## Performance characterization and guidelines for the design of a counter-propagating nonlinear lossless polarizer

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We characterize the performance of a nonlinear lossless polarizer, an all-optical fiber-based device that allows for the control of the state of polarization of an optical signal. The device relies on the lossless polarization attraction generated by the nonlinear interaction between the controlled signal and a controlling pump. Choosing a counterpropagating pump, we quantify its performance by introducing the degree of attraction (DOA), which highlights the trade-off between the average attraction of the signal polarization and the unavoidable degradation of its degree of polarization (DOP). We investigate, by numerical simulations, the dependence of the DOA on the injected power and on the fiber length, thus providing the design guidelines to reach the desired performance. We find that an effective attraction can occur even for strongly unbalanced signal and pump power levels, and that fibers longer than a few kilometers yield only a marginal improvement of the DOA. © 2013 Optical Society of America

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### **1. INTRODUCTION**

Controlling the state of polarization (SOP) of an arbitrarily polarized optical signal is a fundamental task, both for optical signal processing applications and for optical communication systems. Recent works [1-6] have identified the lossless polarization attraction (LPA) as a promising solution to perform an all-optical control of signal polarization. LPA is a Kerr-based phenomenon, generated by the nonlinear crosspolarization modulation (XpolM) interaction between a (possibly depolarized) optical signal and a fully polarized continuous wave (CW) pump beam. Whatever the input signal polarization, the output signal polarization is attracted toward the pump polarization, without any loss of power, due to polarization rotation. The first experimental demonstration of LPA occurring between signals at telecommunication wavelengths was obtained by injecting counter-propagating beams with large power (tens of Watt) into a short (2 m) isotropic fiber [1]. Afterward, LPA was experimentally observed between signals with moderate power (hundreds of milli-Watt), counter-propagating in a long (20 km) birefringent telecom fiber [2]. Exploiting the LPA phenomenon, an alloptical fiber-based device, called nonlinear lossless polarizer (NLP), can be realized that allows an all-optical control of the signal polarization. Several research groups have produced both theoretical [3-5] and numerical analyses [6-8], eventually leading to practical applications showing the potentials of the NLP [9–11].

In addition to LPA, which is the focus of our investigation, other all-optical polarization stabilization approaches have being recently studied, that are mainly based on Raman amplification [12-17]. In [18] and [19], instead, the polarization

pulling of the field relies on stimulated Brillouin scattering and on photo-refractivity, respectively.

All the works in [1-10] focus on the LPA generated by means of a counter-propagating pump beam (i.e., injected into the opposite fiber end, with respect to the signal). With this configuration, complete attraction is only an asymptotic condition, since the transient time of LPA is in the microsecond scale [6], hence the attraction of the mean signal SOP occurs at the expense of its degree of polarization (DOP), which degrades partially [6–8]. Using a co-propagating pump beam (i.e., injected into the same fiber end as the signal), the transient of LPA can be dramatically reduced [20-22]. The reason is that, within the copropagating configuration, the transient time depends not only on the strength of the nonlinear polarization interactions, i.e., on power, but also on the extra degree of freedom provided by the relative pump-signal propagation speed [22], i.e., on the walk-off delay. As a consequence, the response of the NLP can be optimized, acting on the walk-off delay, by tuning the pump wavelength [22]. However, such a technique can be exploited up to a limited extent, since the optimal pump wavelength depends on the signal duration [22]. Hence, to control the SOP of "long" signals (e.g., bit-packets in the microsecond scale), the pump should be placed at unpractical distant wavelengths. Thus, the counter-propagating configuration for the NLP is still the most practical choice in a packet switched scenario (e.g., the Ethernet network), where the duration of signals (bit-packets) can reach the microsecond scale, i.e., the same order of magnitude as the transient time of LPA.

In this paper, we refer to the latter scenario and analyze the performance of a counter-propagating NLP. Since the relative propagation speed between pump and signal is fixed and relativistic (i.e., equal to the speed of light), the main design parameters for the NLP are just the injected pump and signal power and the nonlinear fiber length. We thus characterize the LPA phenomenon as a function of both power and fiber length, providing the design guidelines to select their values so that the NLP achieves the desired performance. To tackle this problem in a sound way, we first introduce the degree of attraction (DOA) [7], then we quantify it by simulation, resorting to a recently introduced counter-propagation algorithm [8].

## 2. HOW TO MEASURE THE EFFECTIVENESS OF POLARIZATION ATTRACTION

In order to measure the performance of a NLP device, we need to evaluate the similarity between the polarization of two optical fields, with time-varying amplitude, phase, and polarization. Let the real Stokes vectors  $\vec{s}_p(t) = s_{0p}(t)\hat{s}_p(t)$  and  $\vec{s}_s(t) = s_{0s}(t)\hat{s}_s(t)$  represent the optical fields of the pump and signal, respectively, where the scalar quantities  $s_{0p}(t)$ and  $s_{0s}(t)$  are the instantaneous field intensities, while the unit magnitude Stokes vectors  $\hat{s}_p(t)$  and  $\hat{s}_s(t)$  represent the instantaneous field SOPs. We define the degree of attraction (DOA) as the maximum normalized cross-correlation between the Stokes vectors of the two interacting signals [7]

$$\text{DOA} \triangleq \max_{\tau} \left\{ \frac{\langle \vec{s}_p(t+\tau) \cdot \vec{s}_s(t) \rangle}{\langle s_{0p}(t+\tau) s_{0s}(t) \rangle} \right\},\tag{1}$$

where the dot stands for scalar product and the angular brackets denote time-averaging, i.e.,  $\langle \cdot \rangle = (1/T) \int_T \cdot dt$ , extended over the whole signal duration *T*. Since  $\tau$  is a time offset between pump and signal, accounting for their mutual delay, we assume in the following, without loss of generality, that the optimal  $\tau = 0$ . The DOA is a ratio of time-averages, hence *T* simplifies in Eq. (1) and  $\langle s_0(t) \rangle$  can be seen as the energy of a signal with instantaneous intensity  $s_0(t)$  (collected over *T*). Factoring out the Stokes vector magnitudes, DOA can be written as

$$\text{DOA} = \left\{ \frac{s_{0p}(t)s_{0s}(t)}{\langle s_{0p}(t)s_{0s}(t) \rangle} \hat{s}_p(t) \cdot \hat{s}_s(t) \right\} = \langle w(t)\cos(\varphi(t)) \rangle, \quad (2)$$

where we defined  $\varphi(t) = \arccos(\hat{s}_p(t) \cdot \hat{s}_s(t))$  as the angle between pump and signal SOPs, while  $w(t) = s_{0p}(t)s_{0s}(t)/\langle s_{0p}(t)s_{0s}(t)\rangle$  is a positive and normalized  $(\langle w(t) \rangle = 1)$  "weight function," representing the time distribution of the joint signal intensities. Thus, the DOA physically represents the time-average of the angle  $\varphi(t)$  between the instantaneous signal SOPs, weighted by their joint intensity. By definition, DOA  $\in [-1; 1]$  and the extrema correspond to constantly orthogonal  $[\hat{s}_s(t) = -\hat{s}_p(t)]$  or constantly parallel  $[\hat{s}_s(t) = \hat{s}_p(t)]$  signal SOPs. The latter condition yields DOA = 1 and identifies the case of an ideal polarization attraction.

The above definition of DOA, first introduced in [7,10], stems from classical communication theory. In other works, the effectiveness of polarization attraction is quantified in different ways: the fraction of signal energy co-polarized with the attracting pump is measured in [1], while the DOP is adopted in [9,21,22]. All these quantities are closely related to each other, as we show next.

#### A. Completely Polarized CW Pump

Due to the transient behavior of LPA [6], the SOP and intensity of the attracting pump should be stable in time. Indeed, all the literature on LPA assumes a completely polarized CW pump [2,4,6–9], so that the Stokes vector of the pump is timeindependent,  $\vec{s}_p = s_{0p}\hat{s}_p$ , and Eq. (2) consequently simplifies

$$DOA = \frac{\langle \vec{s}_s(t) \rangle}{\langle s_{0s}(t) \rangle} \cdot \hat{s}_p = DOP_s \times MSA.$$
(3)

In Eq. (3), we used the standard definition [23] of the timeaveraged DOP of the signal (as pointed out by the subscript),  $\text{DOP}_s = \|\langle \vec{s}_s(t) \rangle \| / \langle \| \vec{s}_s(t) \| \rangle$ , and introduced the mean SOP attraction

$$MSA = \frac{\langle \vec{s}_s(t) \rangle}{\|\langle \vec{s}_s(t) \rangle\|} \cdot \hat{s}_p = \hat{m}_s \cdot \hat{s}_p = \cos(\chi).$$
(4)

The MSA has a simple geometrical meaning: since  $\langle \vec{s}_s(t) \rangle$  is the mean signal Stokes vector,  $\chi$  is the angular distance between the attracting (constant) pump SOP  $\hat{s}_p$  and the mean (power-averaged) signal SOP  $\hat{m}_s$ . The factorization in Eq. (3) is a conceptually remarkable result, since it highlights the trade-off inherent in the LPA process, where an originally completely polarized signal becomes depolarized (i.e., its DOP<sub>s</sub> decreases) as its average SOP  $\hat{m}_s$  moves closer to the pump SOP  $\hat{s}_p$  [6,8]. Hence, an effective attraction occurs only if the increase in MSA is larger than the DOP<sub>s</sub> decrease. Supposing that the input signal is completely polarized, i.e.,  $\vec{s}_s(t) = s_{0s}(t)\hat{s}_s$ , the initial DOA, evaluated before LPA takes place, from Eq. (3) is DOA<sub>in</sub> =  $\hat{s}_s \cdot \hat{s}_p = \cos(\chi_{in})$ , where  $\chi_{in}$  is the angle between the input signal and pump SOPs, in Stokes space.

Besides its geometrical interpretation, the DOA also has a precise physical meaning. It is supposed to filter the signal through an ideal polarizer, aligned with the pump SOP. The energy output from such a filter can be expressed, in terms of Stokes vectors, as  $(1/2)T\langle s_{0s}(t) + \vec{s}_s(t) \cdot \hat{s}_p \rangle$  (again, *T* is the averaging period) [23], compared with the input signal energy  $T\langle s_{0s}(t) \rangle$ . Hence, from Eq. (3), the ratio of signal energies detected after and before filtering is  $\rho = (1/2)(1 + \text{DOA})$ . The quantity  $\rho$  can be measured, and was used in [1] to experimentally quantify the amount of LPA.

#### **B.** Input Signals with Random SOP

Referring to a practical application of the NLP, while the pump SOP at the input of the device can be controlled freely, the hypothesis of an input signal with a deterministic SOP is unrealistic, e.g., due to the polarization impairments brought about by the optical link where the signal propagates. Consequently, the SOP of the signal at the NLP input, hence its angular distance  $\chi_{in}$  from the input pump SOP, and the corresponding DOA value at the NLP output, are random variables. Thus, we consider a random input signal SOP (uniformly distributed on the Poincaré sphere), and evaluate the NLP performance by (statistically) averaging the DOA with respect to the signal SOP realizations

$$\overline{\text{DOA}} = E\left[\frac{\langle \vec{s}_s(t) \rangle}{\langle s_{0s}(t) \rangle} \cdot \hat{s}_p\right] = \frac{E[\langle \vec{s}_s(t) \rangle]}{\langle s_{0s}(t) \rangle} \cdot \hat{s}_p, \tag{5}$$

where  $E[\cdot]$  represents statistical averaging, while  $\langle s_{0s}(t) \rangle$  and  $\hat{s}_p$  are independent of the input signal SOP.

Note the relationship between  $\overline{\text{DOA}}$  in Eq. (5) and the (time- and statistically averaged) definition of the degree of polarization, as applied to nonergodic signals:  $\text{DOP} = ||E[\langle \vec{s}_s(t) \rangle]||/\langle s_{0s}(t) \rangle$ . In particular, the authors in [21] show that  $\overline{\text{DOA}}$  and DOP coincide when LPA occurs within a fiber with a small PMD coefficient, such as, e.g.,  $D_{\text{PMD}} = 0.05 \text{ ps/km}^{1/2}$  (a value typical of modern fibers). In this case, it is thus  $\overline{\text{DOA}} \in [0; 1]$ , although negative values are allowed for the unaveraged  $\text{DOA} \in [-1; 1]$ .

#### **3. SYSTEM SETUP**

We simulated a counter-propagating NLP composed by a nonlinear, nonzero dispersion-shifted fiber (NZ-DSF), with Kerr coefficient  $\gamma = 1.99 \text{ W}^{-1} \text{ km}^{-1}$  and attenuation  $\alpha =$ 0.2 dB/km, and a fully polarized (CW) pump laser, with power  $P_p = s_{0p}$ , as shown in Fig. <u>1</u>. The randomly birefringent fiber, with length *L* km, has a PMD coefficient  $D_{\text{PMD}} =$ 0.05 ps/km<sup>1/2</sup>, so that propagation is governed by the Manakov equation [<u>4,24</u>]. Hence, the Kerr effect is isotropic on the Poincaré sphere and polarization attraction occurs toward any fixed pump SOP [<u>4,8</u>], here chosen as linear horizontal, i.e.,  $\hat{s}_p = \hat{s}_1$  is the first Stokes axis.

We assumed that the input signal consists of a single intensity-modulated pulse, with duration 1 µs and power  $P_s = s_{0s}$ , placed at the fiber zero-dispersion wavelength (zdw). Indeed, such a pulse represents an OOK-modulated bit packet (e.g.,  $10^4$  bits at 10 Gbit/s). In fact, as we verified numerically, results do not change when introducing intensity modulation onto the pulse, at fixed mean power [7,10].

As far as the signal propagates at the zdw, chromatic dispersion has no effect on counter-propagating LPA (as opposed to co-propagating LPA [20–22]), hence a different fiber type can be used, provided that  $D_{\text{PMD}}$  is small enough to ensure propagation in the Manakov limit [20]. Highly non-linear fibers are desirable, in the design of a NLP, since power acts on LPA directly through  $\gamma$ .

As highlighted by the box in Fig. 1, the DOA was measured, according to Eq. (3), based on the output signal and on the input pump SOP  $\hat{s}_p$  (dashed line in Fig. 1). We assumed that the changes in signal polarization are either due to a switch of the input bit-packet (pulse) or they are brought about by the birefringence and PMD of the preceding optical link, hence are slowly varying, on a time scale longer than the pulse period. We thus assumed a completely polarized input signal,  $\bar{s}_s^{in}(t) = s_{0s}^{in}(t)\hat{s}_s^{in}$ , where  $\hat{s}_s^{in}$  is constant over the whole duration, and lies at an angular distance  $\chi_{in}$  (on the Poincaré sphere) from the input pump SOP.



Fig. 1. System setup of the NLP. The NLP is composed of the fiber along with the (fully polarized) pump laser, with power  $P_p$ , and the optical circulators.

Counter-propagation was solved by the iterative SCAOS algorithm [8], implemented within the open-source optical simulator Optilux [25]. Thanks to its efficiency and speed, we could analyze the dynamics of LPA in detail (see Section 6), by varying system parameters. As a drawback, SCAOS should be applied to signals with limited duration (a few  $\mu$ s), propagating in fibers with limited length (a few tens of kilometers), otherwise its convergence may slow significantly.

## 4. ROLE OF FIBER LENGTH

Being LPA driven by the nonlinear XpolM induced by the pump, one can expect that its effect is proportional to the nonlinear phase rotation (NPR)  $\phi_{\rm NL} = \gamma P L_{\rm eff}$  [rad], where *P* is the power and  $L_{\rm eff} = (1 - \exp(-\alpha L))/\alpha$  is the effective fiber length. The NPR  $\phi_{\rm NL}$  is a physical parameter that quantifies the strength of the nonlinear Kerr interaction. It is thus natural to analyze how the effectiveness of LPA depends on power and fiber length.

Figure 2 shows the contour plots of the DOA [Fig. 2(a)] and of its factors, defined in Eq. (3), DOP<sub>s</sub> and MSA [Figs. 2(b) and 2(c), respectively], as a function of both power and effective length. Results were obtained by launching the same power for pump and signal ( $P_p = P_s = P$ ), while the maximum value  $L_{\text{eff}} = 13$  km, in the figures, corresponds to a physical fiber length L = 20 km, beyond which the nonlinear effects have decayed significantly. As explained in Section 2, results depend on the launched signal polarization, and in particular on the angular distance  $\chi_{\text{in}}$  between pump and signal SOPs. Here, we chose the linear horizontal and vertical polarization components of the input signal  $\vec{s}_s^{\text{in}}(t)$  with a random phase offset and with equal power, which determines an angle  $\chi_{\text{in}} = 90^{\circ}$  between  $\hat{s}_s^{\text{in}}$  and the linear horizontal pump polarization  $\hat{s}_p$  (not depending on the phase offset).

The equilateral hyperbola plotted onto the DOA contours in Fig. 2(a) with a solid magenta line highlights the locus of points with constant  $\phi_{\text{NL}}$ . We see that DOA is not directly proportional to  $\phi_{\text{NL}}$ , thus denying the intuitive hypothesis formulated above. For a fixed  $\phi_{\text{NL}}$ , LPA is more effective when powerful signals propagate in a short fiber. Geometrically, the DOA contours tend to "flatten," as a function of the effective length, meaning that the DOA increases little by further lengthening of the fiber.

Note, however, that for the signal SOP launched here, the initial value is  $\text{DOA}_{in} = \cos(\chi_{in}) = 0$ , thus, from a null value, the DOA in Fig. 2(a) increases monotonically with the strength of the nonlinear interaction. Such a result is considerable since the DOA in Eq. (3) is affected by an unavoidable  $\text{DOP}_s$  degradation, entailed in the dynamics of LPA [6,8], as shown in Fig. 2(b), at intermediate values of the nonlinear phase  $\phi_{\text{NL}}$ . Anyway, Figs. 2(b) and 2(c) show that the DOP<sub>s</sub> decrease is more than compensated by the growth of MSA, so that their product is increasing monotonically.

So far, only signal SOPs with an intermediate angular distance from the pump,  $\chi_{in} = 90^{\circ}$ , have been considered. To enlarge the picture, we show in Fig. <u>3</u> the DOA [Fig. <u>3(a)</u>], and its factors [Figs. <u>3(b)</u> and <u>3(c)</u>], obtained for different input signal SOPs: curves, with different symbols and colors, correspond to (top to bottom) an increasing angular distance  $\chi_{in}$  from the input pump SOP  $\hat{s}_p = \hat{s}_1$ , ranging from 0° to 180°, in 30° steps. For the moment, we analyze only the dependence on  $L_{\text{eff}}$ , deferring the dependence on power to the next



Fig. 2. LPA effectiveness as a function of equal pump and signal power (*P*) and of effective fiber length ( $L_{\rm eff}$ ) for (a) DOA, (b) DOP<sub>s</sub>, and (c) MSA. The angular distance between the input signal and pump SOPs is  $\chi_{\rm in} = 90^{\circ}$  (on the Poincaré sphere).

section. In Fig. 3, the curves were obtained with equal signal and pump power P = 2 W, a large value chosen to stress polarization attraction.

The extreme  $\chi_{in}$  values, plotted with symbols  $\nabla$  (dark green) and  $\Delta$  (magenta), refer to a signal SOP equal or orthogonal to the input pump SOP, i.e.,  $\hat{s}_s^{in} = \pm \hat{s}_p$ . In this case, signals propagate without any change in polarization, since their nonlinear interaction is of a scalar type, reducing to a simple cross-phase modulation (XPM). Hence, the DOA remains constant and equal to its input value. For every other input signal SOP, the DOA increases with increasing effective length, along with the amount of nonlinear interaction.



Fig. 3. LPA effectiveness as a function of the effective fiber length  $(L_{\rm eff})$ , with equal signal and pump power P = 2 W for (a) DOA, (b) DOP<sub>s</sub>, and (c) MSA. Lines refer to an angular distance between the input signal and pump SOPs that varies (top to bottom) from  $\chi_{\rm in} = 0^{\circ}$  to  $\chi_{\rm in} = 180^{\circ}$ , in 30° steps.

In the right side of each curve in Fig. <u>3(a)</u>, DOA values tend to "saturate," for effective lengths above 8 km. The DOA value at which saturation occurs is smaller for input signal SOPs that are further away from the input pump SOP. Note that such a saturation phenomenon is due solely to the depolarization of the signal. In fact, while the MSA values in Fig. <u>3(c)</u> are all close to 1 (except for orthogonal input SOPs), the depolarization of the signal, in Fig. <u>3(b)</u>, is larger for input signal SOPs further away from the pump. Anyway, even for such large  $\chi_{in}$  values, results show that the DOA increases most within the first  $L_{\text{eff}} = 8 \text{ km}$  (i.e., L = 10 km), a length after which the performance of the LPA process does not significantly improve. The interest in using short fibers is due to PMD, since, in a randomly birefringent fiber, a large PMD coefficient can spoil polarization attraction, if the fiber is too long, due to the incoherent polarization evolution of pump and signal, located at different wavelengths [21]. We thus conclude that L = 10 km is a good compromise, to maximize LPA performance when the impairments due to PMD are not severe. In the remainder of this work, the fiber length is consequently fixed at L = 10 km, while we concentrate on the impact of optical power on the LPA effectiveness.

# 5. LPA DEPENDENCE ON SIGNAL AND PUMP POWER

To characterize the effectiveness of LPA as a function of power, Fig. <u>4</u> shows the contour plots of the DOA and of its factors, DOP<sub>s</sub> and MSA, in the case of a fiber with length L = 10 km, obtained by independently varying the pump and signal input power,  $P_p$  and  $P_s$ , in a range of practical interest, between 0.2 and 2.2 W. Here, as in Fig. <u>2</u>, the input signal SOP  $\hat{s}_s^{in}$  lies at an angular distance  $\chi_{in} = 90^\circ$  from the input pump SOP ( $\hat{s}_p = \hat{s}_1$ ).

Again, we see that the DOA [Fig. 4(a)] increases mono tonically with power, despite the initial decrease of DOP<sub>s</sub> [Fig. 4(b)], at low powers, which is more than compensated by the MSA increase [Fig. 4(c)].

The noteworthy result revealed by Fig. <u>4</u> is that all contour plots overlap with equilateral hyperbolae, as can be seen in Fig. <u>4(a)</u>, where three hyperbolae with solid thick lines (red, blue, and green) are superimposed on the DOA contour plots. Consequently, in the tested range of power, the DOA, the DOP<sub>s</sub>, and the MSA all depend on the pump–signal power product. We verified numerically that this is true for any launched signal SOP, hence a plot of the DOA (and of its factors) as a function of  $P = (P_s P_p)^{1/2}$  (defined as the geometric mean of pump and signal power) contains all the necessary information. As a practical implication, even the polarization of a weak signal can be effectively attracted toward the pump polarization, provided that the pump is powerful enough.

Relying on this result, Fig. 5 shows the dependence of the DOA, the DOP<sub>s</sub>, and the MSA on the geometric mean power *P* for a launched signal SOP with an increasing angular distance  $\chi_{in}$  from the input pump SOP, ranging from 0° to 180°, in 30° steps.

From Fig. 5, three different "regimes of operation" can be identified for LPA. At low power ( $P \le 0.5$  W), signals propagate in a quasi-linear regime, where the input signal SOP is almost unchanged and, from Eq. (3), the DOA is close to its initial value  $\text{DOA}_{in} = \cos(\chi_{in})$ . At intermediate power (0.5 W < P < 1.5 W), signals propagate in a nonlinear regime, where the output signal SOP tends to align, on average, to the input pump SOP [see the increase in MSA in Fig. 5(c)], at the expense of its DOP  $[DOP_s \text{ decreases in Fig. 5(b)}]$ , as we already pointed out. In this region, the DOA versus power curves, in Fig. 5(a), show the largest slope, hence the Kerr effect, and in particular the XPolM, is maximally effective in terms of polarization attraction. At large power (P > 1.5 W), signals propagate in a strongly nonlinear regime, where the average output signal SOP has become aligned with the pump SOP (MSA  $\simeq$  1), regardless of input signal polarization (except in the case of an almost orthogonally polarized input signal,  $\chi_{\rm in} \simeq 180^\circ$ ), and its DOP<sub>s</sub> start to increase slowly.



Fig. 4. LPA effectiveness as a function of signal and pump power for (a) DOA, (b) DOP<sub>s</sub>, and (c) MSA. The angular distance between the input signal and pump SOPs is  $\chi_{in} = 90^{\circ}$  (on the Poincaré sphere).

This is due to a repolarization of the signal around its average SOP, i.e., around the pump SOP. However, the dynamics of such a repolarization are slow, hence the lowest  $DOP_s$  that the signal reaches sets a practical limit to the attainable DOA values.

#### 6. DYNAMICS OF LPA

To better clarify the dynamics of LPA described above, we can visualize the quantities defined so far. Figure <u>6</u> shows the signal SOP, at the fiber output, plotted on the Poincaré sphere. The depolarization traces, visible in red in Figs. <u>6(a)-6(c)</u>, represent the time evolution of the signal pulse's SOP,  $\hat{s}_s(t) = \vec{s}_s(t)/s_{0s}(t)$ . The Stokes vector of each time sample is normalized to its power, so that the depolarization traces lie on the Poincaré sphere. The inner (red) vector represents



Fig. 5. LPA effectiveness as a function of the geometric mean power  $P = (P_s P_p)^{1/2}$  for (a) DOA, (b) DOP<sub>s</sub>, and (c) MSA. Lines refer to an angular distance between the input signal and pump SOPs that varies (top to bottom) from  $\chi_{\rm in} = 0^{\circ}$  to  $\chi_{\rm in} = 180^{\circ}$ , in 30° steps.

the power-averaged signal SOP  $\langle \vec{s}_s(t) \rangle / \langle s_{0s}(t) \rangle$  that appears in the definition of DOA Eq. (3), which is clearly related to the depolarization trace (although it is not equal to its mean value). Its magnitude is equal to the DOP<sub>s</sub>, as per Eq. (3), while its direction is the unit magnitude vector  $\hat{m}_s$ , appearing in Eq. (4), hence  $\chi$  is the angle that it forms with the input pump SOP, represented by the unit magnitude (blue) vector aligned with  $\hat{s}_1$ . In Fig. 6, the angle between the input signal and pump SOPs is  $\chi_{in} = 90^\circ$ , while the mean signal power *P* is set in the nonlinear regime [1 W, in Fig. 6(a)], and at the onset [1.6 W, in Fig. 6(b)], or deeply in the strongly nonlinear regime [2.2 W, in Fig. 6(c)]. These three system configurations are marked by circles, on the black line with  $\Box$ symbols, in Fig. 5(a).



Fig. 6. SOP of the signal at the output of the NLP, for an equal signal and pump power of (a) 1 W, (b) 1.6 W, and (c) 2.2 W. The angular distance between the input signal and pump SOPs is  $\chi_{\rm in} = 90^{\circ}$ . The angle between the average output signal SOP (red) and input pump SOP  $\hat{s}_1$  (blue) is (a)  $\bar{\chi} = 40^{\circ}$ , (b)  $\bar{\chi} = 8^{\circ}$ , and (c)  $\bar{\chi} = 4^{\circ}$ , while the output signal DOP<sub>s</sub> is (a) 0.89, (b) 0.82, and (c) 0.89.

As pointed out in [8], the leading edge of the signal pulse is never attracted toward the pump SOP, but rather rotates around it. In fact, all the depolarization traces in Figs. <u>6</u> start on the  $(\hat{s}_2, \hat{s}_3)$  circle. Instead, the following portions of the pulse move toward the pump SOP, thus giving rise to the depolarization trace. The stronger the signal power, the larger the time extension of that trailing part of the signal pulse attracted toward the pump SOP. In the strongly nonlinear regime of Fig. <u>6(c)</u>, where the average signal SOP is already very close to the pump SOP, the increased power implies that the signal repolarizes around the input pump SOP, hence DOP<sub>s</sub> increases, as can be seen by comparing the (identical)  $\text{DOP}_s$  values related to Figs. <u>6(a)</u> and <u>6(c)</u>. In Fig. <u>6(a)</u>, most of the pulse has a SOP close to that of its leading edge, while in Fig. <u>6(c)</u>, most of the pulse has a polarization close to that of the attracting pump SOP.

The discussion above leads to an important conclusion concerning the maximum value obtainable for the DOA (theoretically equal to 1). We can have DOA = 1 only when both MSA and DOP<sub>s</sub> are both equal to 1, i.e., when the output signal pulse is fully polarized, with the same polarization as the input pump. Since the leading edge (hence, for physical continuity, the initial portion) of the signal pulse will never be attracted toward the pump SOP, neither the MSA nor the DOP<sub>s</sub> can reach their theoretical limit, unless the input signal already has the same SOP as the pump, i.e., in the trivial case  $\chi_{in} = 0^{\circ}$ . Consequently, DOA = 1 only represents an asymptotic value for any signal polarization (except the one coinciding with the pump SOP  $\hat{s}_p$ ), as can be seen in Fig. 5(a). Of course, for longer pulses, the portion closer to the leading pulse edge has less influence on the overall evaluation of  $\text{DOP}_s$ . The analysis of LPA dynamics clarifies the reason for which there exists a transient in LPA and short (picoseconds) pulses are not effectively attracted in a NLP in the counter-propagating configuration [6], so that a co-propagating configuration is required [21], which poses other constraints.

#### 7. AVERAGE PERFORMANCE OF LPA

The analysis performed so far assumes an input signal SOP that is deterministic, at least with respect to the angular distance from the attracting pump SOP, equal to a given  $\chi_{in}$ . As discussed in Section 2.B, the performance of LPA should instead be assessed with no prior knowledge on the input SOP, hence resorting to the ensemble-averaged DOA introduced in Eq. (5). Thus, we performed a statistical study of the DOA and

of its factors (DOP<sub>s</sub> and MSA), as a function of the (geometric) mean power P, defined above, in order to evaluate the average performance of LPA.

Figures  $\underline{7(a)}-\underline{7(c)}$  show the dependence of the ensemble average of DOA, DOP<sub>s</sub>, and MSA versus *P*, while Figs.  $\underline{7(d)}-\underline{7(f)}$  report the standard deviation of the same quantities. The two expected values, mean and standard deviation (first- and second-order moments), were computed numerically from the "deterministic" curves in Fig. 5, that were averaged over the unknown angle  $\chi_{in}$ , i.e., weighted by the distribution of  $\chi_{in}$ . For an input signal SOP uniformly distributed over the Poincaré sphere, the probability density function (pdf) of the angular pump–probe distance is  $f(\chi_{in}) =$  $(1/2) \sin(\chi_{in})(0 \le \chi_{in} \le 180^{\circ})$  [26], hence  $\chi_{in} = 90^{\circ}$  is its mean value. The curve reported in Fig. 7(a) is, at least from a practical viewpoint, the most important result in this work, since it yields the rule for setting the power levels, once the desired average-performance for the NLP is given.

As already noted in Section 2.A, Fig. 7(a) shows that  $\overline{\text{DOA}} \in [0; 1]$ , which can be easily demonstrated as follows. When power tends to zero, the optical fields propagate in a linear regime, no polarization attraction occurs, and DOA coincides with its input value,  $\text{DOA}_{\text{in}} \simeq \cos(\chi_{\text{in}})$ , with  $0 \le \chi_{\text{in}} \le 180^{\circ}$ . Hence, its average value can be calculated exactly as follows:

$$\overline{\text{DOA}}_{\text{in}} = E[\cos(\chi_{\text{in}})] = \int_0^{\pi} \cos(\chi_{\text{in}}) \frac{\sin(\chi_{\text{in}})}{2} \, \mathrm{d}\chi_{\text{in}} = 0.$$

When power increases, the DOA curves in Fig. 5 increase monotonically toward the asymptotic value 1 (for all but the orthogonal input SOP case, for which DOA is always null), hence  $\overline{\text{DOA}}$  values cannot become negative.



Fig. 7. First- and second-order moments of the quantities used to characterize the performance of a NLP, as a function of (geometric) mean power for (a) and (d) DOA, (b) and (e) DOP<sub>s</sub>, and (c) and (f) MSA. In (a) and (d), thick dashed lines report the first- and second-order statistics evaluated by assuming that DOP<sub>s</sub> and MSA are uncorrelated random variables.

Regarding the DOA standard deviation in Fig. <u>7(d)</u>, it is decreasing monotonically with power and its maximum value, obtained when power tends to zero, can be again calculated exactly from the DOA variance at the input,  $\sigma_{\text{DOA}_{in}}^2 = E[(\text{DOA}_{in})^2] - (\overline{\text{DOA}}_{in})^2$ , as follows:

$$\sigma_{\rm DOA_{in}}^2 = E[\cos^2(\chi_{\rm in})] = \int_0^{\pi} \cos^2(\chi_{\rm in}) \frac{\sin(\chi_{\rm in})}{2} d\chi_{\rm in} = \frac{1}{3}$$

from which the maximum DOA standard deviation results,  $\sigma_{\text{DOA}_{in}} = (1/3)^{1/2} \simeq 0.577$ . The same values are obtained for the first- and second-order statistics of the MSA [Figs. 7(c) and 7(f)] when power tends to zero, since we assume a fully polarized input signal, for which  $\text{DOP}_s^{in} = 1$  always [see Figs. 7(b) and 7(e)], hence MSA = DOA in this limit.



Fig. 8. Graphical picture of LPA toward the input pump SOP  $\hat{s}_1$  (blue), for 100 random input signal SOPs. Average output signal SOPs (red) are plotted for increasing transmitted power for (a) 600 mW, (b) 1.6 W, and (c) 2.2 W.

Even the curves in Fig.  $\underline{7}$  seem to suggest the existence of three different operating regimes for LPA. In particular, in the strongly nonlinear propagation regime,  $\overline{\text{DOA}}$  and its standard deviation remains almost constant, meaning that a further increase of power would not lead to an appreciable enhancement of the performance of a NLP.

Since the DOA is the product between the MSA and the  $DOP_s$ , its average is  $\overline{DOA} = E[MSA \times DOP_s]$  and its variance is  $\sigma_{DOA}^2 = E[MSA^2 \times DOP_s^2] - \overline{DOA}^2$ . Assuming, for the moment that the MSA and the DOPs are statistically uncorrelated random variables, we evaluated the average  $\overline{DOA}^{unc} =$  $E[MSA] \times E[DOP_s]$  and the standard deviation  $\sigma_{DOA}^{unc}$  accordingly, and plotted these quantities in Figs. 7(a)-7(d), with thick dashed red lines. In particular, the dashed curve in Fig. 7(a) is the product of the two curves in Figs. 7(b) and 7(c). The dashed curves match very well with the real curves of the moments of DOA (solid black lines), suggesting that MSA and DOP<sub>s</sub> are indeed almost statistically uncorrelated. This would be a weird-though not impossible-fact, since the only random parameter in the system is the angle  $\chi_{in}$ , from which both MSA and DOP<sub>s</sub> deterministically stem from the propagation equation, thus being a transformation of the same random variable.

To give another, more immediate, representation of the average performance of a NLP, Fig. <u>8</u> shows the average output signal SOPs  $\hat{m}_s$  (red circles) obtained for 100 random input SOPs (with uniform distribution on the Poincaré sphere), in the case of a (geometric) average power *P* equal to (a) 0.6 W, (b) 1.6 W, and (c) 2.2 W. The DOA, evaluated by using Monte Carlo simulation over the 100 realizations of the input signal SOP, results in  $\overline{DOA} = 0.27$ ,  $\overline{DOA} = 0.75$ , and  $\overline{DOA} = 0.83$ , for the tested power levels, respectively.

Although we use a small number of input signal SOPs in order to evaluate the (weighted-) average performance of the NLP, this method provides results very close to those obtained with Monte Carlo averaging over 100 SOPs: namely, the two methods yield results that differ at most by 0.03.

## 8. CONCLUSION

We characterized, by numerical simulation, the performance of a nonlinear lossless polarizer, when its free parameters (signal and pump power; nonlinear fiber length) are varied. The DOA, introduced here, highlights the trade-off between the mean SOP attraction and an inevitable DOP degradation. We find that the attraction of the signal polarization toward that of a counter-propagating CW pump increases with the pump-signal power product, which allows the designer to trade power between the signal and pump. The results found on the average attraction of randomly polarized signals yield the rule for setting the power levels. Although longer fibers increase the performance of the device, length should be limited by the possible presence of PMD. Results show that fiber lengths beyond 10 km only yield a marginal improvement on performance.

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