Tutorial - Dynamic Link Budget Optimisation for Telemetry Links

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***Abstract*—Telemetry links in non-geostationary orbit have a signal-to-noise ratio (SNR) that changes over time due to the change of geometry. This paper is a tutorial on dynamic link budget and provides a mathematical framework for analysing the link as function of time for trading-off bitrate vs transmission time with the aim of maximising the amount of downloaded data to ground.**

I. NOTE BEFORE READING

The view expressed herein is based totally on author personal studies and views, and is not related to his profession know-how, work projects, and related activities.

This work has been done during the author (few) free time and has been never subject to peer review and could contain typos, discrepancies, etc.

II. INTRODUCTION

The demand of high bitrates in telemetry applications is continuously increasing and today's engineering is making every effort for scratching the last bit from the bottom of the barrel. Basically, the maximisation of the bitrate can be done in two ways. The first approach is *designing transceivers with coding and modulation formats with high spectral efficiency*. For instance, this approach has been followed for the definition of the new CCSDS standard for telemetry communications [1] and the definition of turbo codes for telemetry applications [2], [3]. The second approach is the *optimisation of the link budget* parameters with the aim of maximising the signal to noise ratio (SNR) at the receiver (and thus the maximum bitrate). In this paper we focus on the second approach.

Link budget is typically done by means of a table that shows gains and losses for the computation of the SNR, and thus the corresponding maximum bitrate. In the past, the link budget table was filled with the worst case value for each entry. Clearly, this approach did not last too long: it was clear that the event of having all measurements equal to their worst case was extremely unlikely, resulting in a overdesign of the link margin. In 70s, Joseph Yuen proposed a statistical approach [4] that associates a probability density function to each entry of the link budget and applies a margin policy that guarantees link availability with probability higher than 99%. This technique is still employed today for the design of several communication systems (for instance, the ones following ECSS standards [3]).

Although Yuen approach tries to avoid the use of worst case values, sometimes we are implicitly still using the worst case for those parameters related to the geometry. In particular the path loss, the antenna pattern, the gain to noise temperature ratio, etc., are often considered in the specific time instant when the spacecraft becomes visible to ground. In other words, we do a *static link budget*. However, it is easy to realise that if we are focusing our analysis on a specific time instant we could draw wrong conclusions because we did not look at the whole picture of the problem. For instance, if we consider a link budget as function of time (i.e. a *dynamic link budget*), we could decide to use higher bitrates that could increase the overall amount of transmitted data although the time interval of transmission will be lower.

The remainder of this paper is organized as follows: Section III provides the mathematical model that will be adopted through the paper. Section IV describes optimisation techniques for a fixed bitrate, whereas Section V extends the problem to variable bitrate systems. Finally in Section VI numerical results are reported.

III. MATHEMATICAL MODEL

We consider a spacecraft (S/C) that has to download data to a ground station (G/S). The SNR at the receiver is given by the received power to noise spectral density ratio that reads [5]

$$\frac{P_{\mathrm{RX}}}{N_0} = \frac{\mathrm{EIRP}}{\rho L M k_{\mathrm{B}}} \left(\frac{G_{\mathrm{A}}}{T_{\mathrm{A}}}\right) \quad [\mathrm{Hz}],$$

where EIRP is the equivalent isotropically radiated power, k_{B} is the Boltzman constant, L is the path loss, $G_{\mathrm{A}}/T_{\mathrm{A}}$ the gain to noise temperature ratio of the G/S antenna, and M the margin (defined according to a given policy). Other losses (like atmospheric, polarization, etc.) are all included in the parameter ρ.

During the mission all the parameters that are related to the geometry change as function of time. Hence, it holds that $P_{\mathrm{RX}}/N_0 = P_{\mathrm{RX}}/N_0(t)$ is also a function of time t. We then define

$$\mathrm{SNR}(t) \triangleq \begin{cases} \frac{P_{\mathrm{RX}}}{N_0}(t) & \text{S/C visible to the G/S} \\ 0 & \text{S/C not visible to the G/S} \end{cases} .$$

For a given coding and modulation format there is a E_b/N_0 (being E_b the energy per bit) such that detection and decoding of the signal is guaranteed for a specific target error rate (e.g. the Reed Solomon code in [6] has $E_b/N_0 = 6.8$ dB for frame error rate 10^{-5} over the AWGN channel). The maximum achievable bitrate (before coding) is given by

$$R(t) = \frac{\mathrm{SNR}(t)}{E_b/N_0} \quad [\mathrm{bit/s}] \tag{1}$$

that is also a function of time. The function $R(t)$ is the key element of our model, and we will call it *achievable bitrate*

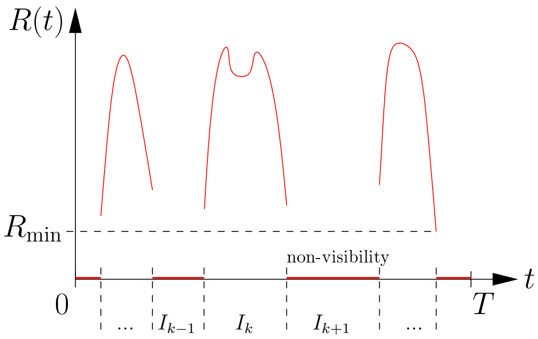

Fig. 1: Example of shape for the $R(t)$ function.

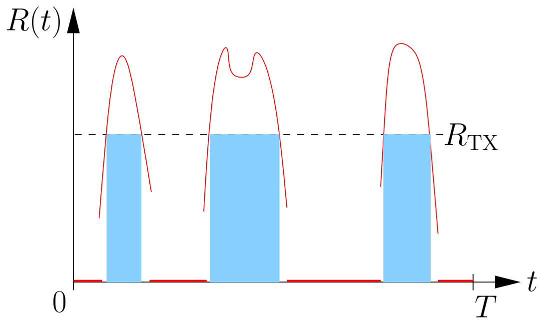

Fig. 2: Graphical interpretation for the amount of downloaded data for a given $R(t)$ function and R_{TX}.

function since it shows instantaneously what is the maximum bitrate at any time of the mission. We assume that $R(t)$ has the following properties:

- It is a piecewise function defined in the interval $[0, T]$ (i.e. the mission starts at $t = 0$ and ends at $t = T$).
- The number of constituent functions of $R(t)$ is finite.
- Each constituent function is defined on a subinterval I_k with dimension greater than zero, and is differentiable on that interval.
- Each constituent function is equal to zero, or has codomain bounded between 0 and a finite value.

We also define the non-zero minimum of $R(t)$ as

$$R_{\min} \triangleq \inf \min_{R(t)>0} R(t)\,.$$

Under these conditions, $R(t)$ has a shape like the one shown in Figure 1 where visibility and non-visibility intervals are staggered along the time. Notice that each constituent function can have more local maxima such that several phenomena (like change of attitude, etc.) can be taken into account. Although not shown in the sketch, there are no constraints on the order of visibility and non-visibility intervals, so that $R(t)$ can model also a change of G/S, or polarization, etc.

If the S/C transmitter adopts a bitrate equal to $R_{\mathrm{TX}} \geq 0$, clearly the communication with the G/S is possible only when $R(t) \geq R_{\mathrm{TX}}$. Then, the average downlink bitrate R_{avg} for the mission reads

$$R_{\mathrm{avg}}(R_{\mathrm{TX}}) = R_{\mathrm{TX}} \frac{1}{T} \int_0^T \mathrm{I}(R(t) \geq R_{\mathrm{TX}}) \mathrm{d}t \quad [\mathrm{bit/s}] \tag{2}$$

where $\mathrm{I}(\cdot)$ is an indicator function, i.e. it is equal to 1 if the argument is true, 0 otherwise. By definition, the amount of downloaded data during the mission is given by

$$R_{\mathrm{avg}}(R_{\mathrm{TX}})T \quad [\mathrm{bit}]\,. \tag{3}$$

Equation (3) has a nice graphical interpretation as sketched in Figure 2. The amount of downloaded data is the area of the boxes with height R_{TX} that are inscribed in the function $R(t)$.

It must be noticed that for $R_{\mathrm{TX}} = R_{\min}$ Equation (2) simplifies to

$$\begin{aligned} R_{\mathrm{avg}}(R_{\min}) &= R_{\min} \frac{1}{T} \int_0^T \mathrm{I}(R(t) > 0) \mathrm{d}t \\ &= R_{\min} \frac{T_{\mathrm{V}}}{T} \end{aligned}$$

where we defined $T_{\mathrm{V}} \triangleq \int_0^T \mathrm{I}(R(t) > 0)\mathrm{d}t$. The value T_{V} is the amount of time for which the S/C is visible to the G/S. Thus, the ratio T_{V}/T is the fraction of visibility of the S/C.

IV. Dynamic Link Budget Optimisation

With the classical approach, when designing a transmitter for a S/C we want to know the maximum bitrate such that the link can be established every time that G/S is visible. Thus, we do a static link budget and we derive $R_{\min}$ and set $R_{\mathrm{TX}} = R_{\min}$. But do we really need to guarantee the link all the time? Obviously the answer depends on the specific mission, but in many cases (the one that we are interested in) is No. In this section we show how to select R_{TX} that maximise such amount. The problem will be solved without constraints and, later on, under a bandwidth and minimum download per G/S pass constraint.

A. Unconstrained maximisation of the downloaded data

We can define the increase of downloaded data by using R_{TX} instead of $R_{\min}$ as

$$\begin{aligned} G(R_{\mathrm{TX}}) &\triangleq \frac{R_{\mathrm{avg}}(R_{\mathrm{TX}})T}{R_{\mathrm{avg}}(R_{\min})T} \\ &= \frac{R_{\mathrm{TX}}}{R_{\min}} \frac{1}{T_{\mathrm{V}}} \int_0^T \mathrm{I}(R(t) \geq R_{\mathrm{TX}}) \mathrm{d}t\,. \end{aligned} \tag{4}$$

The optimisation problem becomes

$$R_{\mathrm{opt}} = \arg\max_{R_{\mathrm{TX}}} G(R_{\mathrm{TX}})\,. \tag{5}$$

From a graphical point of view this is equivalent to maximise the area of the boxes in Figure 2 by changing R_{TX}. We have the following theorem:

Theorem 1. *Let $R(t)$ be an achievable bitrate function. Then, $G(R_{\mathrm{TX}})$ is a continuous function $\forall R_{\mathrm{TX}} \in [0, \infty)$ and the optimal value R_{opt} is such that $G(R_{\mathrm{opt}}) \geq 1$.*

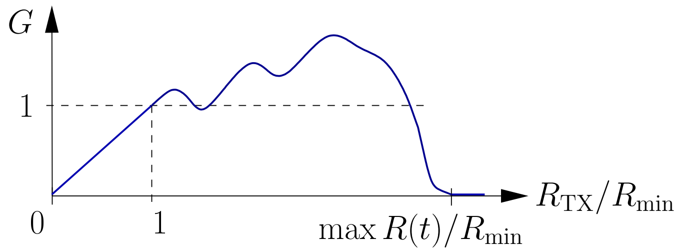

Fig. 3: Example of shape for the $G(R_{\mathrm{TX}})$ function.

The proof is provided in Appendix A. In few words, Theorem 1 guarantees what intuitively we can expect: $G(R_{\mathrm{TX}})$ can be computed for any R_{TX} and we can find R_{opt} by performing a full search on its domain. Moreover, the optimal bitrate allows to download more data then using $R_{\min}$ or, in the worst case, it provides exactly the same amount. A sketch of G shape that we can expect is shown in Figure 3. For $R_{\mathrm{TX}} \in [0, R_{\min}]$ the function G increases linearly whereas for $R_{\mathrm{TX}} \in [R_{\min}, \max R(t)]$ there are no constraints on its codomain. This shape if plotted as function of $R_{\mathrm{TX}}/R_{\min}$ does not depend on the E_b/N_0 adopted, since it is just a scaling factor that simplifies when comparing amounts of downloaded data. Unfortunately, nothing can be stated on the convexity of G and the optimal bitrate must be found by doing a full plot (i.e. a brute force search) or by means of suboptimal techniques, e.g. *simulated annealing* [7].

Maximising G has another consequence: the decrease of the communication time. With some manipulation of (2), we can derive the percentage of time (w.r.t. T_{V}) for which the transmitter sends data as

$$\tau(R_{\mathrm{TX}}) \triangleq \frac{R_{\mathrm{avg}}(R_{\mathrm{TX}})T}{R_{\mathrm{TX}}T_{\mathrm{V}}}\,, \tag{6}$$

and it is easy to show that $\tau \leq 1$, for any $R_{\mathrm{TX}} \geq 0$. Typically for the optimal bitrate it holds $\tau(R_{\mathrm{opt}}) < 1$. Therefore, by optimising G we are also decreasing the average power consumption of the communication subsystem, with possible savings on the sizing of the power subsystem. Moreover since the G/S is used less, we have also savings in terms of ground operations.

B. Constrained maximisation of the downloaded data

The optimisation problem (5) can be subject to one or more constraints depending on mission needs. In case of bandwidth constraint, the problem can be straightforwardly solved as follows. Let W the maximum bandwidth available, then the constraint reads

$$R_{\mathrm{TX}} \leq \eta W$$

where η is the spectral efficiency[1] of the coding and modulation format in $[\mathrm{bit/s/Hz}]$. Thus, the optimisation problem in (5) can be solved as previously, but over a limited domain $[0, \eta W]$.

[1]The spectral efficiency must be properly defined according a bandwidth definition, e.g. 99% of the power.

Instead, in case of minimum download D per G/S pass, the constraint can be defined as

$$\left(R_{\mathrm{TX}} \int_{I_k} \mathrm{I}(R(t) \geq R_{\mathrm{TX}})\mathrm{d}t \right) \geq D \quad \forall I_k\,. \tag{7}$$

If we define $G_k(R_{\mathrm{TX}})$ as the gain function in (4) but over the interval I_k, we can rewrite the constraint (7) as

$$G_k(R_{\mathrm{TX}}) \geq \frac{D}{R_{\min}|I_k|} \quad \forall I_k\,,$$

where $|I_k|$ is the duration of the interval. Hence, the optimisation problem under minimum download per G/S constraint is equivalent to maximise G under the constraint that each G_k is over a minimum threshold. Thus, the problem cannot be solved with a single plot of G as previously, and nonlinear optimisation technique shall be adopted.

V. Variable Bitrate

We considered till now dynamic link budgets where a single coding and modulation format was adopted with a fixed bitrate for the whole mission. We now relax the constraint assuming a variable bitrate that can be obtained by either changing the bandwidth or by using adaptive coding and modulation (ACM).

A. Variable bitrate limit

We define the achievable rate function $R(t)$ as previously but using the lowest E_b/N_0 available among all the bitrates. This time the S/C will use a bitrate $R_{\mathrm{TX}} = R_{\mathrm{TX}}(t)$ that is a function of time[2]. Consequently, the average downlink bitrate R_{avg} is functional of the instantaneous bitrate as

$$R_{\mathrm{avg}}[R_{\mathrm{TX}}(t)] = \frac{1}{T}\int_0^T R_{\mathrm{TX}}(t)\mathrm{I}(R(t) \geq R_{\mathrm{TX}}(t))\mathrm{d}t\,. \tag{8}$$

If we define G as in (4) but with (8) in place of $R_{\mathrm{avg}}(R_{\mathrm{TX}})$, the optimisation problem becomes

$$R_{\mathrm{opt}}(t) \quad = \quad \arg\max_{R_{\mathrm{TX}}(t)} G[R_{\mathrm{TX}}(t)] \tag{9}$$

We can now state the following theorem:

Theorem 2. *Let $R(t)$ be an achievable bitrate function. Then, a solution of (9) is $R_{\mathrm{opt}}(t) = R(t)$, and the amount of downloaded data is*

$$R_{\mathrm{avg}}[R_{\mathrm{opt}}(t)]T = \int_0^T R(t)\mathrm{d}t\,. \tag{10}$$

The proof is provided in Appendix B. The theorem states that the optimal strategy to be adopted during a mission is a continuous change of the bitrate. Unfortunately, when moving from the mathematical world to the real one, we have to face two hitches: first, the number of adopted bitrates can be only finite. Second, variable bitrate systems have always an overhead. Thus the amount of downloaded data in (10) must be considered as a theoretical *variable bitrate limit*. However, Theorem 2 still provides an interesting result, and it can be understood why in CCSDS 26 ACM have been designed [1].

[2]The transmitted bitrate function is assumed to have same mathematical properties of achievable rate functions.

B. *Finite variable bitrate schemes*

For having a more realistic mathematical model, we consider now only $R_{\mathrm{TX}}(t)$ functions that can adopt a finite number of bitrates. Let us define N_{VBR} as the number of available bitrates that can be adopted during the mission. Each bitrate can be obtained by changing the bandwidth (and thus the channel symbol rate) or the ACM, or even both. For each bitrate we define the increase of bitrate γ_k w.r.t. the first one as

$$\gamma_k \triangleq \frac{\mathcal{I}_k R_{\mathrm{CHS},k}}{\mathcal{I}_1 R_{\mathrm{CHS},1}}, \quad k = 1, \ldots, N_{\mathrm{VBR}}$$

where $\mathcal{I}_k$ is the number of information bit per transmitted channel symbol, and $R_{\mathrm{CHS},k}$ is the channel symbol rate. Similarly, for each bitrate we also define the required increase of SNR as

$$\Delta_{\mathrm{SNR},k} \triangleq \frac{(E_b/N_0)_k}{(E_b/N_0)_1}, \quad k = 1, \ldots, N_{\mathrm{VBR}}\,.$$

Clearly it holds that $\gamma_1 = 1$ and $\Delta_{\mathrm{SNR},1} = 1$. We also assume that bitrates are properly selected and ordered[3] such that $\gamma_k > \gamma_{k-1}$ and $\Delta_{\mathrm{SNR},k} > \Delta_{\mathrm{SNR},k-1}$ hold for $k = 2, \ldots, N_{\mathrm{VBR}}$. We can now define the bitrate function for a *finite variable bitrate scheme* as

$$\begin{aligned} R_{\mathrm{TX}}(t) &= R_{\mathrm{TX}} \\ &+ \sum_{k=2}^{N_{\mathrm{VBR}}} (\gamma_k - \gamma_{k-1}) R_{\mathrm{TX}} \mathrm{I}\left(R(t) \geq \Delta_{\mathrm{SNR},k}\gamma_k R_{\mathrm{TX}}\right) \end{aligned} \quad (11)$$

where R_{TX} is (with a slight abuse of notation) the first bitrate, i.e. the one for $k = 1$. Although (11) looks very complicated, the stategy hidden in the formula is to select instantaneously the highest bitrate that the link budget allows, or in other terms at any time t to find the largest γ_k that satisfies $R(t) \geq \Delta_{\mathrm{SNR},k}\gamma_k R_{\mathrm{TX}}$ and to set $R_{\mathrm{TX}}(t) = \gamma_k R_{\mathrm{TX}}$. The optimisation problem becomes now

$$R_{\mathrm{opt}}(t) = \arg\max_{R_{\mathrm{TX}}} G\left[R_{\mathrm{TX}}(t)\right] \quad (12)$$

where $R_{\mathrm{TX}}(t)$ is in the form (11), and we just do the optimisation over the scalar value R_{TX}. Thus, we can solve the problem as was done in Section IV.

Again, there is an interesting graphical interpretation of the optimisation process as shown in Figure 4. The amount of donwloaded data by a finite variable bitrate scheme is equal to the area of a set of boxes that lie under $R(t)$. Thus, by changing R_{TX} we try to maximise the area and possibly achieve the variable bitrate limit. We now show under which conditions a finite bitrate scheme can achieve the limit.

Theorem 3. *Let $R_{\mathrm{TX}}(t)$ be a finite bitrate function with equally spaced bitrates between $R_{\min}$ and $\max R(t)$, and with $\Delta_{\mathrm{SNR},k} = 1$ for $k = 1, \ldots, N_{\mathrm{VBR}}$. Then, for $R_{\mathrm{TX}} = R_{\min}$*

$$G[R_{\mathrm{TX}}(t)] \to \frac{1}{R_{\min} T_{\mathrm{V}}} \int_0^T R(t)\mathrm{d}t$$

for $N_{\mathrm{VBR}} \to \infty$, i.e. it achieves the variable bitrate limit. If instead $R_{\mathrm{TX}}(t)$ has all $\Delta_{\mathrm{SNR},k} > 1$, then

$$G[R_{\mathrm{TX}}(t)] < \frac{1}{R_{\min} T_{\mathrm{V}}} \int_0^T R(t)\mathrm{d}t$$

for any $R_{\mathrm{TX}} \in [0, \infty)$.

The proof is provided in Appendix C. Translating the theorem from mathematical terms to practical terms, we have some important consequences. With a close look to Figure 4 we can notice that if $\Delta_{\mathrm{SNR},k} > 1$ the boxes will be not inscribed in $R(t)$, making the variable bitrate limit unreachable even for an infinite number of bitrates. On the other hand, this is no longer true when $\Delta_{\mathrm{SNR},k} = 1$. But this condition can be reached only by using the ACM with the lowest E_b/N_0 and (an ideal) change of the bandwidth that does not interrupt the link.

Therefore, for any variable bitrate system there is a maximum limit that cannot be reached. Moreover,it does not make any sense in using too many ACMs. This behaviour has been also experienced in [8] for a laboratory demonstrator exploiting DVB-S2 with ACM.

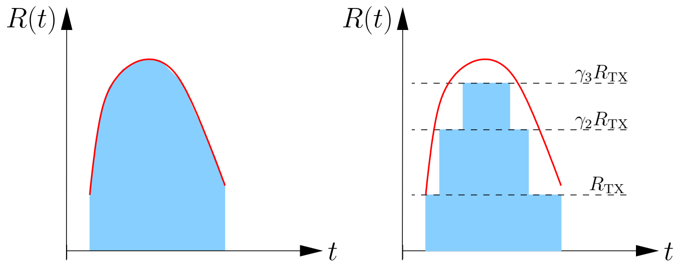

Fig. 4: Graphical interpretation for the amount of downloaded data for (left) the theoretical variable bitrate limit and (right) a finite variable bitrate scheme.

VI. Numerical results

A. *fixed bitrate*

Let us consider a scenario with a S/C in a 500 km circular orbit around Earth, inclination angle 5 deg, and right ascension of the ascending node (RAAN) equal to 0 deg. The assumed G/S is the *Luigi Broglio* located in Malindi, Kenya. The orbit projection on Earth surface and G/S position is shown in Figure 5. Let us assume that we have already decided frequencies, antennas, modulation and coding format, etc., and we do a link budget in the classical way as shown in Table I. As it can be seen from the figure, the outcome of the classical approach is that the maximum bitrate is 2 Mbps. By using a time-dynamic position and attitude simulator (for instance AGI STK) we find that the total connection time in 5 days with the G/S is roughly 11.2 hours that correspond to ~ 80.5 Gbit of downloaded data. From this analysis it seems we cannot do much more, unless we increase the EIRP or we resort to more powerful coding format. Instead, we now show that using the dynamic link budget and the proposed optimisation method this is not true.

[3]Notice that the inequalities can be satisfied by any variable bitrate scheme by means of an expurgation of the bitrates that do not provide higher γ_k although the higher $\Delta_{\mathrm{SNR},k}$. The expurgation does not imply any loss in the optimisation.

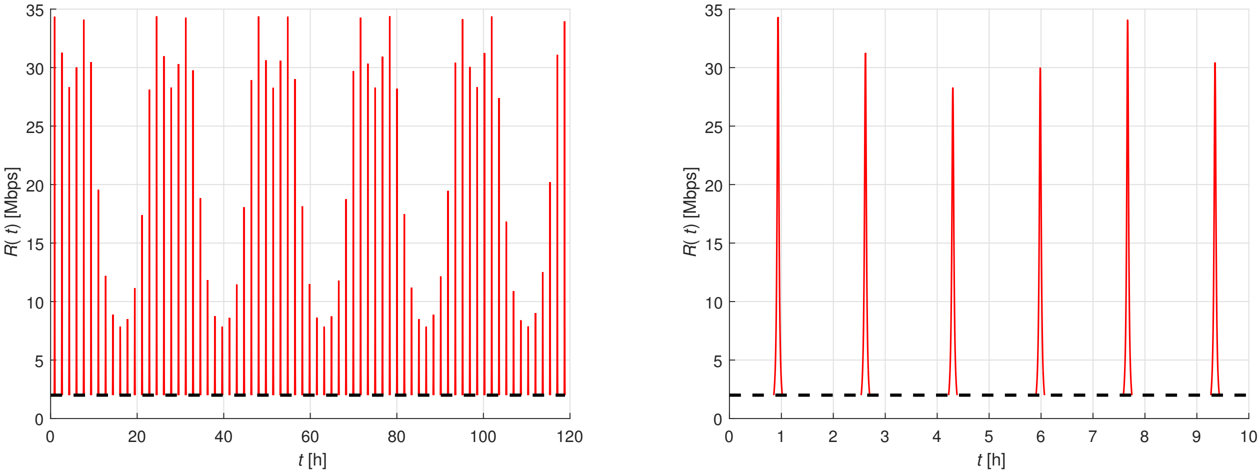

Fig. 6: Achievable bitrate function for the S/C in circular orbit and Malindi G/S.

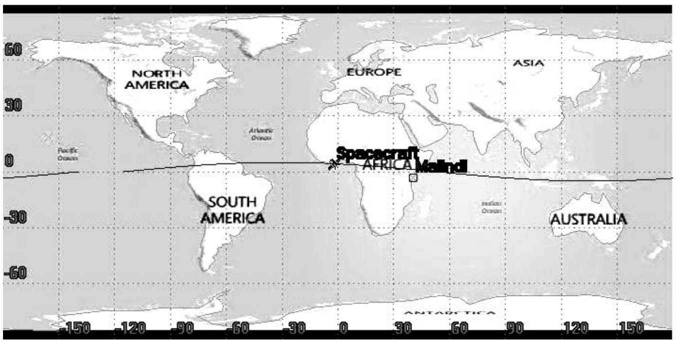

Fig. 5: S/C orbit projection on Earth surface and considered G/S for optimisation of the link budget.

TABLE I: Static link budget for a S/C with altitude 500 km and Malindi G/S.

PARAMETER	Value	Notes
Altitude [km]	500.00	
Elevation angle [deg]	5.00	
Slant range [km]	2077.94	Computed
Frequency [MHz]	8500.00	X-Band for Space Research
TX power [W]	2.09	
TX antenna gain [dB]	-3.00	
TX losses [dB]	4.00	3dB coupler + cables
TX EIRP [dBW]	-3.80	Computed
Path loss [dB]	177.38	Computed
Atmospheric loss [dB]	4.90	99% availability
RX G_A/T_A [dBK^{-1}]	31.80	Malindi
Demodulation loss [dB]	1.50	
Modulation loss [dB]	0.00	Suppressed carrier modulation
Required E_b/N_0 [dB]	6.80	CCSDS Reed Solomon
Margin [dB]	3.00	ECSS margin requirement
Bitrate [Mbps]	2.00	Computed

Figure 6 shows the achievable bitrate function over five days. A zoom of the first ten hours is also shown for reference. Red curves are $R(t)$, whereas the black dashed line is $R_{\min}$ that is equal to 2 Mbps (as founded by using the static link budget). We can notice that for some instants the S/C link could transmit at 30 Mbps, and thus we can expect that the optimal bitrate R_{opt} will be higher than 2 Mbps. Figure 7 shows G for all possible R_{TX}. We can see that the optimal bitrate is $\sim 3.5 R_{\min}$ that allows to download an extra 46% of data. Depending on the chosen R_{TX}, we have also a reduction of the communication time τ as shown in Figure 8. For the optimal bitrate, we have $\tau \cong 0.4$. In summary, we found that using 7 Mbps instead of 2 Mbps allows a total download of 117.5 Gb instead of 80.5 Gb, and to communicate only for 40% of the visibility time, that implies a 60% reduction of the average power consumption and of the G/S use.

Since gains depends strictly on the geometry, the optimisation has been repeated for other scenarios. We report the case of i) a sun synchronous orbit with altitude equal to 800 km, and using the G/S facility in Kiruna, Sweden, and ii) an elliptical orbit with eccentricity 0.52, apogee altitude 16430 km, RAAN and inclination equal to 0 deg, and transmitting to Malindi. For case i), results are shown in Figure 7 and 8, and we can see that the increase of downloaded data is only 4% by using $R_{\text{opt}} = 1.6 R_{\min}$. Although the G is not impressive, the result could be still of interest since the communication time can be decreased by 36%. For case ii), results are shown in Figure 9. Thanks to the large variance of the S/C slant range, results show an impressive gain. We obtained $R_{\text{opt}} = 86.1 R_{\min}$, $G = 2.33$ (i.e. 133% more) and τ equal to 3%. In few words, it is more convenient to transmit only when the S/C has a slant range lower than $\sim$2300 km than transmitting all the time.

B. Variable bitrate

Let us consider again the scenario with a S/C in 500 km circular orbit and G/S in Malindi. The transmitter of the S/C uses a finite variable bitrate scheme as in (11). In particular

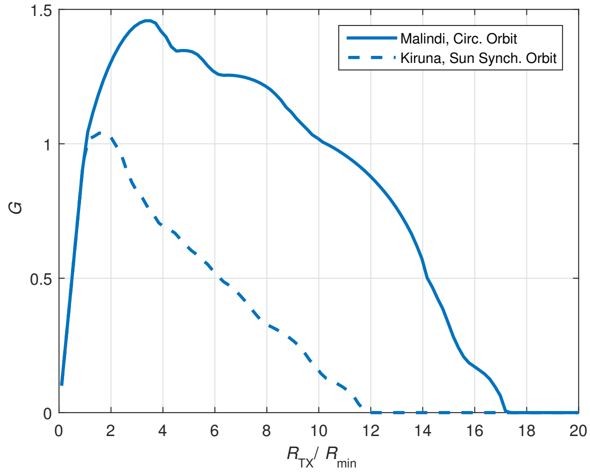

Fig. 7: Increase of the amount of downloaded data as function of $R_{\mathrm{TX}}/R_{\mathrm{min}}$.

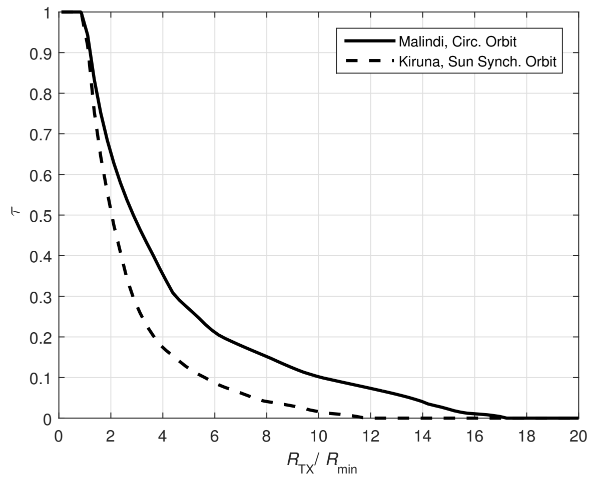

Fig. 8: Decrease of the communication time as function of $R_{\mathrm{TX}}/R_{\mathrm{min}}$.

we assume that the S/C can adopt 10 ACM as in Table II, where we reported the absolute values E_b/N_0 and $\mathcal{I}_k$ and the corresponding normalised values $\Delta_{\mathrm{SNR},k}$ and γ_k.

Figure 10 shows G as function of the basic bitrate R_{TX} when only ACM 1–6 or 1–10 are adopted. For reference the variable bitrate limit and the use of a fixed bitrate are also shown. We can see that the variable bitrate scheme with 6 ACM is able to provide a maximum gain of 115% versus the 46% found for the fixed bitrate. However, the gain is still far from the variable bitrate limit that suggests a gain till 317%. Unfortunately, as discussed in Section V, even increasing the number of ACM to 10 we cannot see any advantage due to the required increase of SNR.

Different results are obtained for systems that use a variable bitrate by changing the signal bandwidth. Figure 11 shows G when the transmitter adopts 10 bitrates with γ_k as in Table II

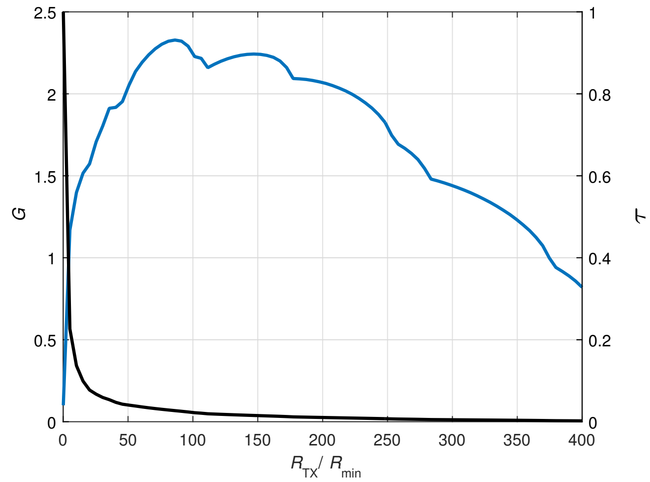

Fig. 9: Increase of the amount of downloaded data and decrease of the communication time for an elliptical orbit.

TABLE II: Adopted ACM at the transmitter.

ACM	E_b/N_0 [dB]	$\mathcal{I}_k$	$\Delta_{\mathrm{SNR},k}$	γ_k
1	0.90	0.71	1.00	1.00
2	1.08	0.86	1.04	1.21
3	1.44	1.04	1.13	1.46
4	1.87	1.21	1.25	1.70
5	2.39	1.39	1.41	1.96
6	3.00	1.63	1.62	2.30
7	3.63	1.84	1.88	2.59
8	4.53	2.10	2.31	2.96
9	5.46	2.37	2.86	3.34
10	6.69	2.64	3.79	3.72

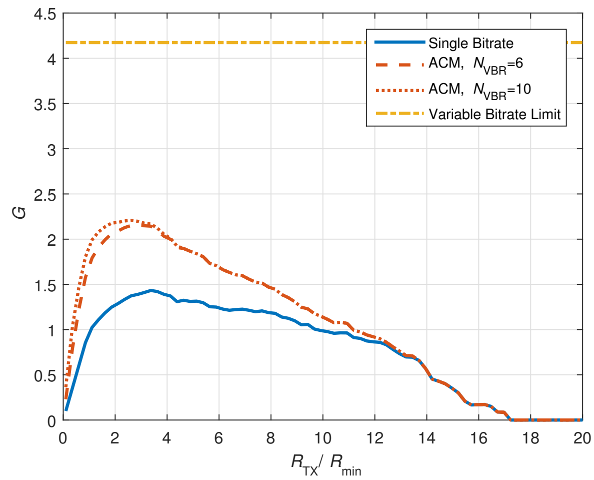

Fig. 10: Increase of the amount of downloaded data as function of $R_{\mathrm{TX}}/R_{\mathrm{min}}$ when different ACM are adopted.

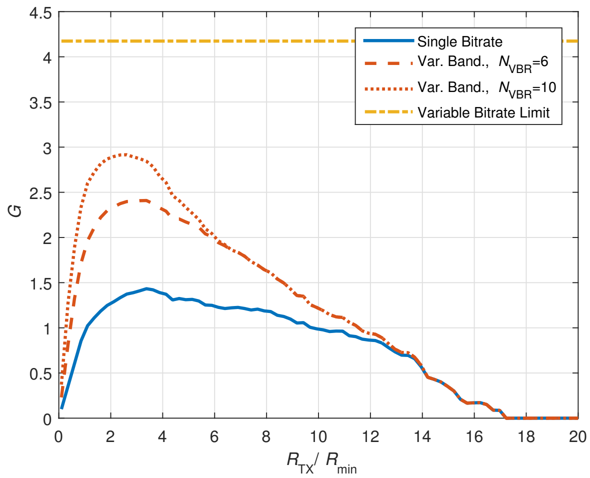

Fig. 11: Increse of the amount of downloaded data as function of $R_{\mathrm{TX}}/R_{\min}$ by changing the bandwidth of the signal.

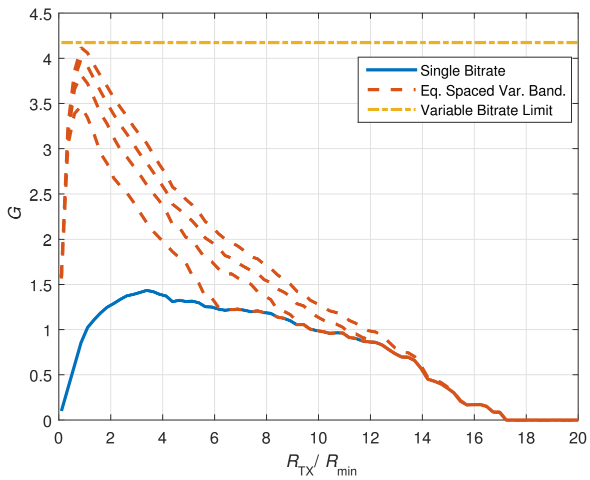

Fig. 12: Increse of the amount of downloaded data as function of $R_{\mathrm{TX}}/R_{\min}$ by changing the bandwidth of the signal and using equally spaced bitrates.

but $\Delta_{\mathrm{SNR},k} = 1$. We can see that using 10 bitrates provides a gain of 190% versus 140% with 6 bitrates and 46% with a single bitrate.

We finally show numerically that the variable bitrate limit can be achieved by increasing the number of bitrates as stated by Theorem 3. Figure 12 shows the effect of having $\Delta_{\mathrm{SNR},k} = 1$ and equally spaced bitrates between $R_{\min}$ and $\max R(t)$ for different values of N_{VBR}. We can see that as N_{VBR} increases the gain tends to the limit as provided by the theorem.

VII. Conclusions

A tutorial on the dynamic link budget analysis and optimisation has been provided.

Appendix A

Proof. For the proof we can limit the study to the function $R_{\mathrm{avg}}(R_{\mathrm{TX}})$ that dictates the dependence of G on the variable R_{TX}. We first consider $R_{\mathrm{avg}}(R_{\mathrm{TX}})$ on the interval $[0, R_{\min}]$. By definition it holds that

$$R_{\mathrm{avg}}(R_{\mathrm{TX}}) = R_{\mathrm{TX}}\frac{T_{\mathrm{V}}}{T} \quad \forall R_{\mathrm{TX}} \in [0, R_{\min}]$$

that is clearly a continuous linear function. It also straightforward to see that $R_{\mathrm{avg}}(R_{\mathrm{TX}}) = 0$ for any $R_{\mathrm{TX}} \in [\max R(t), \infty)$. We now pick any point $R_{\mathrm{TX}} \in [R_{\min}, \max R(t)]$. Say K the number of constituent functions, the achievable rate function can be rewritten as

$$\begin{aligned} R_{\mathrm{avg}}(R_{\mathrm{TX}}) &= \frac{R_{\mathrm{TX}}}{T}\sum_{k=1}^{K}\int_{I_k} \mathrm{I}(R(t) \geq R_{\mathrm{TX}})\mathrm{d}t \\ &= \frac{R_{\mathrm{TX}}}{T}\sum_{k=1}^{K}\sum_{m=1}^{M_k}(t_{E_k,m} - t_{S_k,m}) \qquad (13) \end{aligned}$$

where in the last equation we expressed the integral as the difference of a pair of instants, i.e. the starting and ending instants $t_{S_k,m}$ and $t_{E_k,m}$. Notice that each subinterval can have a number of pairs equal to $M_k = 0$ (in case the integral is null on the I_k), or $M_k \geq 1$. We now consider a small $|\epsilon| > 0$, and consider $R_{\mathrm{avg}}(R_{\mathrm{TX}} + \epsilon)$. Since the constituent functions are continuous, all the starting points will become

$$t_{S_k,m} + \begin{cases} \epsilon\left(\frac{\mathrm{d}R(t_{S_k,m})}{\mathrm{d}t}\right)^{-1} + \mathcal{O}(\epsilon) & \text{if } R(t_{S_k,m}) = R_{\mathrm{TX}} \\ 0 & \text{if } R(t_{S_k,m}) \neq R_{\mathrm{TX}} \end{cases}$$

and similarly the ending points. But then, it exists $\alpha \neq 0$ such that $R_{\mathrm{avg}}(R_{\mathrm{TX}} + \epsilon)$ reads

$$\frac{R_{\mathrm{TX}} + \epsilon}{T}\left(\alpha\epsilon + \mathcal{O}(\epsilon) + \int_0^T \mathrm{I}(R(t) \geq R_{\mathrm{TX}})\mathrm{d}t\right).$$

Then, for $\epsilon \to 0$ it follows $R_{\mathrm{avg}}(R_{\mathrm{TX}} + \epsilon) \to R_{\mathrm{avg}}(R_{\mathrm{TX}})$, and hence the function is continuous in R_{TX}. Finally, since $R_{\min}$ is part of the domain, clearly it holds that the maximum of G will be greater than or equal to the unity. $\square$

Appendix B

Proof. Using the notation of the previous section, Equation (8) can be upper bounded as

$$\begin{aligned} R_{\mathrm{avg}}[R_{\mathrm{TX}}(t)] &= \frac{1}{T}\sum_{k=1}^{K}\sum_{m=1}^{M_k}\int_{t_{S_k,m}}^{t_{E_k,m}} R_{\mathrm{TX}}(t)\mathrm{d}t \qquad (14) \\ &\leq \frac{1}{T}\sum_{k=1}^{K}\sum_{m=1}^{M_k}\int_{t_{S_k,m}}^{t_{E_k,m}} R(t)\mathrm{d}t \qquad (15) \\ &\leq \frac{1}{T}\int_0^T R(t)\mathrm{I}(R(t) > 0)\mathrm{d}t \qquad (16) \\ &= \frac{1}{T}\int_0^T R(t)\mathrm{d}t \qquad (17) \end{aligned}$$

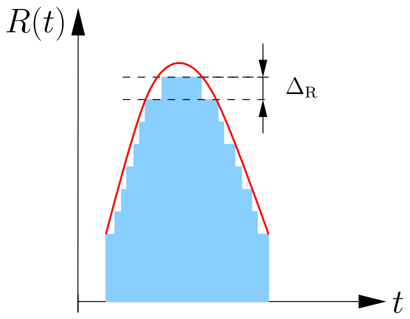

Fig. 13: Graphical proof of Theorem 3.

Inequality (15) holds with equality if and only if $R(t) = R_{\mathrm{TX}}(t)$ for all $t \in \bigcup_{k,m}[t_{S_k,m}, t_{E_k,m}]$. Moreover, since these intervals are disjoint, (16) holds with equality if and only if

$$\bigcup_{k,m}[t_{S_k,m}, t_{E_k,m}] \cup \{t : R(t) = 0\} \equiv [0, T] \, ,$$

or equivalently

$$\bigcup_{k,m}[t_{S_k,m}, t_{E_k,m}] \equiv \{t : R(t) > 0\} \; .$$

Hence, equality in (16) can be achieved if and only if $R_{TX}(t) = R(t)$ for all $t \in \{t : R(t) > 0\}$. Clearly, $R_{TX}(t) = R(t)$ over all the domain satisfy this condition and (10) is found. □

Appendix C

Proof. The proof is straightforward and can be done graphically. Under the mentioned conditions it holds that $R_{\mathrm{TX}} = R_{\mathrm{min}}$ and the bitrate increments are constant as

$$(\gamma_k - \gamma_{k-1})R_{\mathrm{min}} = \frac{\max R(t) - R_{\mathrm{min}}}{N_{\mathrm{VBR}}}$$

that we can redefine as Δ_{R}. Obviously Δ_{R} decreases as N_{VBR} increases and the amount of downloaded data is the one shown in Figure 13. But then, for $N_{\mathrm{VBR}} \to \infty$ that amount of donwloaded data is the Lebesgue integration [9] of $R(t)$, and thus the variable bitrate limit is achieved.

Conversely, if all $\Delta_{\mathrm{SNR},k} > 1$, each box will be scaled down by $\Delta_{\mathrm{SNR},k}$. Thus the overall area will be strictly lower than the integral of $R(t)$.

□

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