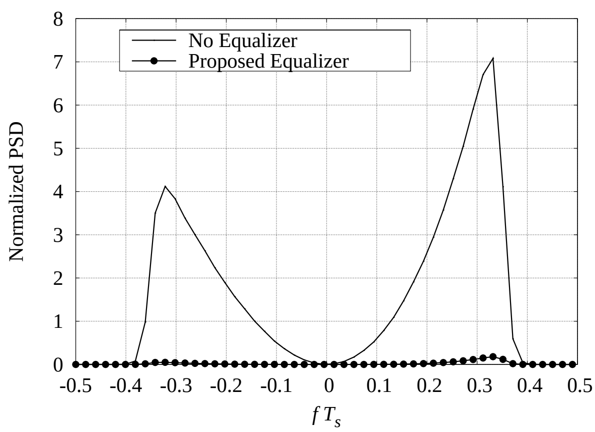

Fig. 7. Power spectral density of the difference among the signal as it should be after an ideal slope compensator and the signal as it really is in the absence/presence of the proposed equalizer.

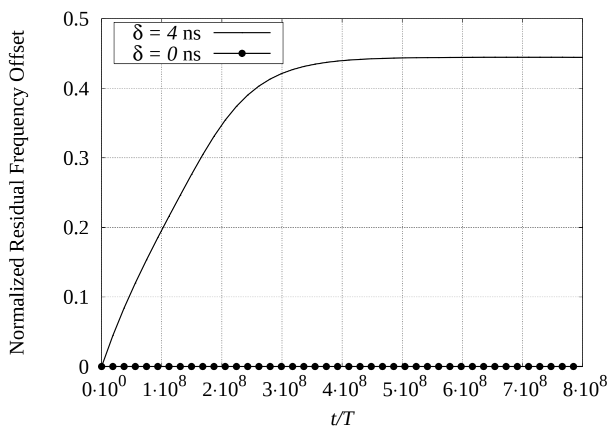

Fig. 8. Residual frequency offset, normalized with respect to the symbol rate, after an AFC loop implementing the ML algorithm, in the cases of strong slope ($\delta = 4$ ns) and null slope ($\delta = 0$ ns).

In Fig. 6 the behavior of the proposed slope equalizer is shown, in the case of white Gaussian noise as input signal, limited in bandwidth by means of a 6$^{\text{th}}$-order Butterworth lowpass filter in order to match (13), and sampled at the Nyquist rate. It is clear that, after an acquisition stage of about 10^6 sampling intervals, the equalizer reaches its steady state, and that it is unbiased as expected after analyzing the S-curve (11). The reported results refer to $n = 1$, which implies $A_n = 1$ in this case. In order to fasten the acquisition stage without degrading the performance of the tracking stage, a time-varying step-size was adopted in the loop: the value of γ in (7) is linearly decreased at every sampling epoch, such that it is reduced from the initial value $3 \cdot 10^{-4}$ to the steady-state value 10^{-6} in a temporal interval of $4 \cdot 10^6 \cdot T_s$.

Once achieved the tracking condition, the performance of the slope equalizer can be evaluated in terms of PSD of the error due to the slope, that is the difference among the signal as it should be after an ideal compensator and the signal as it really is after the equalizer. Referring to the same scenario of Fig. 6, the results reported in Fig. 7 show that the proposed solution is noticeably effective in reducing both the peak values and the overall power of the error.

V. SLOPE EQUALIZER FOR BROADBAND SATELLITE SYSTEMS

Let us now consider a communication system where the complex envelope of the received signal, if the slope effect is neglected, can be written as

$$r(t) = e^{j(2\pi\nu t+\theta)} \sum_i c_i p(t - iT - \tau) + w(t) \tag{14}$$

where $\{c_i\}$ are complex data symbols, $p(t)$ is the equivalent shaping pulse at the receiver, T is the symbol interval, ν, θ, and τ are the frequency, phase, and timing offsets, and $w(t)$ is complex white Gaussian noise. We assume that $\{c_i\}$ are zero-mean and independent, and that $p(t)$ is the inverse Fourier transform of $P(f)$, that is a raised-cosine function with known roll-off α. Moreover, the unknown offsets ν, θ, and τ are assumed constant.

In order to perform a fully-digital coarse frequency synchronization, $r(t)$ is passed through an anti-alias filter (AAF) and sampled at a proper rate, and the resulting samples are processed by an automatic frequency control (AFC) loop. The frequency recovery algorithms commonly implemented through AFC closed loops completely neglect the slope distortion, but at large baudrates, namely from 20 Mbaud ahead, the slope effect heavily alters the spectral properties which they exploit. In Fig. 8, it is shown that an AFC loop implementing the maximum-likelihood (ML) algorithm [2], which is unbiased in the absence of slope distortion ($\delta = 0$ ns), exhibits a bias of about 45% of the symbol rate when $\delta = 4$ ns. These results refer to the case of symbol rate of 45 Mbaud, 16-APSK modulation, signal-to-noise ratio of 13 dB, null ν and τ, and strong phase noise as described in [5]. Let us recall that biases of the frequency estimation larger than about 20% of the symbol rate cannot be tolerated by any timing recovery algorithm [2] and, consequently, dramatically affect the performance of the whole system. Hence, a proper slope equalizer must be placed before the AFC loop.

It is well known [6] that the PSD $S_r(f)$ of the received signal (14) satisfies the following relation

$$S_r(f) \propto |P(f-\nu)|^2 + C \tag{15}$$

where C is a positive constant depending on the signal-to-noise ratio. It is worth pointing out that the function $S_r(f)$ has even symmetry only if $\nu = 0$, whereas the proposed slope equalizer was derived in the hypothesis of an input signal which, before being distorted, has a real-valued autocorrelation or, equivalently, an even PSD function. Hence, the equalizer cannot work in the presence of non-zero frequency offset, as it actually is since the equalizer should be placed before the AFC loop. In order to solve this issue, we exploit the fact that the function $S_r(f)$ is flat over the interval $\nu - \frac{1-\alpha}{2T} < f < \nu + \frac{1-\alpha}{2T}$. Let us pass $r(t)$ through an ideal rectangular lowpass filter $H_{LP}(f)$ with bandwidth B_{LP}, obtaining the output $x(t)$. If the following condition is satisfied

$$B_{LP} < \tfrac{1-\alpha}{2T} - |\nu| \tag{16}$$

the resulting PSD of $x(t)$ is

$$S_x(f) \propto \Big[|P(0)|^2 + C\Big] H_{LP}(f) \tag{17}$$

which has even symmetry, as desired. This choice would ensure the effectiveness of the compensator, but it is not viable since the useful signal would be distorted by $H_{LP}(f)$. The final solution is thus to place the lowpass filter in the feedback path, as shown in Fig. 9, so that the filter $H_{LP}(f)$ does not affect the signal path. All these considerations are valid even if the rectangular filter is replaced by any lowpass filter with even symmetry—in particular, we adopted a 5$^{\text{th}}$-order Butterworth filter in the computer simulations. Let us point out that, when the system parameters are such that (16) results critical, the only solution is to place the slope equalizer inside the AFC loop, so that (16) can be satisfied after a transient stage.

Let us again consider the ideal rectangular filter $H_{LP}(f)$; by applying the inverse Fourier transform to (17), the following relation results

$$R_x(nT_s) \propto \mathrm{sinc}(2B_{LP}nT_s) \tag{18}$$

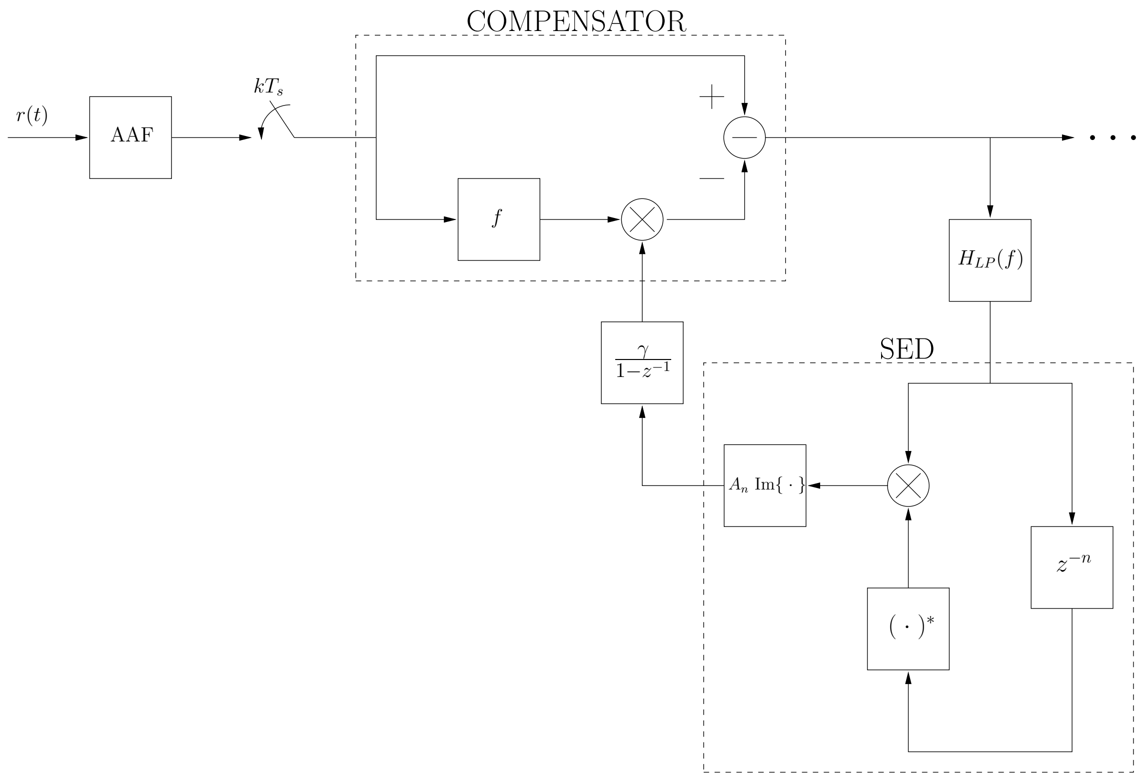

Fig. 9. Slope equalizer for systems where the received signal can be written as in (14).

where $\mathrm{sinc}(x) = \sin(\pi x)/(\pi x)$. Given any value of B_{LP} and T_s, since $R'_x(nT_s)$ must be non-zero, we can just choose n such that this condition is verified. After properly choosing the value of T_s, the choice to set $n = 1$ and $A_n = 1$ can be always adopted when the received signal can be written as in (14). Many other results similar to the ones shown in Fig. 6 and Fig. 7 confirm the effectiveness of the proposed slope equalizer in various scenarios of interest.

VI. Conclusions

We have presented a novel equalizer for particular amplitude distortions affecting the consumer-grade receivers in broadband satellite communications. The equalizer has been analytically derived by exploiting low-order polynomial approximations of the distortion filter, modeled as an exponential function of the frequency. We have shown that the proposed solution is unbiased and that no false locks can occur in the bandwidth of interest. Since the ideal equalizer requires a complex compensation filtering, an alternative low-complexity solution, practically ensuring the same performance, has also been described. The effectiveness of the proposed solutions has been proved by reporting some computer simulations results relative to various scenarios of interest.

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